EECS 16B
Designing Information Devices and Systems II

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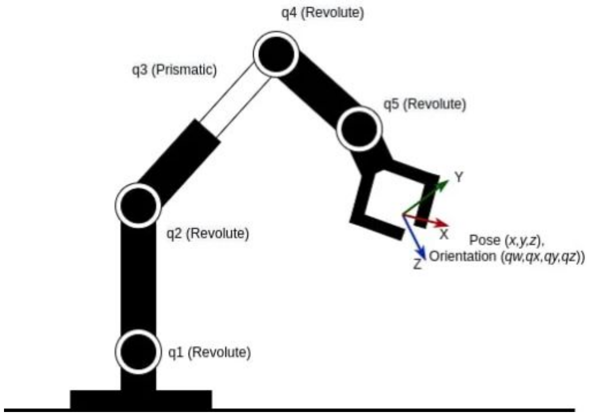
Announcements

- Midterm #1
  - We have a no-penalty final exam clobber policy. (Requires midterm redo.)
- Lab
  - Midterm lab report is due @11:59 on Wednesday 3/6
  - Buffer lab this week and next

Today

- Review
  - state space modeling
  - eigenvectors and eigenvalues
  - natural response ($\dot{x} = Ax$)
    - uncoupled case (A is diagonal)
    - general case (need to diagonalize A => $A=V\Lambda V^{-1}$)
- Matrix exponentials
- The “order” of a system / converting to and from state space form

Systems and Controls - State Space Modeling



Analysis: Given inputs and initial state, find state trajectory.

Path Planning: Given initial state and desired final state, find feasible (or optimal) state trajectory.

Control: Given desired state trajectory, find inputs.

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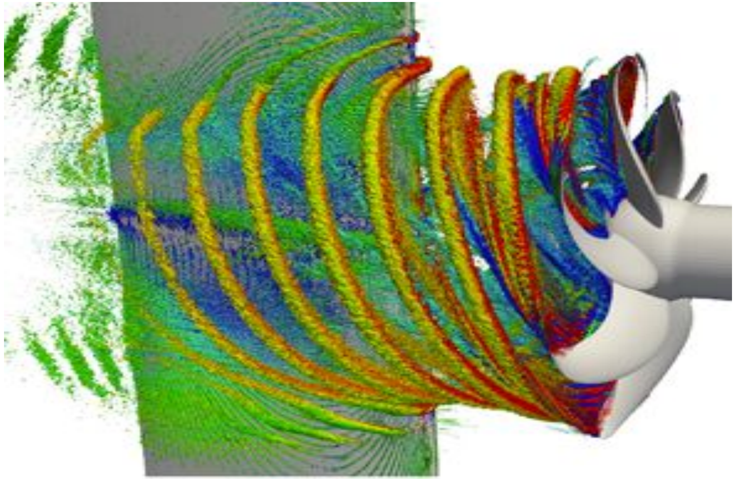

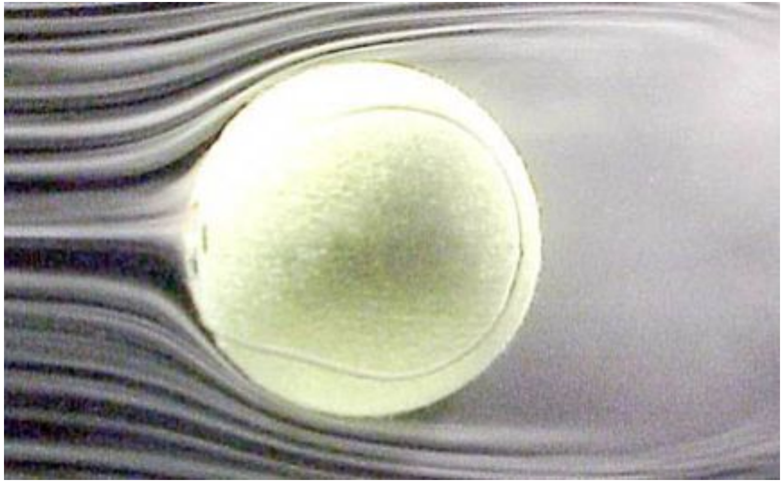

Fluid Dynamics:

- model to capture velocity, temperature, pressure, etc, for each small volume of fluid
- Are state variables coupled or uncoupled?
- How many will I need?

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9. Distinct λ_i's => eigenvectors are independent
10. $A = A^T$ (***A*** is “symmetric”) => eigenvectors are orthogonal, eigenvalues are real

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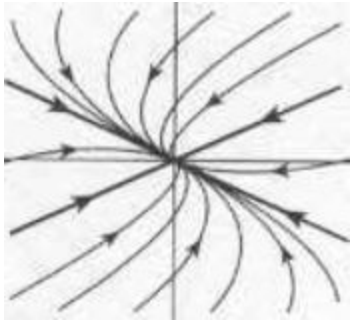
True or False: Square matrix $\boldsymbol{A}$ is invertible if and only if 1 is an eigenvalue of $\boldsymbol{A}$.

1. True
2. False

True or False: Square matrix $\boldsymbol{A}$ and its transpose $\boldsymbol{A}^{T}$ share the same eigenvectors.

1. True
2. False

What can say about the eigenvalues of the below system?



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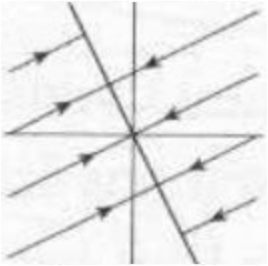

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