Midterm 2

This is a preview of the published version of the quiz

Started: Nov 15 at 3:16am

Quiz Instructions

Midterm 2 is open book. You are allowed to use any lecture/course notes, homeworks, discussions, or websites (except those for collaborative documents or forums). In addition to this, we will allow the use of a calculator and a Python File or Notebook. You may not access or post on any collaborative documents (e.g. Google Docs) or forums (e.g. Chegg). **Collaboration with other students is prohibited.**

Assuming you do not have an approved time extension, you will have 1 hour (60 minutes) to complete the Midterm and you may begin the Midterm at any point during the window of 7:10-8:30 pm. However, the Midterm will close at 8:30 pm, meaning that you must start by 7:30 pm to have the full 1 hour. **We are not Zoom proctoring.**

We will not clarify anything during the exam so please do your best with the information provided. If you have an issue during your exam please email us at eecs16b-fa20@berkeley.edu (mailto:eecs16b-fa20@berkeley.edu) and CC the professors (seth.sanders@berkeley.edu (mailto:seth.sanders@berkeley.edu) and mlustig@eecs.berkeley.edu (mailto:mlustig@eecs.berkeley.edu)).

Good luck!

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**Question 1**

Consider the following continuous-time system:

\[
\frac{d}{dt} \mathbf{x}(t) = \begin{bmatrix} 1 & -1 \\ -6 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ \alpha \end{bmatrix} u(t).
\]
For which values of $\alpha$ is this system controllable?

Mark all the correct options.

- 0: yes
- -1: yes
- 2: yes
- 3: no

Question 2

Suppose we have a system $\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1) \\ f_2(x_2) \end{bmatrix}$ with $f_1$ and $f_2$ plotted below.
How many equilibrium points are there with $0 \leq x_1 \leq 5$ and $0 \leq x_2 \leq 5$?

9  

For each of the following, state whether the system is stable or unstable when linearized about this point:

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>Unstable</td>
</tr>
</tbody>
</table>
Question 3

Let us model a biological system as the following set of differential equations:

\[
\frac{d}{dt} m = k_1 - d_1 m \\
\frac{d}{dt} p = k_2 m - d_2 p
\]

Where \( k_1, k_2, d_1, d_2 \) are all constants, with units appropriate to the situation. If a biological system like this is allowed to persist for a long time, \( m \) and \( p \) will converge to a unique equilibrium. What are the values of \( m \) and \( p \) at this equilibrium?

\[
m = \begin{cases} 
(i) & \frac{k_1}{d_1} \\
(ii) & -\frac{k_1}{d_2} \\
(iii) & 0 \\
(iv) & -\frac{k_1 k_2}{d_1 d_2} 
\end{cases}
\]

\[
p = \begin{cases} 
(i) & \frac{k_1 k_2}{d_1 d_2} \\
(ii) & -\frac{k_1 k_2}{d_1 d_2} \\
(iii) & 0 \\
(iv) & \frac{k_2}{d_2} 
\end{cases}
\]

Question 4

We have the following discrete time system:

\[
\ddot{x}(t + 1) = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} \ddot{x}(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t),
\]
\[ \ddot{x}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}. \]

Mark if the following statements about this system are True/False.

- It is possible to design an input sequence \( u(0), u(1), u(2), u(3) \) to reach any \( \ddot{x}^* \in \mathbb{R}^2 \) at \( t = 4 \). False

- It is possible to design an input sequence \( u(0), u(1) \) to reach \( x(2) = \begin{bmatrix} 1 + \alpha \\ -2 - \alpha \end{bmatrix} \) for any scalar \( \alpha \in \mathbb{R} \). True

- This system is controllable. False

- In a single time step, we can reach \( x(1) = \begin{bmatrix} 1 + \alpha \\ -2 - \alpha \end{bmatrix} \) for any scalar \( \alpha \in \mathbb{R} \). False

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**Question 5**

Consider the following discrete time linear system:

\[ \ddot{x}(t + 1) = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \ddot{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t). \]

For feedback control \( u(t) = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \ddot{x}(t) \), which of the following values of \( k_2 \) make the eigenvalues of the resulting closed loop system sum to zero?

- -2

- 2
Question 6

Taejin is trying to identify an unknown linear discrete-time system of the form
\[
\begin{bmatrix}
x(t+1) \\
y(t+1)
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix} + \begin{bmatrix}
b_{11} \\
b_{21}
\end{bmatrix} u(t)
\]

To do this, he applies the following sequence of scalar inputs
\[u(0), u(1), \ldots, u(10), u(11)\]
and observes the following states
\[x(0), y(0), x(1), y(1), \ldots, x(11), y(11), x(12), y(12)\]
Which of the following are valid set-ups that can be used to solve for the \(A\) and \(B\) matrices?

\[
\begin{bmatrix}
x(0) & y(0) & u(0) & u(0) \\
x(1) & y(1) & u(1) & u(1) \\
\vdots & \vdots & \vdots & \vdots \\
x(11) & y(11) & u(11) & u(11)
\end{bmatrix} \begin{bmatrix}
a_{11} & a_{21} \\
a_{12} & a_{22} \\
b_{11} & b_{21} \\
b_{21} & b_{22}
\end{bmatrix} = \begin{bmatrix}
x(1) & y(1) \\
x(2) & y(2) \\
\vdots & \vdots \\
x(12) & y(12)
\end{bmatrix}
\]
Given an over-constrained system of equations

\[ D\vec{p} = \vec{y} \]
Which of the following statements must be true in order to create a unique Least-Squares estimate $\hat{p}$.

- $D$ has linearly independent columns. True
- $D^T D$ has linearly independent columns. True
- $D^T D$ has strictly positive eigenvalues. True
- The error $\tilde{y} - D\hat{p}$ is orthogonal to $\text{Col}(D)$. True

**Question 8** 1 pts

Consider the following dynamical system with $a, d, \lambda, \mu > 0$,

$$
\begin{align*}
\frac{d}{dt} x_1(t) &= x_1(t)(a - bx_2(t) - \lambda x_1(t)) \\
\frac{d}{dt} x_2(t) &= x_2(t)(-d + cx_1(t) - \mu x_2(t))
\end{align*}
$$

Which of the following is the $A$ matrix when you linearize around the equilibrium of the form $(\frac{a}{\lambda}, 0)$.

- $\begin{bmatrix} -a & -\frac{ab}{\lambda} \\ 0 & -d + \frac{ac}{\lambda} \end{bmatrix}$
- $\begin{bmatrix} a + \frac{bd}{\mu} & -\frac{ab}{\lambda} \\ 0 & d \end{bmatrix}$
- None of the above
Question 9

\( \begin{bmatrix} \alpha & 0 \\ \frac{cd}{\mu} & -d + \frac{ac}{\lambda} \end{bmatrix} \)

\( \begin{bmatrix} a + \frac{bd}{\mu} & 0 \\ -\frac{cd}{\mu} & d \end{bmatrix} \)

\( A \in \mathbb{R}^{2\times2} \) is a diagonal matrix with singular values \( \sigma_1 = 0.5, \ \sigma_2 = 0.1 \).

Complete the following statements:

When \( A \) is the matrix in: \( \ddot{x}(t + 1) = A\dot{x}(t) + Bu(t) \),

system is definitely sta _✓_.

When \( A \) is the matrix in \( \frac{d}{dt} \ddot{x}(t) = A\dot{x}(t) + Bu(t) \),

system’s stability cannot _✓_.

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