EECS 16B Designing Information Devices and Systems II
Spring 2021 Note 7A: Controls Overview

1 Overview

We have seen how to model a continuous-time system (for example, defined by a system of differential equations $\frac{d}{dt}\vec{x}_c(t) = A_c\vec{x}_c(t) + B_c\vec{u}_c(t)$) by choosing a time interval Δ, deciding to apply piecewise constant inputs for durations of length $\Delta > 0$ seconds, and then sampling every Δ seconds to get a new discrete time system:

$$\vec{x}_d[i+1] = A_d\vec{x}_d[i] + B_d\vec{u}_d[i],$$

where $\vec{x}_d[i]$ is the value of $\vec{x}_c(i\Delta)$ and $\vec{u}_d[i]$ is the value of $\vec{u}_c(t)$ for the time interval $t \in [i,(i+1)\Delta)$.

A Side Note on Continuous vs. Discrete: Note that it is perfectly fine if what follows here doesn't make sense yet; we will cover it in more detail in discussion 7A. Note that the A_d matrix for the discrete time system is related to the original A_c matrix of the continuous-time system. It is easiest to see the exact relationship when there exists a coordinate system in which A_c is diagonal — $A_c = V\Lambda V^{-1}$. In the diagonal case, $\frac{d}{dt}\vec{\tilde{x}}_c(t) = \Lambda\vec{\tilde{x}}_c(t) + \widetilde{B}_c\vec{u}_c(t)$, the underlying system is basically a parallel set of scalar differential equations $\frac{d}{dt}\left(\vec{\tilde{x}}_c(t)\right)_k = \lambda_k\left(\vec{\tilde{x}}_c(t)\right)_k + \left(\widetilde{B}_c\vec{u}_c(t)\right)_k$. If $\lambda_k \neq 0$, this discretizes to $\left(\vec{\tilde{x}}_d(i+1)\right)_k = e^{\Delta\lambda_k}\left(\widetilde{x}_d[i]\right)_k + \frac{e^{\Delta\lambda_k}-1}{\lambda_k}\left(\widetilde{B}_c\vec{u}_d[i]\right)_k$. If $\lambda_j = 0$, this discretizes to $\left(\vec{\tilde{x}}_d[i+1]\right)_k = \left(\widetilde{x}_d[i]\right)_k + \left(\Delta\widetilde{B}_c\vec{u}_d[i]\right)_k$. The A and B matrices follow from this.

For the remainder of this module, we will almost entirely disregard the fact that our system may in fact really be continuous, focusing entirely on questions related to our discrete-time model. After all, in reality, most electronic control systems (like the MSP that we will use to develop a "robot car") have some intrinsic Δ in their ability to sample $\vec{x}_c(t)$ and vary their output $\vec{u}(t)$, so even if we knew that $\vec{x}_c(t)$ varied within the interval Δ, we would not be able to measure or react to this variation within the time interval.

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