EECS 16B
Designing Information Devices and Systems II

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Announcements

- Midterm #1 redo is due tomorrow @ 11:59pm
- Please complete our mid-semester survey, due Monday 3/18 @11:59pm
Today

- Review
  - matrix exponential
  - “discretizing” a system
- Stability
- Feedback (“closed loop”) control
  - Feedback stabilization
Review - Matrix Exponential

\[ e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots = \sum_{i=0}^{\infty} \frac{x^i}{i!} \]
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If \( X = \text{diag}(x_{11}, x_{22}, \ldots, x_{nn}) \), then

\[
e^X = \begin{bmatrix} e^{x_{11}} & & \\ & e^{x_{22}} & \\ & & \ddots \\ & & & e^{x_{nn}} \end{bmatrix}
\]
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If \( X = V Y V^{-1} \) is a change of basis of \( X \), then

\[ e^X = Ve^Y V^{-1} \]
Review - “Discretizing” a System

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Review - “Discretizing” a System

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\[
A_d = V e^{A \Delta t} V^{-1} \\
B_d = V (e^{A \Delta t} - I_n) A^{-1} V^{-1} B_c
\]
Stability

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1. State Space ("Internal") Stability
2. BIBO (Bounded Input, Bounded Output) Stability
Stability

u(t) = 0 → Internally Stable → y(t)
Stability

- Internally Stable
  - $u(t) = 0$
  - $y(t)$

- BIBO stable
  - $u(t)$
  - $y(t)$
Stability

System may be:
- stable
- unstable
- marginally stable
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