

Final Part 1

⚠ This is a preview of the published version of the quiz

Started: Jan 27 at 6:50pm

Quiz Instructions

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Question 1

1 pts

Taejin is trying to build an inverter to power up a clock. He considers the following two designs using NMOS and PMOS transistors shown below.

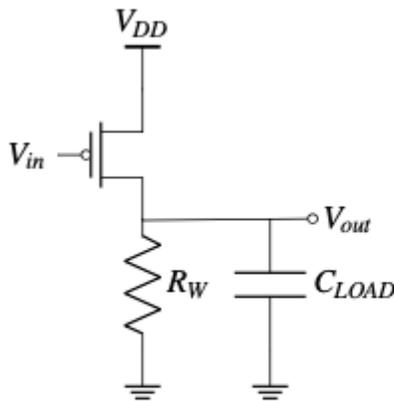


Figure 1: Single PMOS Model

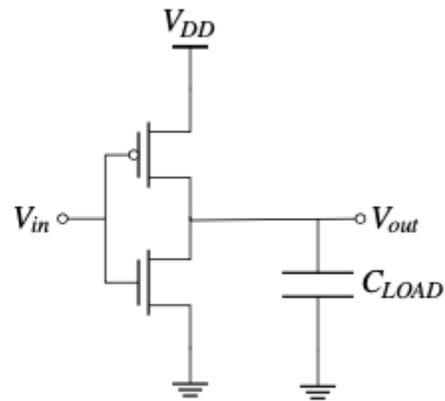
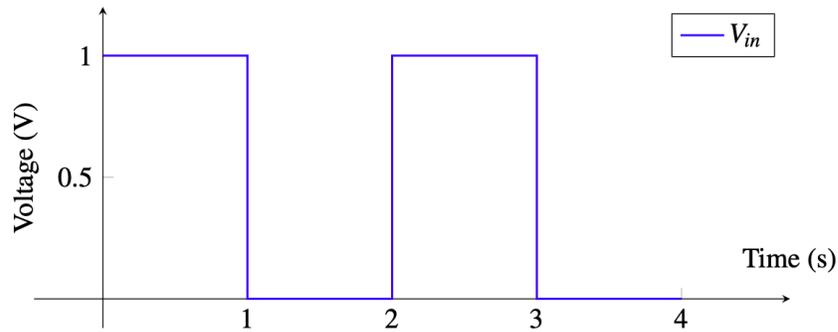


Figure 2: CMOS Inverter Model

Both models contain an output load capacitance C_{LOAD} . In addition, the PMOS inverter model contains a pull-down resistor $R_W = 500\Omega$. All PMOS and NMOS transistors follow the resistor-switch model with switch resistance $R_{NMOS} = R_{PMOS} = 1k\Omega$ and have threshold voltage $|V_{th}| = 0.7V$. All PMOS and NMOS devices have negligible gate capacitance.

Assume that $V_{DD} = 1V$, and that at time $t = 0$, the output voltage $V_{out}(0) = 1V$. To test the models, Taejin applies a square wave input V_{in} to both circuits shown below.



Mark the statements below as True or False.

True



The PMOS Inverter dissipates more energy than the CMOS Inverter in the interval $[0, 4)$.

False



For $t \in [0, 1)$, the CMOS Inverter has a smaller time constant than the PMOS Inverter.

False



The output voltage V_{out} of the PMOS Inverter at $t = 2$ is approximately 1V.

True



The output voltage V_{out} of the CMOS Inverter at $t = 1$ is approximately 0V.

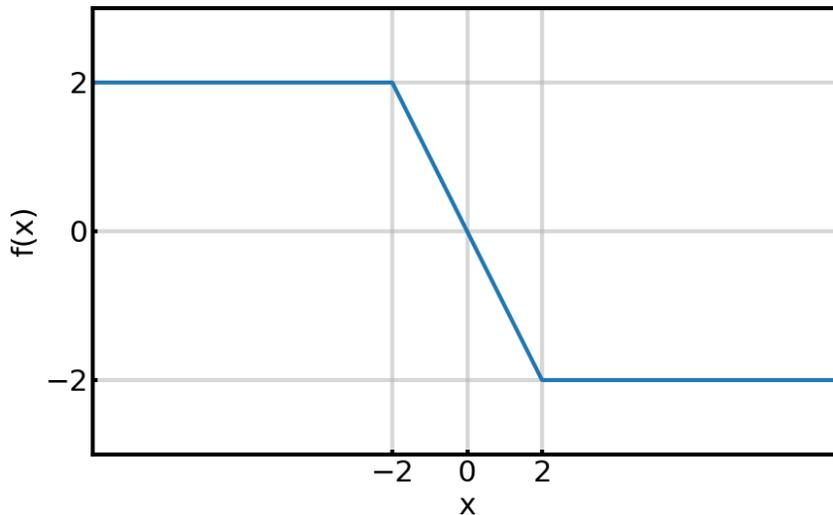
Question 2

1 pts

Consider the first order differential equation

$$\frac{d}{dt}x(t) = f(x),$$

where the function $f(x)$ is as shown in the graph below.



Mark the following statements as **True** or **False**:

True



For any initial condition $x(0)$, $x(t)$ is bounded.

False



There exists an initial condition $x(0)$ for which $\lim_{t \rightarrow \infty} x(t) \neq 0$.

False



If $x(0) = 1$, $x(1) = e^{-2}$.

True



If $x(0) = 4$, $x(1) = 2$.

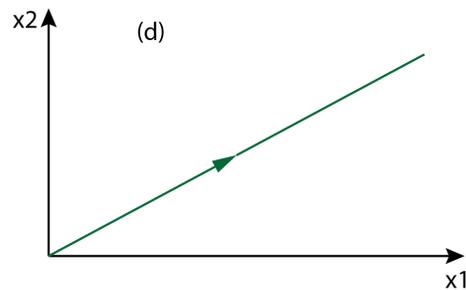
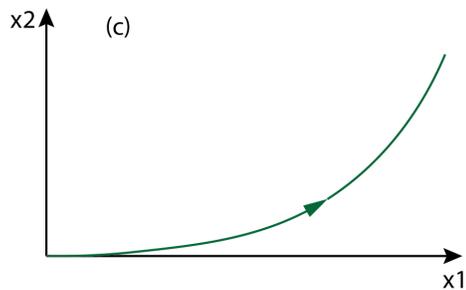
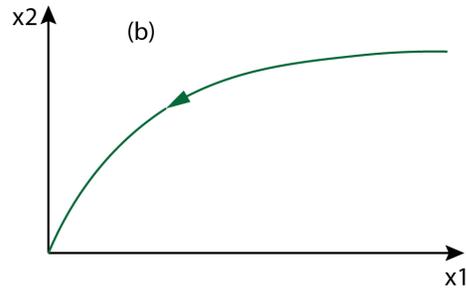
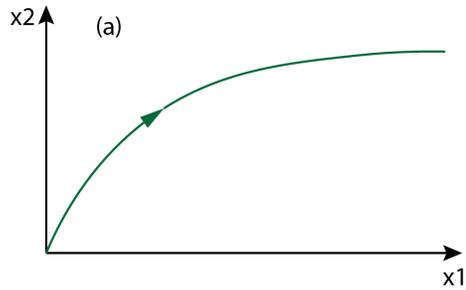
Question 3

1 pts

Consider the following dynamical system with control input $u \in \mathbb{R}$ and state vector $\vec{x} \in \mathbb{R}^2$:

$$\frac{d}{dt} \vec{x}(t) = A \vec{x}(t) + \vec{v}_1 u(t)$$

Where $A \in \mathbb{R}^{2 \times 2}$ and $\vec{v}_1 \in \mathbb{R}^2$ is an eigenvector of A . Suppose $\vec{x}(0) = \vec{0}$, and u is held constant at 1. Which of the following graphs could represent the evolution of the components of $\vec{x}(t)$ as time proceeds?

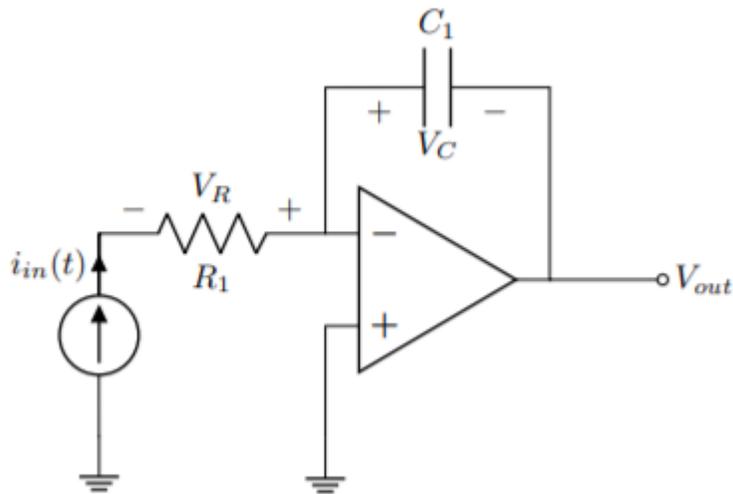


- (d)
- (a)
- (b)
- (c)
- None of the other answers.

Question 4

1 pts

Consider the following circuit with $R_1 = 100 \Omega$, $C_1 = 2 \text{ mF}$, and an ideal op-amp. You may assume the op-amp power supplies do not constrain V_{out} .



Suppose $V_{out} = 0$ at $t = 0$ and $i_{in}(t) = \begin{cases} 0, & t < 0 \\ 1\text{mA}, & t \geq 0 \end{cases}$.

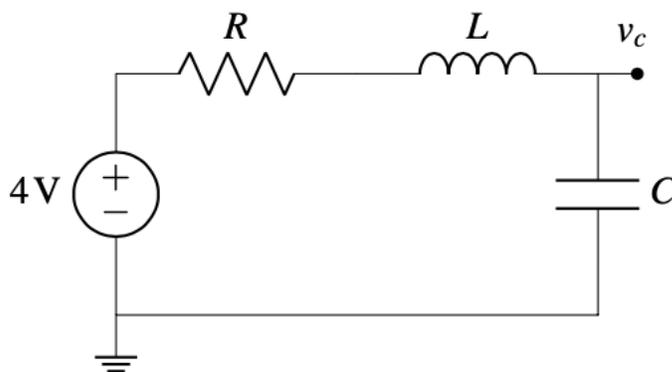
What is V_{out} in volts at $t = 3$ s ?

Provide your answer to 1 decimal place.

Question 5

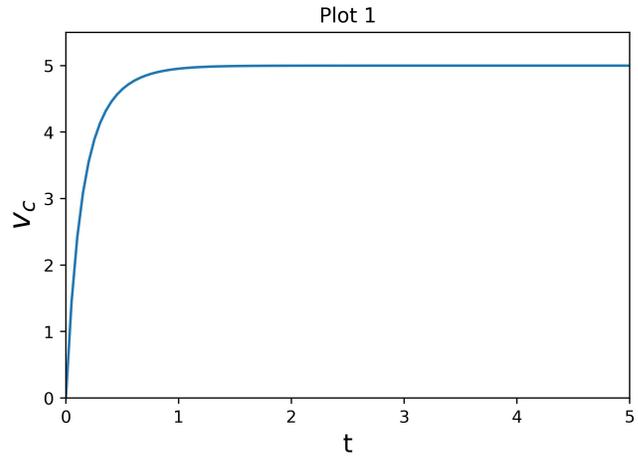
1 pts

Consider the following RLC Circuit with unknown values of R , L , $C > 0$ and unknown initial conditions.

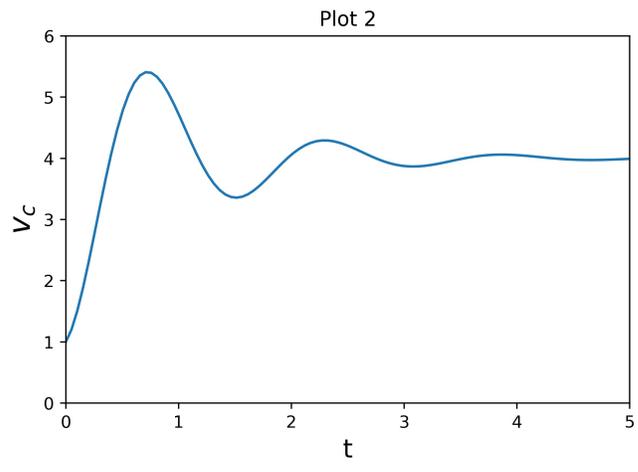


Which of the following plots of $v_c(t)$ are **possible**?

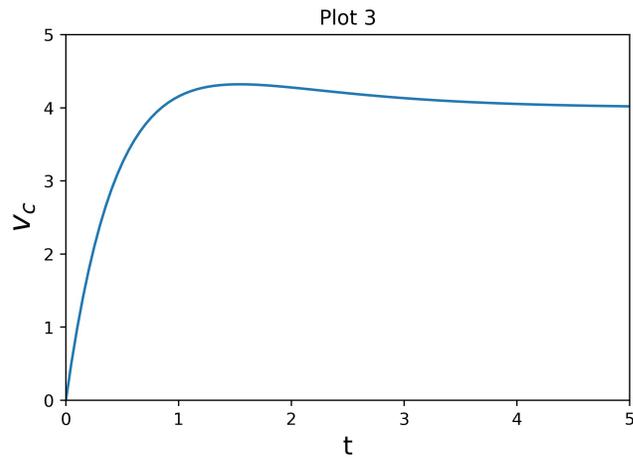
Plot 1 ▼



Plot 2 ▼

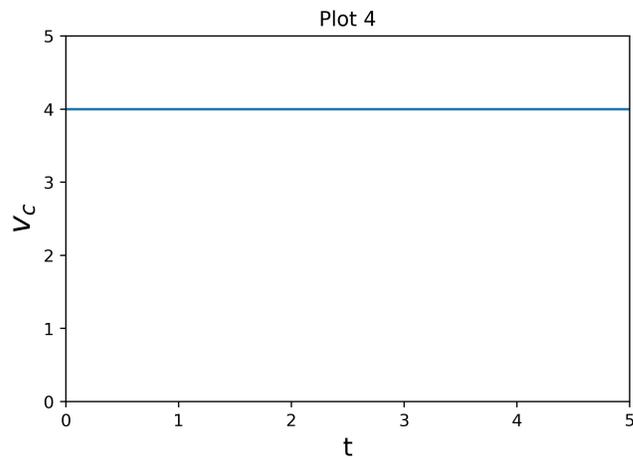


Plot 3 ▼



Plot 4

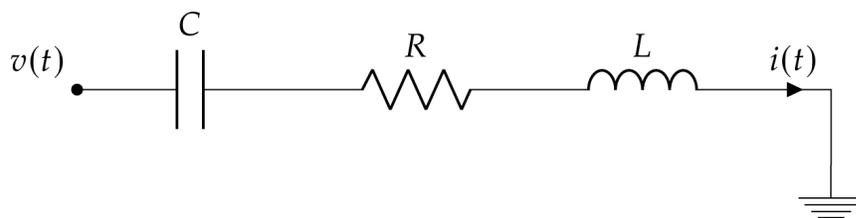
Possible



Question 6

1 pts

Suppose you are given the following circuit.



Steady state time waveforms are given as

$$v(t) = 4 \cos(\omega t)$$

$$i(t) = 2 \cos(\omega t - \pi/6)$$

where the respective units are Volts and Amperes,

$$\omega = 2 \times 10^6 \text{ rad/s}, R = \sqrt{3}\Omega, C = 0.5\mu F.$$

What is the inductance of the inductor?

$5\mu H$

$2\mu H$

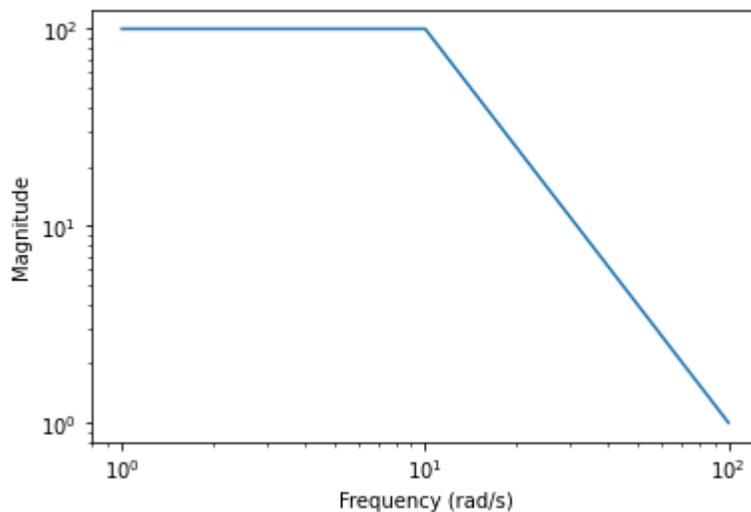
$1\mu H$

$0.5\mu H$

Question 7

1 pts

Consider the following Bode Plot:



Which of the following transfer functions corresponds to this Bode Plot?

$$(a): H(j\omega) = \frac{100}{\frac{j\omega}{10} + 1}$$

$$(b): H(j\omega) = \frac{100}{\left(\frac{j\omega}{10} + 1\right)^2}$$

$$(c): H(j\omega) = \frac{10}{\frac{j\omega}{100} + 1}$$

$$(d): H(j\omega) = \frac{10}{\left(\frac{j\omega}{1} + 1\right)^2}$$

(b)

(a)

(c)

(d)

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Final Part 2

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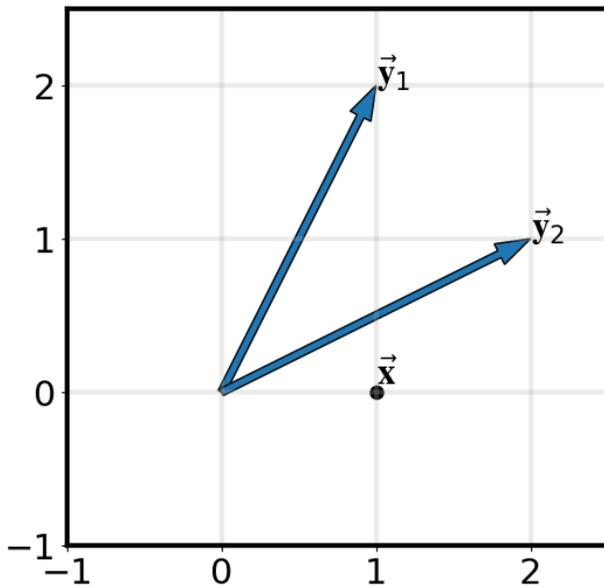
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Question 1

1 pts



Consider vectors $\{\vec{x}, \vec{y}_1, \vec{y}_2\} \in \mathbb{R}^2$. Let α_1 and α_2 be the coordinates of \vec{x} in a (non-orthogonal) basis comprised of \vec{y}_1 and \vec{y}_2 , Mark the following as **True/False**.

- ▾
The coordinates α_1 and α_2 are unique.

- ▾
 $\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = [\mathbf{y}_1 \quad \mathbf{y}_2] \mathbf{x}$.

- ▾
 $\vec{x} = [\vec{y}_1 \quad \vec{y}_2] \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$.

- ▾
 $\alpha_1 = -\frac{1}{3}, \alpha_2 = \frac{2}{3}$.

Question 2

1 pts

Consider the following system

$$\begin{aligned}\frac{dx_1}{dt} &= \sin(\pi x_2) - x_1, \\ \frac{dx_2}{dt} &= x_1 - x_2.\end{aligned}$$

Select the **correct** statement regarding this system's equilibrium points.

- System has exactly 3 equilibrium points.
- System has infinitely many equilibrium points.
- System has exactly one equilibrium.

Question 3

1 pts

$A \in R^{4 \times 3}$ is a **Rank = 1** matrix.

$\vec{x} = \vec{v}_1 - 2\vec{v}_2 + 4\vec{v}_3$, where the vectors \vec{v}_i are the ordered right singular vectors of A (full SVD decomposition).

You are given that $A\vec{x} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$, where $a=-1$, $b=1$, $c=-1$, and $d=-1$

What is σ_1 ? (up to two decimal points. You can use a calculator)

Question 4

1 pts

Consider the following non-linear continuous time system.

$$\frac{dx(t)}{dt} = (1 - y(t))x(t) + u(t)$$
$$\frac{dy(t)}{dt} = (x(t) - y(t))y(t)$$

Linearize the non-linear system around the equilibrium (assuming $u^* = 0$) of the form (x^*, y^*) , $x^* > 0$, $y^* > 0$.

The linearized system is

Stable



The linearized system is

Controllable



Question 5

1 pts

We have the following continuous time system

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u(t)$$

We are trying to use a single-parameter feedback controller

$u(t) = k(x_1(t) + x_2(t))$. Which of the following statements are

True?

False



For $k = 0$ the system is stable.

False



We can change the eigenvalues of the closed loop system with $k \neq 0$.

False



It is possible to stabilize this system using the given feedback controller.

Question 6

1 pts

The following system is a dynamical model of an air bubble in water, with physical constants b , R , P , and γ .

$$\frac{d}{dt} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-3\gamma P}{\rho R^2} & \frac{-b}{\rho R^3} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.$$

Suppose that physical constant b is small. Then, the system's resonant frequency in rad/s is given by which of the following:

(i)



:

(i) $\frac{1}{R} \sqrt{\frac{3\gamma P}{\rho}}$

(ii) $\frac{b}{3\gamma P R}$

(iii) $\frac{1}{R^3} \sqrt{\frac{3\gamma P}{\rho}}$

(iv) $\sqrt{\frac{b}{\gamma P R^2}}$

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Question 1	1 pts

Professor Sanders shoots a video of Professor Lustig's car racing down Sather Gate. When the car is moving forward, its wheels turn in a counterclockwise motion.

However upon watching the video, Archit notices that the car's wheels appear to be moving clockwise at a rate of 2 rev/s. Given that the video is shot at 30 frames per second, and the car's wheels each have 5 identical uniformly spaced spokes, mark all of the possible rates at which the car's wheels could possibly have been moving.

- 8 rev/s clockwise
- 2 rev/s counterclockwise
- 10 rev/s counterclockwise
- 4 rev/s counterclockwise

Question 2

1 pts

You are given 3 LTI systems with the following impulse responses:

$$\text{Sys 1: } h_1[n] = \delta[n - 4] - \delta[n + 4]$$

$$\text{Sys 2: } h_2[n] = U[n - 4] \quad (\text{U}[n] \text{ is a unit step function})$$

$$\text{Sys 3: } h_3[n] = h_2[n] * h_1[n] \quad (\text{the symbol } * \text{ notes a discrete convolution})$$

1) Sys 1 causal, and

BIBO stable.

2) Sys 2 causal, and

BIBO stable.

3) Sys 3 causal, and

BIBO stable.

Question 3

1 pts

Comment on the properties of the following system

$$y[n] = x[n] \cos[n + 1]$$

The system is linear.

The system is time-invariant.

The system is causal.

True



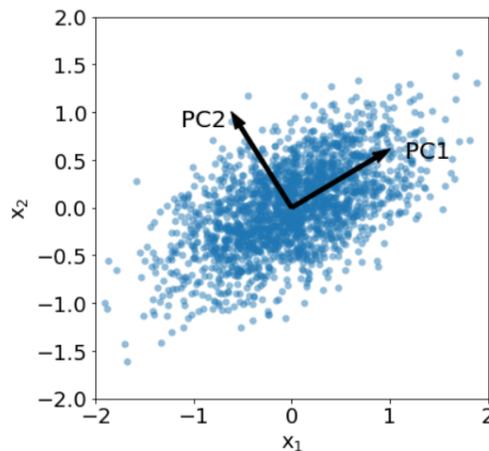
The system is BIBO stable.

Question 4

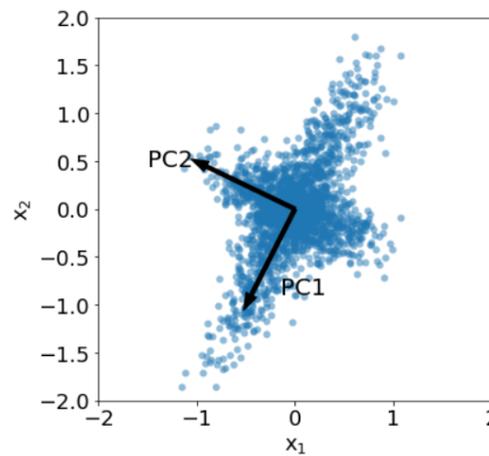
1 pts

Determine whether the following plots of principle components are possible. All data points are two dimensional and are captured by blue dots in the plots. PCA is run on the datasets and the the principle components are presented as black arrows. "PC1" represents the first principle component and "PC2" represents the second principle component.

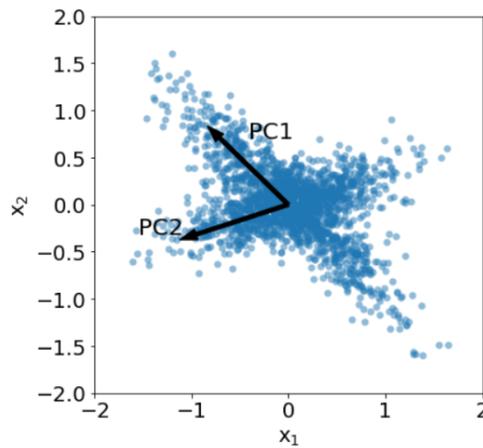
Possible



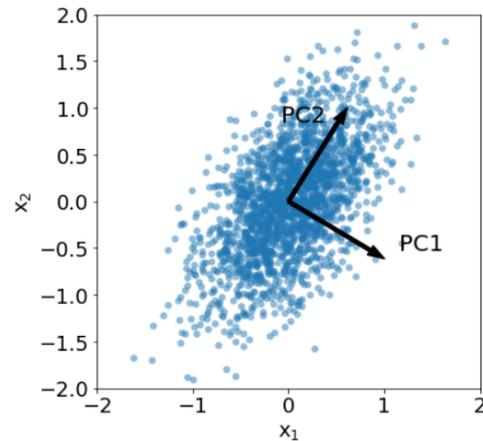
Possible



Impossible



Impossible



Question 5

1 pts

Are the following statements about the Moore Penrose Pseudo-Inverse A^\dagger of some $m \times n$ real matrix A true or false? Assume $m < n$ and the rows of A are linearly independent.

False

It holds that $A^\dagger A = I$, where I is the n by n

identity matrix.

False

$A^T A$ is full rank, and $A^\dagger = (A^T A)^{-1} A^T$

True

AA^T is full rank, and $A^\dagger = A^T (AA^T)^{-1}$

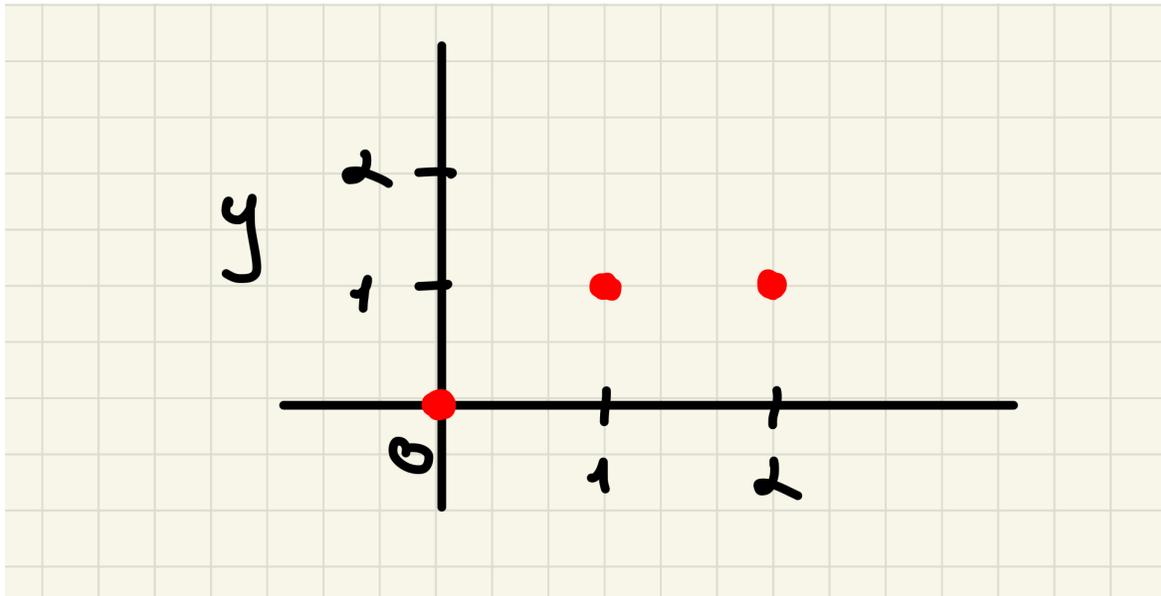
True

∨ A^\dagger has the same rank as A .

Question 6

0.5 pts

Consider the set of points $[y(0), y(1), y(2)] = [0, 1, 1]$



What are the coefficients of the 1st order polynomial

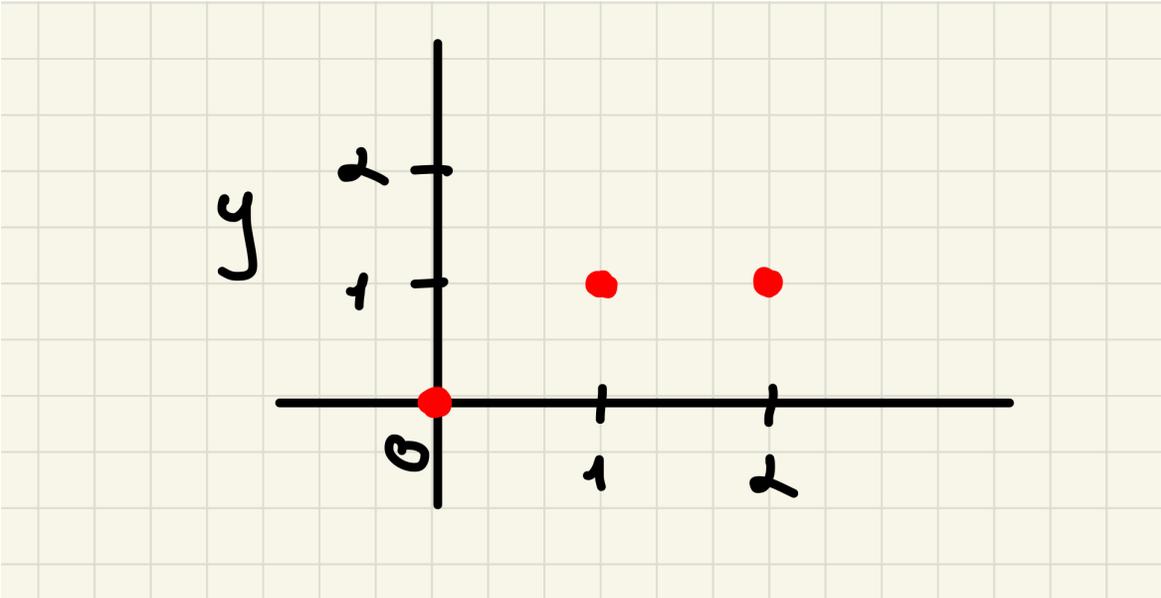
$\tilde{y} = a_1 x + a_0$ that minimize the error $\sum_{n=0}^2 (\tilde{y}[n] - y[n])^2$. (Please round to the closest 1st decimal point)

$a_1 =$, $a_0 =$

Question 7

0.5 pts

Consider the set of points $[y(0), y(1), y(2)] = [0, 1, 1]$



What are the coefficients of the 2nd order polynomial

$\tilde{y} = a_2x^2 + a_1x + a_0$ that fit $y[0], y[1], y[2]$ exactly. (please round to the 1st decimal point)

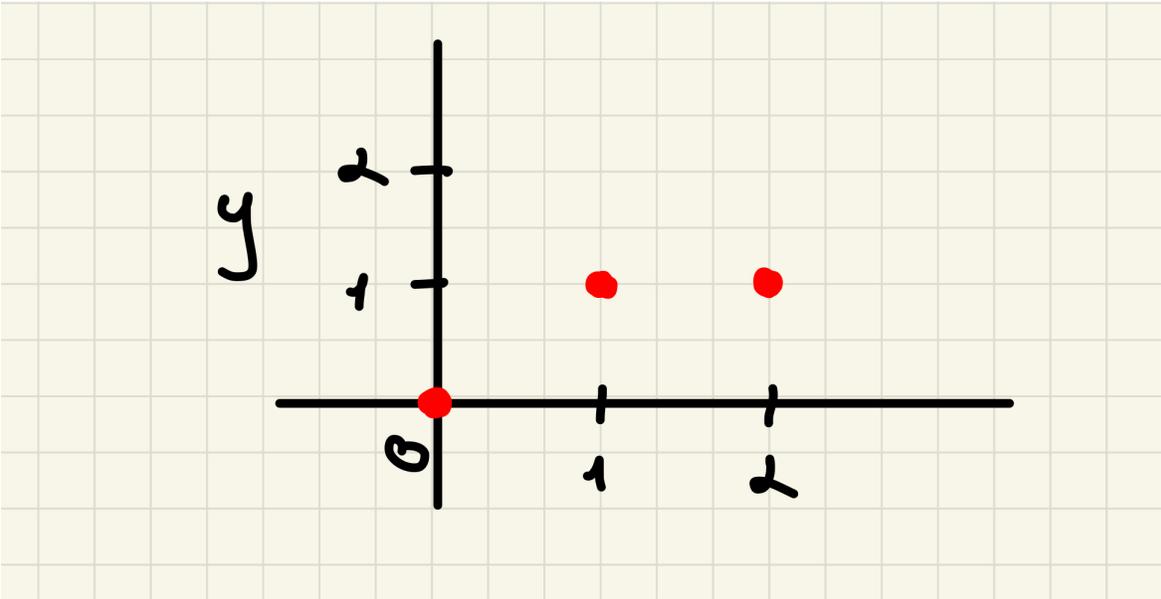
You may find the following useful: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$a_2 =$, $a_1 =$, $a_0 =$

Question 8

0.5 pts

Consider the set of points $[y(0), y(1), y(2)] = [0, 1, 1]$



What are the coefficients of the 3rd order polynomial

$\tilde{y} = a_3x^3 + a_2x^2 + a_1x + a_0$ that fit $y[0]$, $y[1]$, $y[2]$ exactly, and have the smallest norm, ie., minimize $\sum_{n=0}^3 a_n^2$.

You may find the following useful: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

a3= , a2= , a1= , a0=0

Round your answer to 1 or 2 decimals.

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