Announcements

- **Midterm #1**
  - Redo is due Wed 3/13 @ 11:59pm

- **Lab**
  - Midterm lab report due tomorrow @ 11:59pm
  - Buffer lab this week, System Identification next week

- **Lots of Linear Algebra coming up**
  - resource: Gil Strang lectures (YouTube playlist or MIT OCW website)
Today

- Review
  - discrete time and “discretizing” a system
  - least squares optimization (from 16A)
- System Identification
CT vs DT Duality

Continuous Time
\[ \ddot{x} - 3\dot{x} + 2x = u(t) \]
differential equation

Discrete Time
\[ x_i - 3x_{i-1} + 2x_{i-2} = u_i \]
difference equation
CT vs DT Duality

**Continuous Time**

\[ \ddot{x} - 3\dot{x} + 2x = u(t) \]

differential equation

\[ x = \sum_k c_k e^{\lambda_k t} \]
form of homogeneous solution in CT

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characteristic equation in CT

\[
\dot{x} = Ax + Bu
\]
state space form in CT

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state space form in DT
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\[ \ddot{x} = A\dot{x} + B\ddot{u} \]
state space form in CT

\[ \bar{x}(t) = e^{At} \bar{x}(0) + \int_0^t e^{A(t-\tau)} B\bar{u}(\tau) d\tau \]
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state space form in DT

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state space solution in DT
CT vs DT Duality

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“Discretization” of a System

Given a diagonalizable CT system specified by:

\[ A_c = V \Lambda V^{-1} \quad B_c \]

and a timestep \( \Delta \), the DT equivalent system is given by:

\[ A_d = Ve^{\Lambda \Delta} V^{-1} \]
\[ B_d = V(e^{\Lambda \Delta} - I_n)\Lambda^{-1}V^{-1}B_c \]
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