EECS 16B
Designing Information Devices and Systems II

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Announcements

- Midterm #1 redo is due tomorrow @ 11:59pm
- Please complete our mid-semester survey, due Monday 3/18 @11:59pm

Today

- Review
  - matrix exponential
  - “discretizing” a system
- Stability
- Feedback (“closed loop”) control
  - Feedback stabilization

Review - Matrix Exponential

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If $X = \mathrm{diag}(x_{11}, x_{22}, \ldots, x_{nn})$, then

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If $X = VYV^{-1}$ is a change of basis of X, then

$$e^X = Ve^Y V^{-1}$$

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$$A_d = V e^{\Lambda \Delta t} V^{-1}$$

$$B_d = V(e^{\Lambda \Delta t} - I_n)\Lambda^{-1} V^{-1} B_c$$

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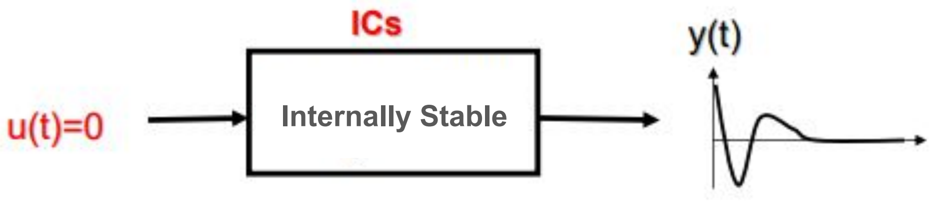
2. BIBO (Bounded Input, Bounded Output) Stability

Stability



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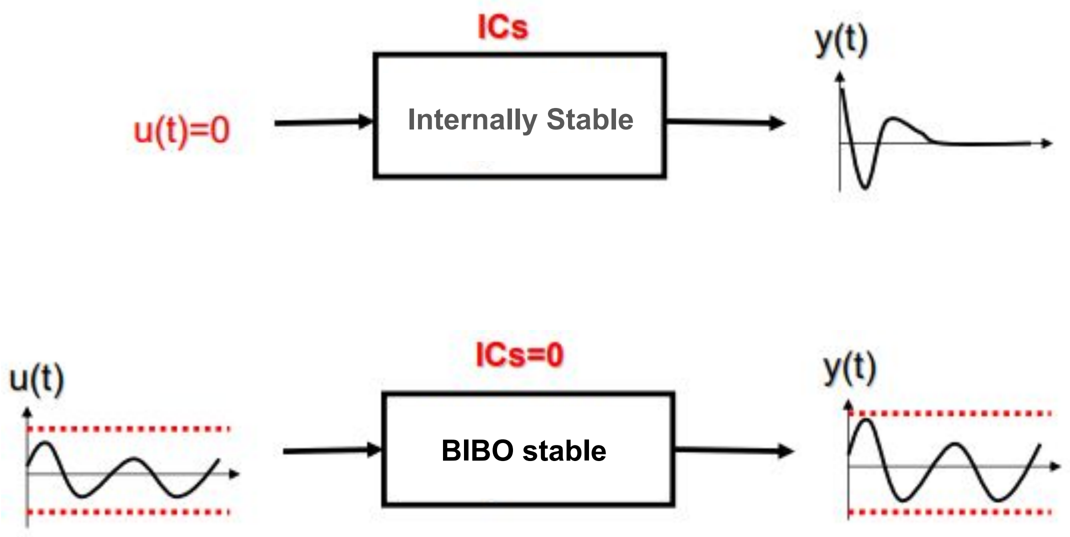

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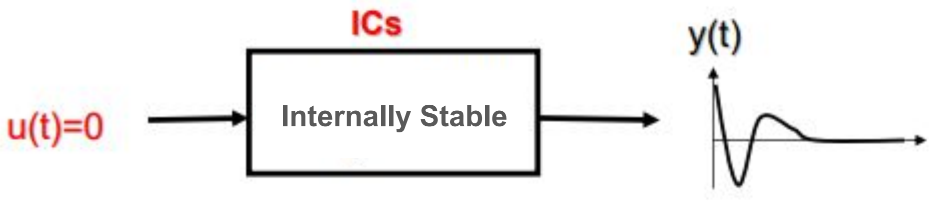

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- marginally stable

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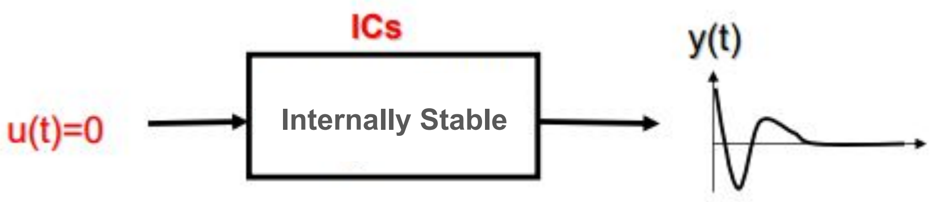

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