EECS 16B Designing Information Systems and Devices II UC Berkeley Fall 2023

Midterm

Exam Location: In Person

PRINT your student ID: ______________________________

PRINT AND SIGN your name: ____________, ____________ ____________

(last) (first) (sign)

PRINT your discussion sections and (u)GSIs (the ones you attend): ______________

Row Number: ____________ Seat Number: ____________

Name and SID of the person to your left: ______________________________

Name and SID of the person to your right: ______________________________

Name and SID of the person in front of you: ______________________________

Name and SID of the person behind you: ______________________________

1. **Honor Code (0 pts.)**

   Please copy the following statement in the space provided below and sign your name.

   As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. I will follow the rules and do this exam on my own.

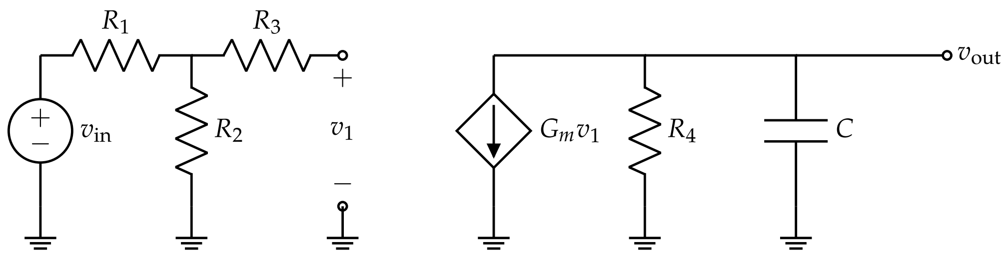
   Note that if you do not copy the honor code and sign your name, you will get a 0 on the exam.

2. **What is your favorite topic of the course so far? (2 pts.)**

Do not turn this page until the proctor tells you to do so.
You can work on the above problems before time starts.

3. RC Circuit with a Dependent Source (30 pts.)

Suppose you have the following circuit with constant input v_{in}, which features a dependent current source.



(a) (10 pts.) **Find the differential equation for** $v_{\text{out}}(t)$ in terms of the component variables and the constant input v_{in}. Note: Recall how dependent sources work; the current through the dependent current source in this case is $G_m v_1$, where v_1 is the voltage defined in the circuit diagram and G_m is some constant associated with the dependent current source.

(b) (2 pts.) **What is the time constant τ of this circuit?** Recall that for a first order differential equation, the time constant will be τ such that $e^{-\frac{t}{\tau}}$ is the relevant exponential present for your system.

(c) (5 pts.) **Find the steady state value of $v_{\text{out}}(t)$ in terms of the component variables and input v_{in}.**

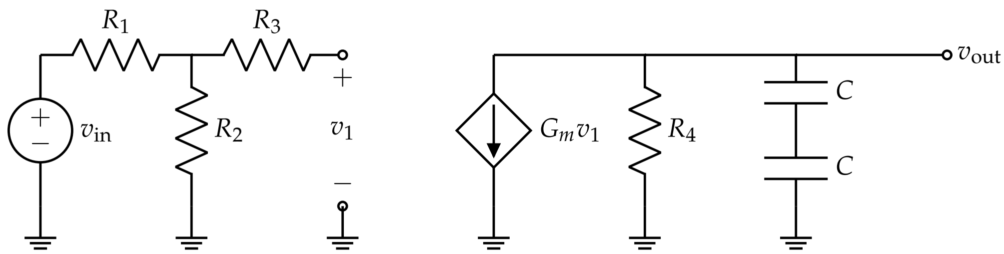
For the rest of the problem, suppose that when we use the component values, we get the following differential equation:

$$\frac{\mathrm{d}v_{\text{out}}(t)}{\mathrm{d}t} = -\frac{1}{2}v_{\text{out}}(t) - 5 \tag{1}$$

Additionally, suppose with component values, we find a steady state solution of -10V and we know that at time $t = 0$, the capacitor is discharged ($v_{\text{out}}(0) = 0$).

(d) (8 pts.) **Find the full solution for $v_{\text{out}}(t)$.**

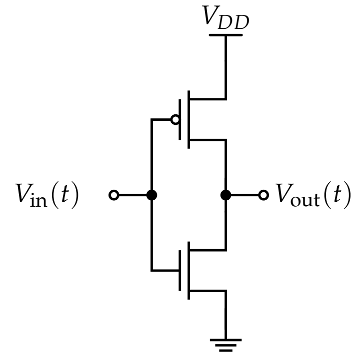
(e) (5 pts.) Suppose that we make a change to the circuit by adding an extra capacitor (with the same capacitance) in series with the current capacitor, as shown in the below circuit diagram:



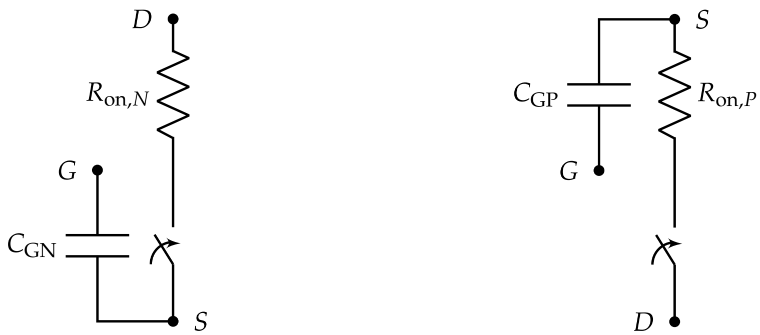
Suppose the previous circuit (one capacitor) had time constant τ_0 and steady state solution V_{p0}. **Find the new circuit's time constant τ and steady state solution V_p in terms of τ_0 and V_{p0}. Briefly explain your answer.**

4. Transistor Inverter (20 pts.)

Suppose we have a transistor inverter, as you have likely seen before:



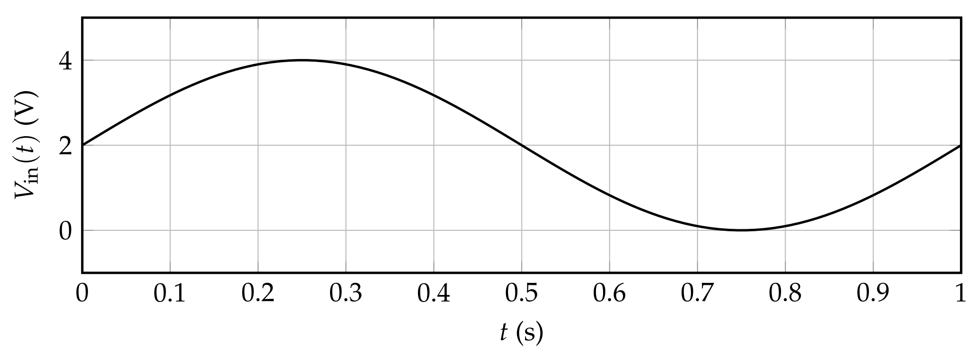
The RC models of NMOS and PMOS transistors we will use are these:



(a) NMOS Transistor RC Model **(b)** PMOS Transistor RC Model

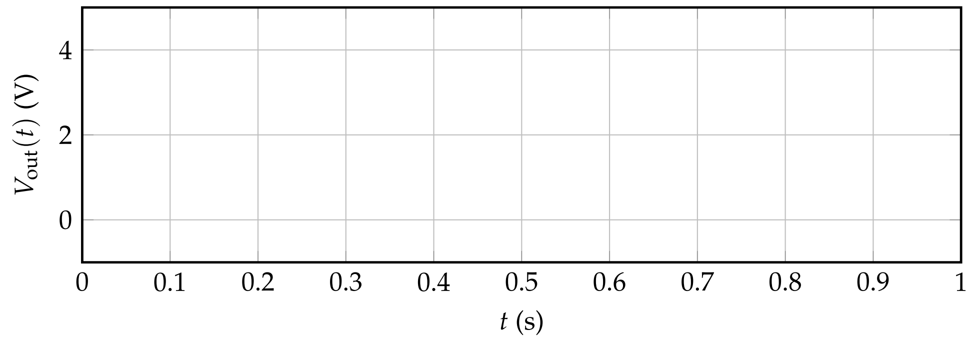
For the entire problem, we will use $V_{DD} = 4\,\text{V}$, threshold voltages $V_{THn} = 0.5\,\text{V}$ (for NMOS transistors) and $|V_{THp}| = 1.5\,\text{V}$ (for PMOS transistors), and resistances $R_{\text{on},N} = 1\,\text{k}\Omega$ (for NMOS transistors) and $R_{\text{on},P} = 3\,\text{k}\Omega$ (for PMOS transistors).

(a) (10 pts.) Suppose the plot of $V_{\text{in}}(t)$ is the sinusoidal function shown below:

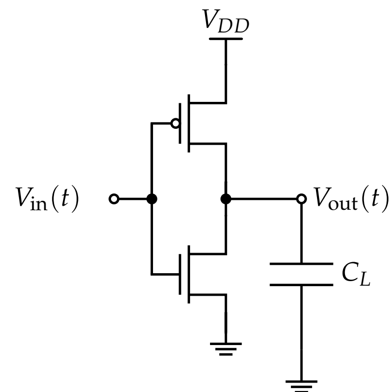


Plot the output voltage $V_{\text{out}}(t)$. You do not need to be exactly precise with the time values.

(HINT: There are three possible situations depending on the input voltage at some specific time: both transistor are on, only the top transistor is on, or only the bottom transistor is on. Try to find what input and output voltages correspond to these three scenarios.)



(b) (10 pts.) Suppose we connect this inverter to some other circuit, represented by a load capacitance C_L:



We are interested in the speed of our inverter for different output voltage transitions. Assume for this part that only one transistor is on at a time for each transition.

With the model values provided at the start of the problem, **will our inverter circuit be faster for pull up transitions (V_{out} from $0\,\text{V} \rightarrow 4\,\text{V}$) or pull down transitions (V_{out} from $4\,\text{V} \rightarrow 0\,\text{V}$)?** Please briefly explain using time constants.

(HINT: First, understand which transistor is on for each type of transition based on what the final voltage for each transition is. Then, think about how to apply the transistor RC model to the situation; which part of the RC model impacts the output time constant?)

5. RLC 2nd Order Differential Equation (30 pts.)

In this problem, you will approach solving the same RLC circuit as the previous problem using your knowledge of 2nd Order Differential Equations.

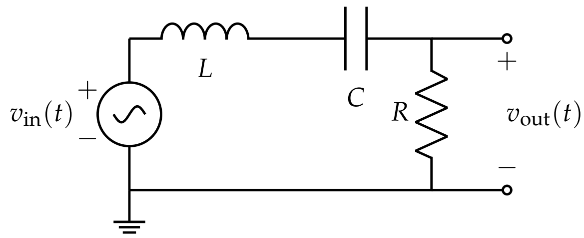


Figure 2: Circuit in "time domain"

(a) (8 pts.) **Find the differential equation for** $i_R(t)$ in the following form (where a, b, and c are constants that may depend on R, L, and C):

$$\frac{\mathrm{d}^2 i_R(t)}{\mathrm{d}t^2} + a\frac{\mathrm{d}i_R(t)}{\mathrm{d}t} + b i_R(t) = c\frac{\mathrm{d}v_{\text{in}}(t)}{\mathrm{d}t} \tag{2}$$

(HINT: Note that $v_{\text{in}}(t)$ is some sinusoidal input and NOT a constant.)

(b) (6 pts.) Suppose that you found the differential equation

$$\frac{\mathrm{d}^2 i_R(t)}{\mathrm{d}t^2} + 5\frac{\mathrm{d}i_R(t)}{\mathrm{d}t} + 6i_R(t) = 2\frac{\mathrm{d}v_{\mathrm{in}}(t)}{\mathrm{d}t} \tag{3}$$

The general *homogeneous* solution $i_{R,h}(t)$ to this differential equation can be written in the following form:

$$i_{R,h}(t) = C_1 \mathrm{e}^{s_1 t} + C_2 \mathrm{e}^{s_2 t} \tag{4}$$

Find numerical values for s_1 **and** s_2 for this differential equation (the order of s_1 and s_2 does not matter).

(c) (6 pts.) Suppose $v_{\mathrm{in}}(t) = V_0 \cos\left(\frac{1}{\sqrt{LC}}t + \frac{\pi}{2}\right)$. Find the **particular solution** $i_{R,p}(t)$ for this differential equation (in terms of R, L, C, and V_0).

(HINT: Remember the relationship between the particular solution and steady state solution. What method can you use to find the steady state solution when the input is sinusoidal?)

(d) (4 pts.) To solve the 2nd Order Differential Equation, we need initial conditions for both $i_R(t)$ and $i_R'(t) = \frac{\mathrm{d}i_R(t)}{\mathrm{d}t}$. Suppose we know that $i_R(0) = 0$ and $v_C(0) = 0$. **Find $i_R'(0)$. Show all your work.**

(HINT: Consider using an intermediate step from part (a) and information from part (c) to help you.)

(e) (6 pts.) Suppose that you found that the particular solution is $i_{R,p}(t) = \sin(\omega_p t)$. Then, you could write your total solution as

$$i_R(t) = i_{R,p}(t) + i_{R,h}(t) = \sin(\omega_p t) + C_1 \mathrm{e}^{s_1 t} + C_2 \mathrm{e}^{s_2 t} \tag{5}$$

Using the initial conditions $i_R(0)$ and $i_R'(0)$, **set up a system of equations to solve for C_1 and C_2** (your equations can contain the constants $i_R(0)$, $i_R'(0)$, ω_p, s_1, and s_2).

6. RLC Vector Differential Equation (30 pts.)

In this problem, you will approach solving the following RLC circuit using *the vector differential equation method*.

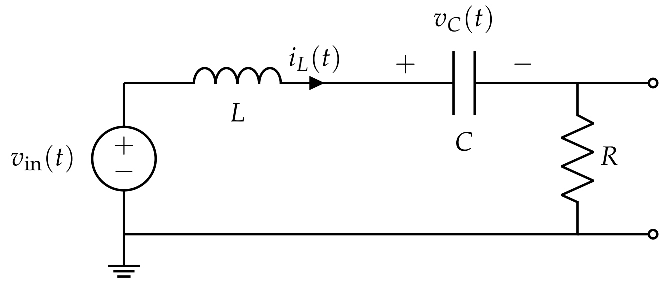


Figure 3: Circuit in "time domain"

(a) (4 pts.) **Derive a pair of differential equations, one for $\frac{\mathrm{d}i_L(t)}{\mathrm{d}t}$ and another for $\frac{\mathrm{d}v_C(t)}{\mathrm{d}t}$ in terms of $i_L(t)$, $v_C(t)$, and any necessary constants..**

(b) (6 pts.) Suppose our vector differential equation was written as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{x}(t) + \begin{bmatrix} e \\ f \end{bmatrix} V_{\text{in}}(t) \tag{6}$$

The state vector is defined as $\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix}$. **Find the values of a, b, c, d, e, and f in terms of R, L, C.**

(c) (6 pts.) Suppose you are now told that the system is given by:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} \vec{x}(t) \tag{7}$$

where $V_{\text{in}}(t)$ is simply zero. **Find the eigenvalues ($\lambda_1 \leq \lambda_2$) and eigenvectors of the given state matrix. Ensure your eigenvectors are in the form $\begin{bmatrix} c \\ 1 \end{bmatrix}$ where c is some constant. Show all your work.**

(d) (4 pts.) Now, we would like to diagonalize our system to more easily solve for the solution of the vector differential equation. We can write our system in the standard basis as $\frac{d\vec{x}(t)}{dt} = A\vec{x}(t)$.

For clarity in the upcoming parts, suppose $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ and V are defined as they have been in class.

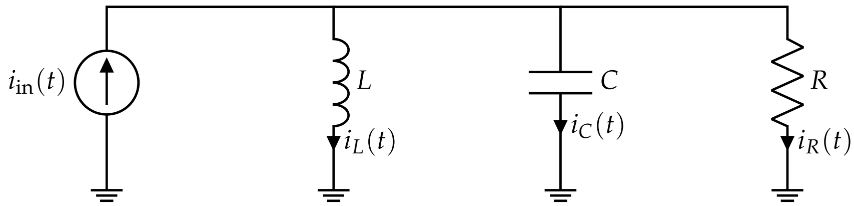
Show what change of basis you would use to diagonalize the system by writing $\vec{x}(t)$ in terms of $\vec{y}(t)$, AND then symbolically write your system in the diagonalized basis in terms of $\frac{d\vec{y}(t)}{dt}$ and $\vec{y}(t)$. You may use any variables defined in this problem part.

(e) (4 pts.) Your initial conditions in the standard basis are given symbolically as $x_1(0) = 2$ and $x_2(0) = 1$. **Find the initial conditions in the diagonalized basis ($y_1(0)$ and $y_2(0)$).**

(f) (6 pts.) **Using your answer to the previous question, solve for $i_L(t)$ and $v_C(t)$**

7. Parallel Resonance (20 pts.)

For this problem, we will examine a parallel resonance circuit, a common resonance circuit just like the series resonance circuit.



(a) (5 pts.) **Show that Z_{eq}, the equivalent impedance of the inductor, capacitor, and resistor as seen by the current source is:**

$$Z_{eq} = \frac{\mathrm{j}\omega L}{1 + \mathrm{j}\omega \frac{L}{R} + (\mathrm{j}\omega)^2 LC} \tag{8}$$

(b) (5 pts.) **Find the transfer function** $H(\mathrm{j}\omega) = \frac{\widetilde{I}_R}{\widetilde{I}_{\text{in}}}$ **for the current through the resistor** in terms of the component variables and frequency ω.

(HINT: The provided expression for Z_{eq} may be helpful here.)

(c) (5 pts.) **Find the frequency** ω_{max} **that maximizes the value of the transfer function.**

Additionally, **draw an equivalent circuit at this frequency**.

(HINT: You can do this problem even if you did not obtain the transfer function in the previous part. What frequency is crucial for RLC circuits?)

(d) (3 pts.) The quality factor for this circuit can be defined as the magnitude of the ratio of current across the capacitor to current across the resistor ($Q_p = \left|\frac{\widetilde{I}_C}{\widetilde{I}_R}\right|$) at the resonant frequency ω_0. **Show that the quality factor for this circuit is $Q_p = \omega_0 RC$.**

(e) (2 pts.) Suppose we want to use this circuit as a filter, but with the current component values we have used, the filter is not sharp enough (too low quality factor).

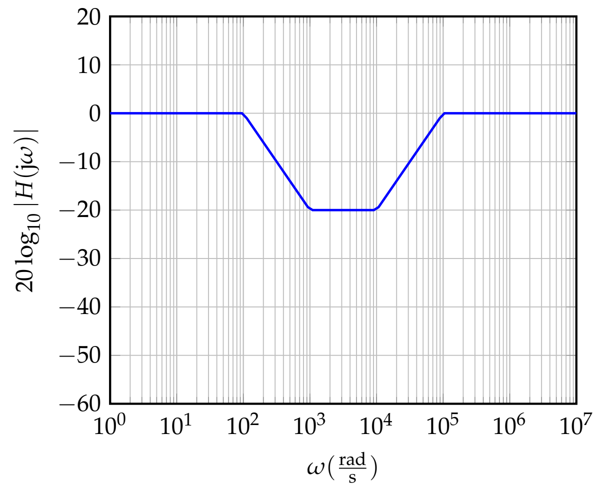
Suppose we have the ability to double or halve exactly one of the component values, with the goal of maximizing our quality factor.

Which component value should we change and should we double it or halve it? Please briefly explain why.

(HINT: The resonant frequency ω_0 will depend on the component values as well so you should consider this factor.)

8. Bode Plot Design (20 pts.)

For this question, we will design a filter based on the following provided Bode plot.



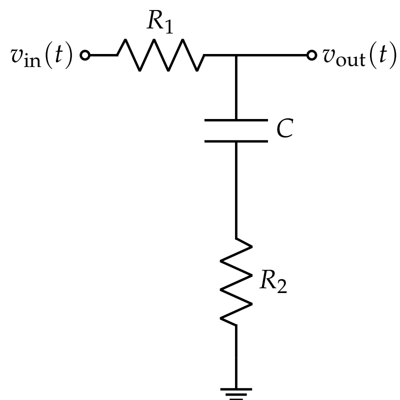
(a) (5 pts.) **What are the pole and zero frequencies of this Bode plot?**

(b) (5 pts.) **Find a transfer function $H(j\omega)$ that would produce the provided magnitude Bode plot.**

(c) (10 pts.) Suppose we had a different Bode plot and found our transfer function to be:

$$H(\mathrm{j}\omega) = \frac{(1+\mathrm{j}\frac{\omega}{10^4})(1+\mathrm{j}\frac{\omega}{10^6})}{(1+\mathrm{j}\frac{\omega}{10^3})(1+\mathrm{j}\frac{\omega}{10^5})} \tag{9}$$

We want to design a filter that will implement this transfer function. We know the transfer function for the following circuit:



The transfer function for this is:

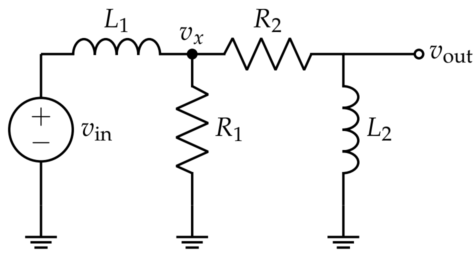
$$H(\mathrm{j}\omega) = \frac{1+\mathrm{j}\omega R_2 C}{1+\mathrm{j}\omega(R_1+R_2)C} \tag{10}$$

Suppose we only have 1 kΩ and 9 kΩ resistors, but we have tunable capacitors (so we can choose any capacitance values we want). We also have op-amps present.

Draw a filter that implements the transfer function. Please label all component values on your diagram.

9. Transfer Functions (40 pts.)

For this problem, we will analyze the following circuit.



We want to find the overall transfer function $H(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out}}}{\widetilde{V}_{\text{in}}}$.

(a) (5 pts.) **What are $H(\mathrm{j}0)$ and $H(\mathrm{j}\infty)$?**

(HINT: You can (and are intended to) do this without finding the whole transfer function.)

(b) (5 pts.) **Find the equivalent impedance of R_1, R_2, and L_2 seen by node v_x.**

Please simplify to the following form:

$$Z_{eq} = Z_0 \frac{1 + \mathrm{j}\omega\tau_1}{1 + \mathrm{j}\omega\tau_2} \tag{11}$$

where Z_0, τ_1, and τ_2 should be in terms of R_1, R_2, and L_2 (this equivalent impedance will not include L_1).

(c) (8 pts.) **Find the transfer function from input to node v_x, $H_1(\mathrm{j}\omega) = \frac{\widetilde{V}_x}{\widetilde{V}_{\text{in}}}$.**

You may use Z_0, τ_1 and τ_2 (along with the component variables) in your answer. Additionally, please simplify your answer to the following form:

$$H_1(\mathrm{j}\omega) = A_0 \frac{1 + \mathrm{j}\omega\tau_3}{1 + \mathrm{j}\omega\tau_4 + (\mathrm{j}\omega)^2\tau_5} \tag{12}$$

where A_0, τ_3, τ_4, and τ_5 should be in terms of L_1, Z_0, τ_1, and τ_2.

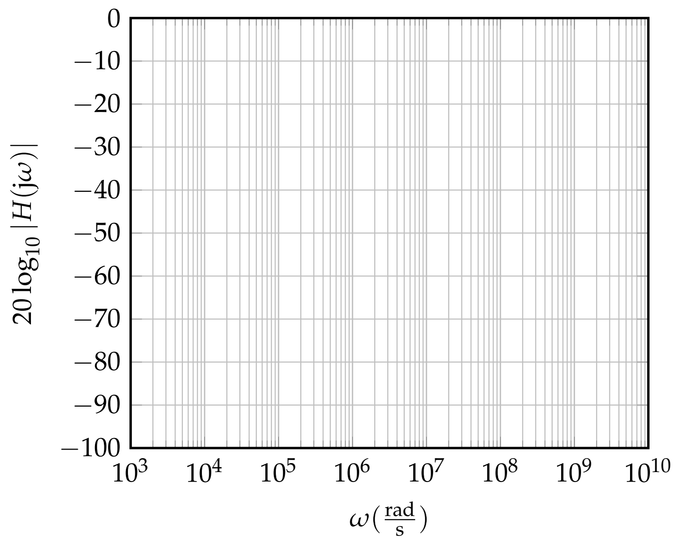
(d) (5 pts.) **Find the transfer function from node v_x to output, $H_2(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out}}}{\widetilde{V}_x}$.**

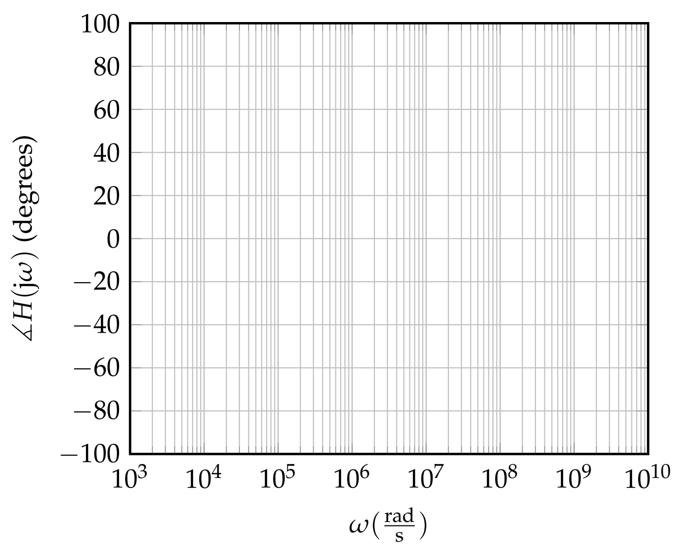
(e) (2 pts.) **What is $H(\mathrm{j}\omega)$ in terms of $H_1(\mathrm{j}\omega)$ and $H_2(\mathrm{j}\omega)$?**

(f) (15 pts.) Suppose that with component values, we find that

$$H(\mathrm{j}\omega) = \frac{\mathrm{j}\omega 10^{-7}}{1 + \mathrm{j}\omega(10^{-8} + 10^{-5}) + (\mathrm{j}\omega)^2 10^{-13}} \tag{13}$$

What are the pole and zero frequencies of this transfer function? Use these to plot the Bode plots for $H(\mathrm{j}\omega)$ on the provided plots below. *(HINT: It may be useful to know that $H(\mathrm{j}10^7) \approx \frac{1}{100}$.)*





10. All-Pass Reverb (20 pts.)

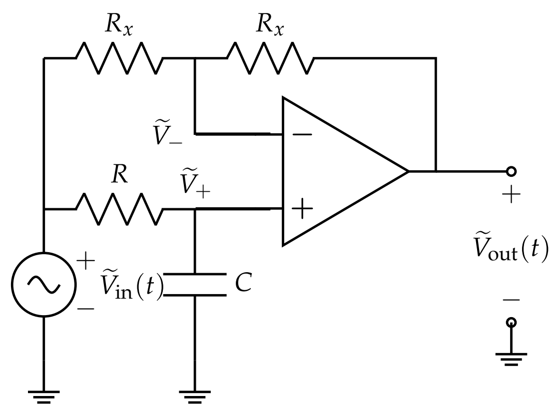
You've been hired as a sound engineer for a music production company.

You are given a request to create a filter that processes audio signals. The filter has the following requirements.

(a) The filter should attenuate all frequencies of audio signals by the same amount (in contrast to something like a low-pass filter which only attenuates low frequency signals)

(b) The filter should allow for precise changes to the inputted audio signal's phase

After consulting your helpful TAs, Chancharik and Nikhil, you receive the suggestion to look into the **all-pass filter**. A circuit diagram of an all-pass filter is shown where it is build using an *ideal* op-amp.



Intrigued, you decide to probe the nature of the transfer function of this filter further using phasors and frequency domain analysis.

(a) (3 pts.) **First, solve for $\widetilde{V}_+$ in terms of $\widetilde{V}_{\text{in}}$, j$\omega$, and any other necessary components.**

(b) (4 pts.) **Write $\widetilde{V}_{\text{out}}$ in terms of $\widetilde{V}_{\text{in}}$, $\widetilde{V}_+$, and any other necessary components.**
(HINT: Properties of ideal op-amps might be helpful here.)

(c) (5 pts.) **Now, find the transfer function, $H(j\omega)$ for this filter.**

(d) (4 pts.) Regardless of your answers to the previous problem part, suppose that the transfer function of the all-pass filter is given by:

$$H(j\omega) = \frac{1 - j\omega RC}{2(1 + j\omega RC)} \tag{14}$$

Find $|H(j\omega)|$ and $\measuredangle H(\mathrm{j}\omega)$. Qualitatively describe what the filter is doing and how it aligns with your goals mentioned previously (one or two sentences maximum).

(e) (4 pts.) Using your expression for $\measuredangle H(\mathrm{j}\omega)$, **Propose a design for your filter (i.e. suggest values for R and C) to shift the phase of a signal with $\omega = 10^3$ by $-\frac{\pi}{2}$.**

[Doodle page! Draw us something if you want or give us suggestions or complaints. You can also use this page to report anything suspicious that you might have noticed.

If needed, you can also use this space to work on problems. But if you want the work on this page to be graded, make sure you tell us on the problem's main page.]

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

