EECS 16B
Designing Information Devices and Systems II

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Announcements

- Midterm #1
    - Redo is due Wed 3/13 @ 11:59pm
- Lab
    - Midterm lab report due tomorrow @ 11:59pm
    - Buffer lab this week, System Identification next week
- Lots of Linear Algebra coming up
    - resource: Gil Strang lectures (YouTube playlist or MIT OCW website)

Today

- Review
  - discrete time and “discretizing” a system
  - least squares optimization (from 16A)
- System Identification

CT vs DT Duality

Continuous Time

$$\ddot{x} - 3\dot{x} + 2x = u(t)$$

differential equation

Discrete Time

$$x_i - 3x_{i-1} + 2x_{i-2} = u_i$$

difference equation

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form of homogeneous solution in CT

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“Discretization” of a System

Given a diagonalizable CT system specified by:

$$A_c = V\Lambda V^{-1} \qquad B_c$$

and a timestep Δ, the DT equivalent system is given by:

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