Final Part 1

⚠️ This is a preview of the published version of the quiz

Started: Jan 27 at 6:50pm

Quiz Instructions

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Question 1

Taejin is trying to build an inverter to power up a clock. He considers the following two designs using NMOS and PMOS transistors shown below.

![Single PMOS Model](image1)

![CMOS Inverter Model](image2)

Both models contain an output load capacitance $C_{LOAD}$. In addition, the PMOS inverter model contains a pull-down resistor $R_W = 500\Omega$. All PMOS and NMOS transistors follow the resistor-switch model with switch resistance $R_{NMOS} = R_{PMOS} = 1k\Omega$ and have threshold voltage $|V_{th}| = 0.7V$. All PMOS and NMOS devices have negligible gate capacitance.

Assume that $V_{DD} = 1V$, and that at time $t = 0$, the output voltage $V_{out}(0) = 1V$. To test the models, Taejin applies a square wave input $V_{in}$ to both circuits shown below.
Mark the statements below as True or False.

[ Select ] ☑️ The PMOS Inverter dissipates more energy than the CMOS Inverter in the interval \([0, 4)\).

[ Select ] ☑️ For \(t \in [0, 1)\), the CMOS Inverter has a smaller time constant than the PMOS Inverter.

[ Select ] ☑️ The output voltage \(V_{out}\) of the PMOS Inverter at \(t = 2\) is approximately 1V.

[ Select ] ☑️ The output voltage \(V_{out}\) of the CMOS Inverter at \(t = 1\) is approximately 0V.

**Question 2**

Consider the first order differential equation

\[
\frac{d}{dt} x(t) = f(x),
\]

where the function \(f(x)\) is as shown in the graph below.
Mark the following statements as True or False:

- For any initial condition \( x(0) \), \( x(t) \) is bounded.
- There exists an initial condition \( x(0) \) for which \( \lim_{t \to \infty} x(t) \neq 0 \).
- If \( x(0) = 1 \), \( x(1) = e^{-2} \).
- If \( x(0) = 4 \), \( x(1) = 2 \).

**Question 3**

Consider the following dynamical system with control input \( u \in \mathbb{R} \) and state vector \( \vec{x} \in \mathbb{R}^2 \):

\[
\frac{d}{dt} \vec{x}(t) = A\vec{x}(t) + \vec{v}_1 u(t)
\]
Where $A \in \mathbb{R}^{2 \times 2}$ and $\vec{v}_1 \in \mathbb{R}^2$ is an eigenvector of $A$. Suppose $\vec{x}(0) = \vec{0}$, and $u$ is held constant at 1. Which of the following graphs could represent the evolution of the components of $\vec{x}(t)$ as time proceeds?

- (a)
- (b)
- (c)
- (d)
- None of the other answers.

**Question 4**

1 pts

Consider the following circuit with $R_1 = 100 \ \Omega$, $C_1 = 2 \ \text{mF}$, and an ideal op-amp. You may assume the op-amp power supplies do not constrain $V_{out}$. 
Suppose $V_{out} = 0$ at $t = 0$ and $i_{in}(t) = \begin{cases} 0, & t < 0 \\ 1\text{mA}, & t \geq 0 \end{cases}$.

What is $V_{out}$ in volts at $t = 3 \text{ s}$?

Provide your answer to 1 decimal place.

---

**Question 5**

Consider the following RLC Circuit with unknown values of $R$, $L$, $C > 0$ and unknown initial conditions.
Which of the following plots of $v_c(t)$ are possible?

Plot 1 [Select]

Plot 2 [Select]

Plot 3 [Select]
Question 6 1 pts

Suppose you are given the following circuit.

Steady state time waveforms are given as
\[ v(t) = 4 \cos(\omega t) \]
\[ i(t) = 2 \cos(\omega t - \pi/6) \]

where the respective units are Volts and Amperes,

\[ \omega = 2 \times 10^6 \text{ rad/s}, \quad R = \sqrt{3} \Omega, \quad C = 0.5 \mu F. \]

What is the inductance of the inductor?

- 5 \mu H
- 2 \mu H
- 1 \mu H
- 0.5 \mu H

**Question 7**

Consider the following Bode Plot:

Which of the following transfer functions corresponds to this Bode Plot?

![Bode Plot](image-url)
(a): $H(j\omega) = \frac{100}{\frac{j\omega}{10} + 1}$

(b): $H(j\omega) = \frac{100}{\left(\frac{j\omega}{10} + 1\right)^2}$

(c): $H(j\omega) = \frac{10}{\frac{j\omega}{100} + 1}$

(d): $H(j\omega) = \frac{10}{\left(\frac{j\omega}{1} + 1\right)^2}$

- (b)
- (a)
- (c)
- (d)
Final Part 2

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Question 1 1 pts
Consider vectors \( \{ \vec{x}, \vec{y}_1, \vec{y}_2 \} \in \mathbb{R}^2 \). Let \( \alpha_1 \) and \( \alpha_2 \) be the coordinates of \( \vec{x} \) in a (non-orthogonal) basis comprised of \( \vec{y}_1 \) and \( \vec{y}_2 \), Mark the following as **True/False**.

- [ Select ]  
  \( \alpha_1 \) and \( \alpha_2 \) are unique.

- [ Select ]  
  \[
  \begin{bmatrix}
  \alpha_1 \\
  \alpha_2 
  \end{bmatrix}
  =
  \begin{bmatrix}
  \vec{y}_1 & \vec{y}_2 
  \end{bmatrix}
  \vec{x}.
  \]

- [ Select ]  
  \( \vec{x} = \begin{bmatrix} \vec{y}_1 & \vec{y}_2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \).

- [ Select ]  
  \( \alpha_1 = -\frac{1}{3}, \alpha_2 = \frac{2}{3} \).

**Question 2**  
1 pts
Consider the following system
\[
\frac{dx_1}{dt} = \sin(\pi x_2) - x_1, \\
\frac{dx_2}{dt} = x_1 - x_2.
\]
Select the correct statement regarding this system's equilibrium points.

- System has exactly 3 equilibrium points.
- System has infinitely many equilibrium points.
- System has exactly one equilibrium.

**Question 3**

\(A \in \mathbb{R}^{4 \times 3}\) is a **Rank = 1** matrix.
\[
\bar{x} = \bar{v}_1 - 2\bar{v}_2 + 4\bar{v}_3, \text{ where the vectors } \bar{v}_i \text{ are the ordered right singular vectors of } A \text{ (full SVD decomposition).}
\]
You are given that \(A\bar{x} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\), where \(a=-1, b=1, c=-1, \text{ and } d=-1\)

What is \(\sigma_1\)? (up to two decimal points. You can use a calculator)
Question 4

Consider the following non-linear continuous time system.

\[
\frac{dx(t)}{dt} = (1 - y(t))x(t) + u(t) \\
\frac{dy(t)}{dt} = (x(t) - y(t))y(t)
\]

Linearize the non-linear system around the equilibrium (assuming \(u^* = 0\)) of the form \((x^*, y^*)\), \(x^* > 0, y^* > 0\).

The linearized system is [Select] .

The linearized system is [Select] .

Question 5

We have the following continuous time system

\[
\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u(t)
\]

We are trying to use a single-parameter feedback controller

\[ u(t) = k(x_1(t) + x_2(t)) \]

Which of the following statements are True?

[Select] For \(k = 0\) the system is stable.
We can change the eigenvalues of the closed loop system with $k \neq 0$.

It is possible to stabilize this system using the given feedback controller.

**Question 6**

The following system is a dynamical model of an air bubble in water, with physical constants $b$, $R$, $P$, and $\gamma$.

\[
\frac{d}{dt} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{3\gamma P}{\rho R^2} & \frac{1}{\rho R^3} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.
\]

Suppose that physical constant $b$ is small. Then, the system's resonant frequency in rad/s is given by which of the following:

- (i) $\frac{1}{R} \sqrt{\frac{3\gamma P}{\rho}}$
- (ii) $\frac{b}{3\gamma PR}$
- (iii) $\frac{1}{R^3} \sqrt{\frac{3\gamma P}{\rho}}$
- (iv) $\sqrt{\frac{b}{\gamma PR^2}}$
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| Question 1 | 1 pts |
Professor Sanders shoots a video of Professor Lustig's car racing down Sather Gate. When the car is moving forward, its wheels turn in a counterclockwise motion.

However upon watching the video, Archit notices that the car's wheels appear to be moving clockwise at a rate of 2 rev/s. Given that the video is shot at 30 frames per second, and the car's wheels each have 5 identical uniformly spaced spokes, mark all of the possible rates at which the car's wheels could possibly have been moving.

- 8 rev/s clockwise
- 2 rev/s counterclockwise
- 10 rev/s counterclockwise
- 4 rev/s counterclockwise

---

**Question 2**

You are given 3 LTI systems with the following impulse responses:

\[
\begin{align*}
\text{Sys 1:} & \quad h_1[n] = \delta[n - 4] - \delta[n + 4] \\
\text{Sys 2:} & \quad h_2[n] = U[n - 4] \quad \text{(U[n] is a unit step function)} \\
\text{Sys 3:} & \quad h_3[n] = h_2[n] * h_1[n] \quad \text{(the symbol * notes a discrete convolution)}
\end{align*}
\]
1) Sys 1 [ Select ] causal, and
   [ Select ] BIBO stable.

2) Sys 2 [ Select ] causal, and
   [ Select ] BIBO stable.

3) Sys 3 [ Select ] causal, and
   [ Select ] BIBO stable.

Question 3

Comment on the properties of the following system

\[ y[n] = x[n] \cos[n + 1] \]

[ Select ] The system is linear.

[ Select ] The system is time-invariant.

[ Select ] The system is causal.
The system is BIBO stable.

**Question 4**

Determine whether the following plots of principle components are possible. All data points are two dimensional and are captured by blue dots in the plots. PCA is run on the datasets and the principle components are presented as black arrows. "PC1" represents the first principle component and "PC2" represents the second principle component.
Question 5

Are the following statements about the Moore Penrose Pseudo-Inverse $A^\dagger$ of some $m \times n$ real matrix $A$ true or false? Assume $m < n$ and the rows of $A$ are linearly independent.

- It holds that $A^\dagger A = I$, where $I$ is the $n$ by $n$ identity matrix.
- $A^T A$ is full rank, and $A^\dagger = (A^T A)^{-1} A^T$
- $AA^T$ is full rank, and $A^\dagger = A^T (AA^T)^{-1}$
Question 6

Consider the set of points \([y(0), y(1), y(2)] = [0, 1, 1]\)

What are the coefficients of the 1st order polynomial

\[\tilde{y} = a_1 x + a_0\]

that minimize the error \(\sum_{n=0}^{2} (\tilde{y}[n] - y[n])^2\). (Please round to the closest 1st decimal point)

\[a_1 = \underline{\hspace{1cm}}, \quad a_0 = \underline{\hspace{1cm}}\]
Consider the set of points \([y(0), y(1), y(2)] = [0, 1, 1]\)

What are the coefficients of the 2\(^{nd}\) order polynomial
\[ \tilde{y} = a_2 x^2 + a_1 x + a_0 \] that fit \(y[0], y[1], y[2]\) exactly. (please round to the 1st decimal point)

You may find the following useful:
\[
\begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix}
  d & -b \\
  -c & a
\end{bmatrix}.
\]

\[ a_2 = \boxed{\phantom{0}} \]
\[ a_1 = \boxed{\phantom{0}} \]
\[ a_0 = \boxed{\phantom{0}} \]

**Question 8**  

Consider the set of points \([y(0), y(1), y(2)] = [0, 1, 1]\)
What are the coefficients of the 3\textsuperscript{rd} order polynomial
\( \tilde{y} = a_3 x^3 + a_2 x^2 + a_1 x + a_0 \) that fit \( y[0], y[1], y[2] \) exactly, and have the the smallest norm, ie., minimize \( \sum_{n=0}^{3} a_n^2 \):

You may find the following useful: 
\[
\begin{bmatrix}
    a & b \\
    c & d
\end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix}
    d & -b \\
    -c & a
\end{bmatrix}.
\]

\[a_3= \quad \quad , \quad a_2= \quad \quad , \quad a_1= \quad \quad , \quad a_0=0\]

Round your answer to 1 or 2 decimals.