EECS 16B
Designing Information Devices and Systems II

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Department of Electrical Engineering and Computer Science

Announcements

- Midterm #1
  - Monday, February 26, 8-10pm
  - New material covered today will not be on MT #1
  - Review session is in Mulford 159, Wednesday 7-10pm
- Lab
  - New and Improved Lab #5 this week
  - No need to bring your car this week!

Today

- Resonant Circuits / Quality Factor - Correction from last time!
- Signals and Systems
- State Space Representation of a System
- Vector Differential Equations

Quality Factor (Q)

- Quantifies resonance, i.e. the degree to which a system that oscillates is underdamped

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- Can be defined in terms of bandwidth

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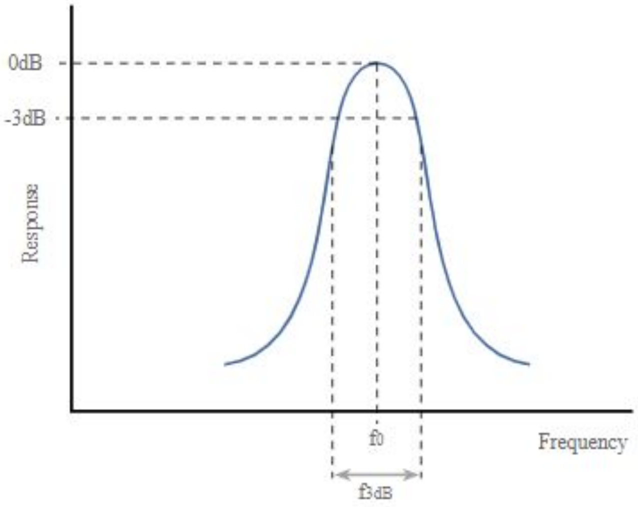
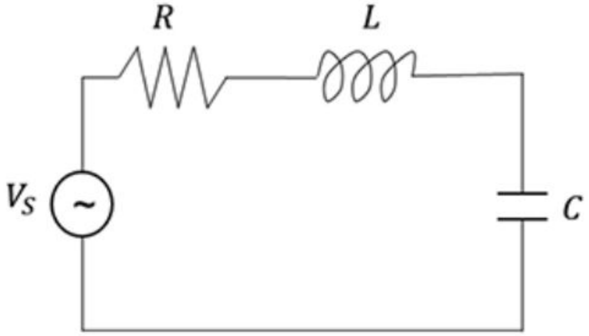


Quality Factor (Q)

- Quantifies resonance, i.e. the degree to which a system that oscillates is underdamped
- Can be defined in terms of bandwidth
- Can also be defined in terms of energy or power

$$Q \triangleq \frac{E_{\text{stored}}}{E_{\text{lost per cycle}}} = \frac{P_{\text{reactive}}}{P_{\text{avg}}}$$



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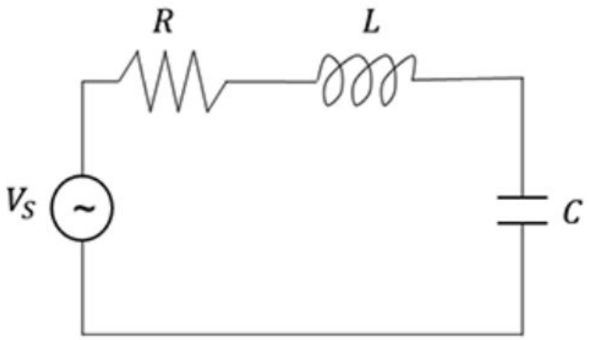

See Note 06C (optional)

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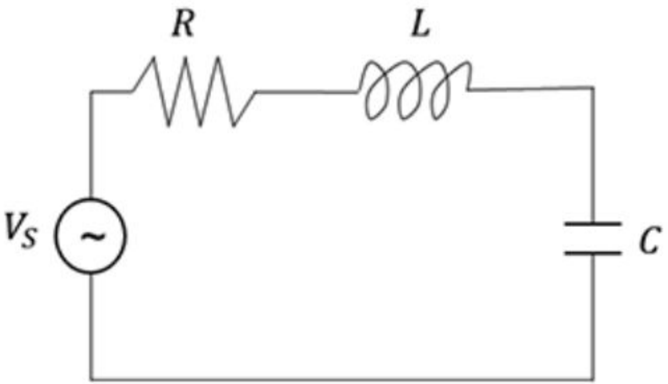
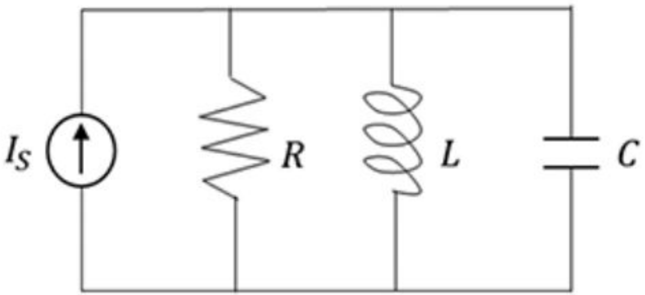


$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$



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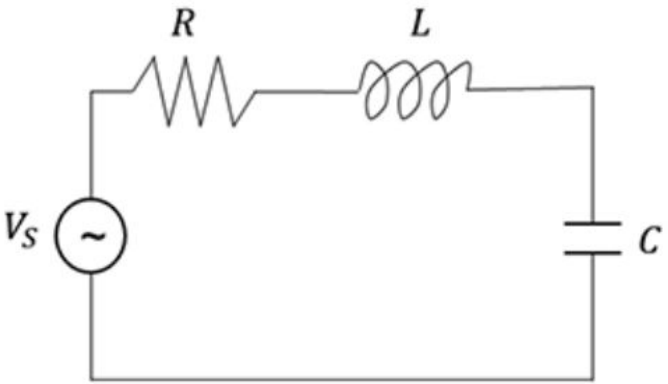

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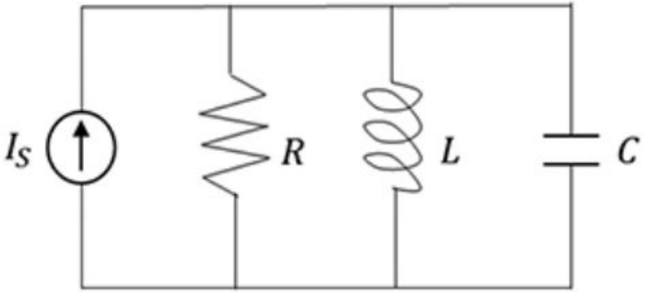

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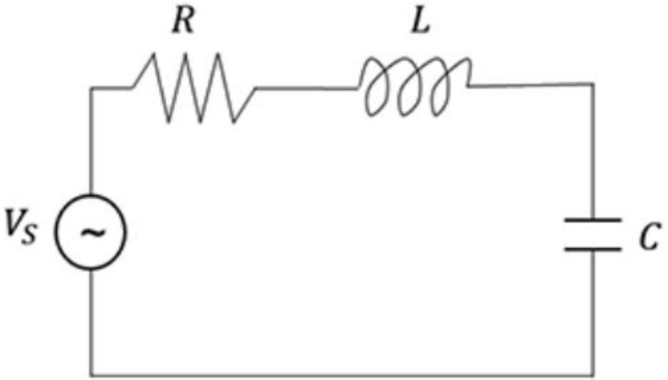

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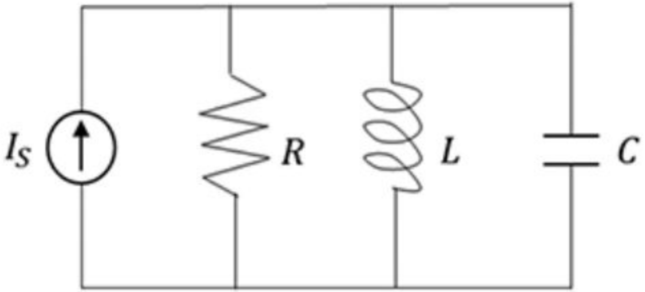

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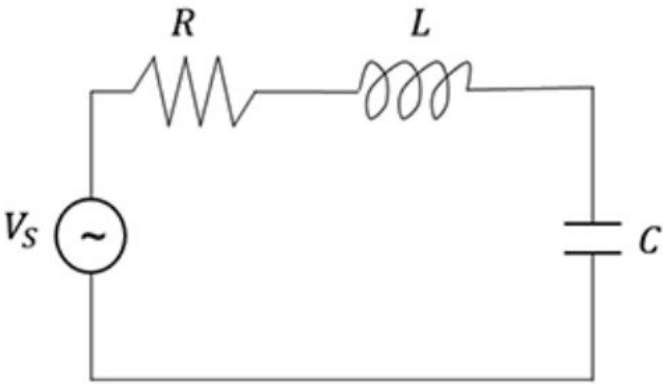

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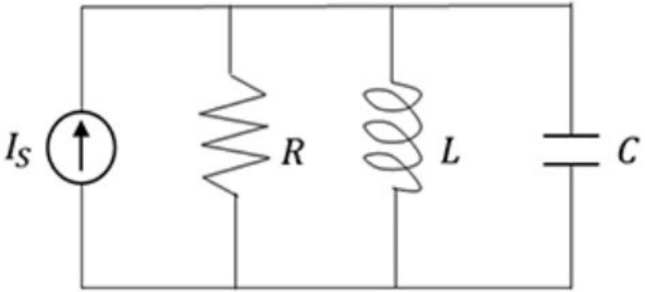

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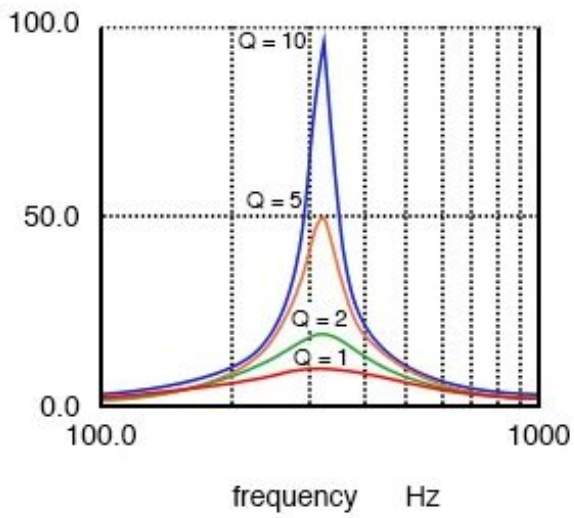

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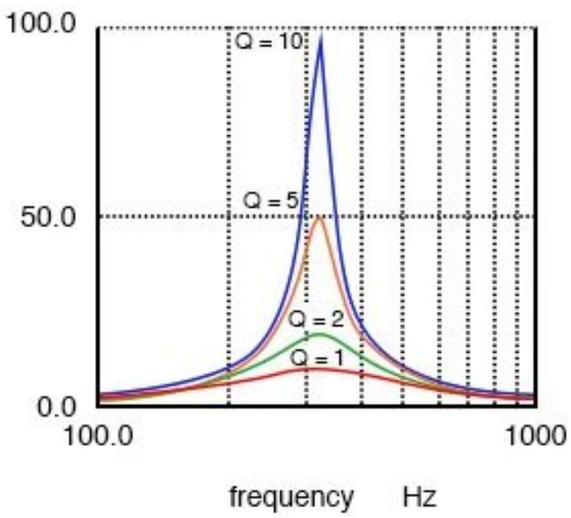

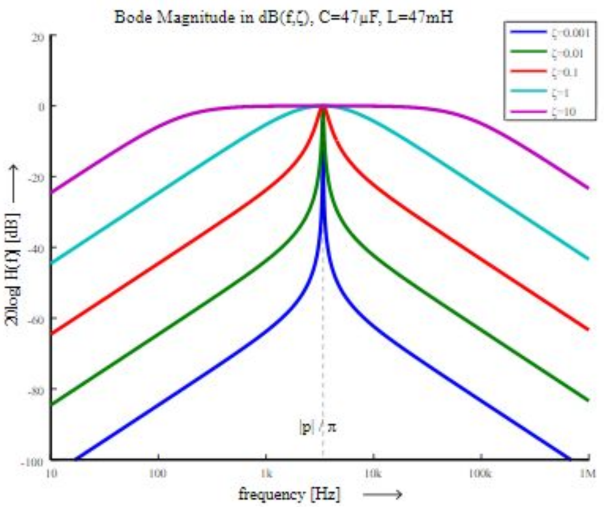

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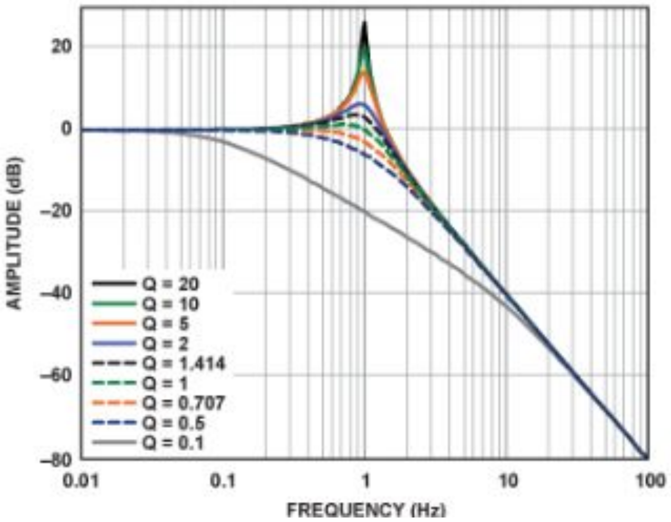

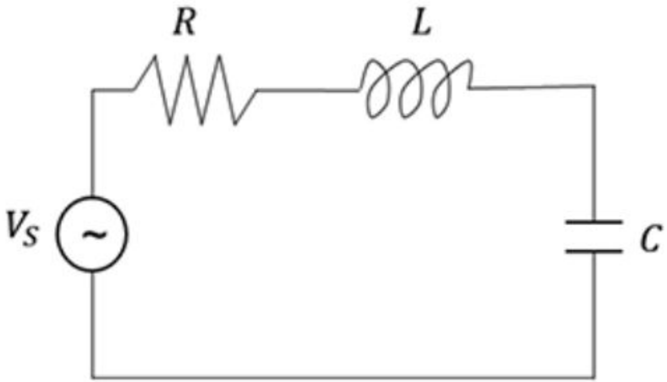

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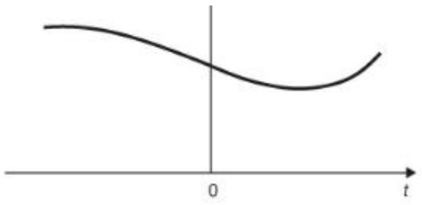
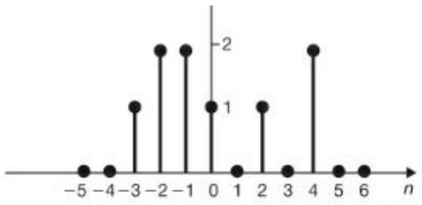
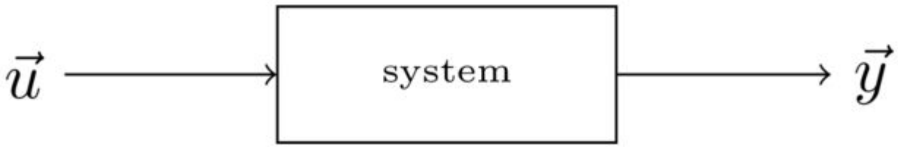
Signals and Systems







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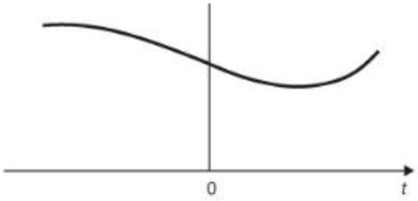
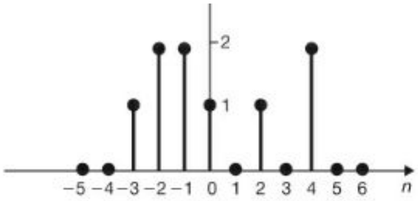
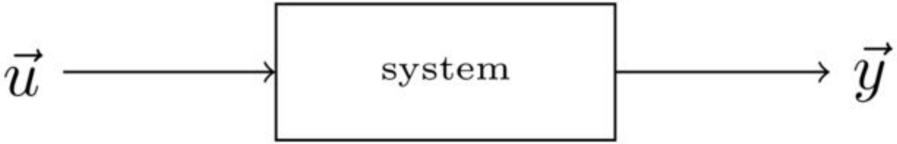


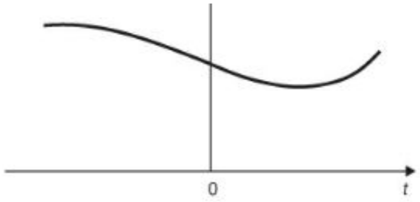


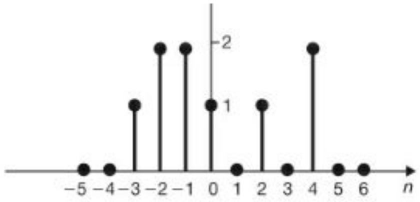


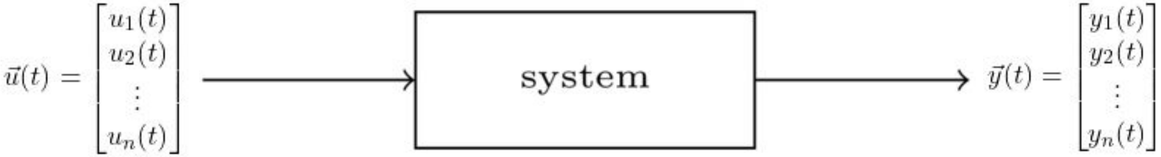
The systems we will study are generally:

- Multiple Input / Multiple Output

Signals and Systems







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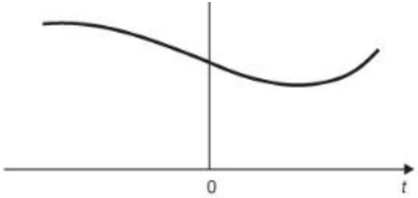

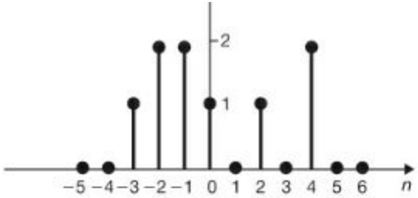

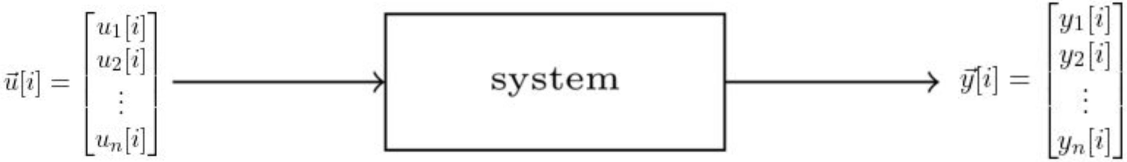

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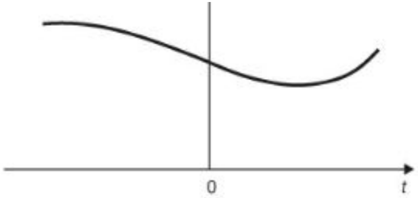
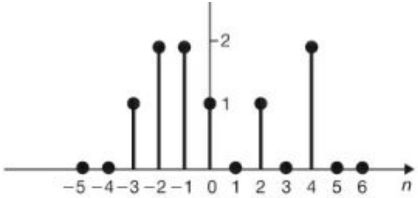
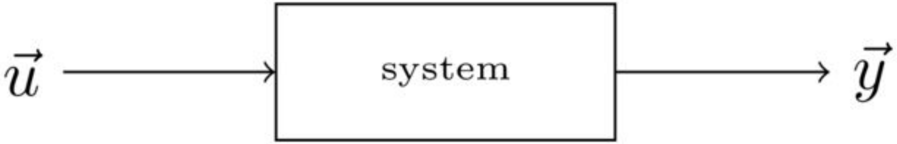


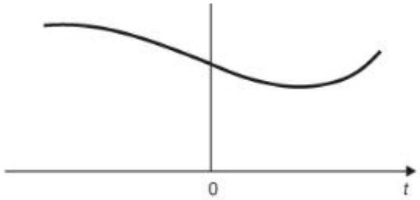


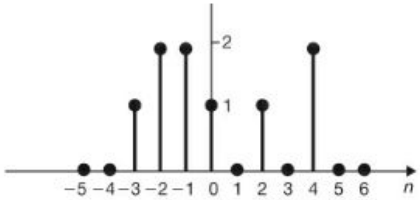


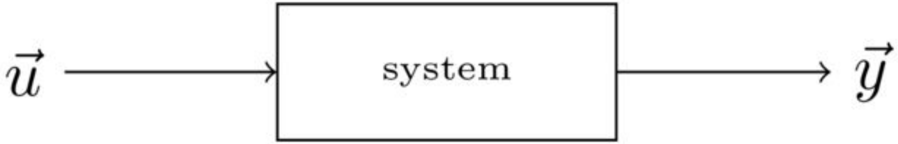
The systems we will study are generally:

- Multiple Input / Multiple Output
- Linear

Signals and Systems





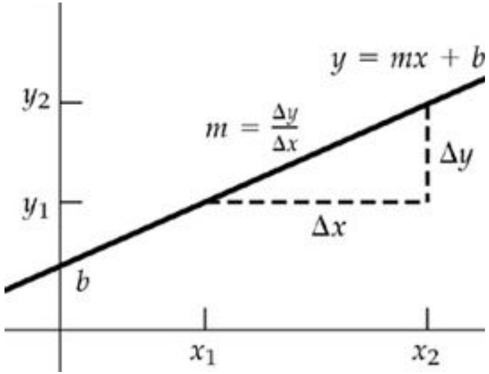


The systems we will study are generally:

- Multiple Input / Multiple Output
- Linear
- Time-Invariant

What does “linear” mean?

High School:



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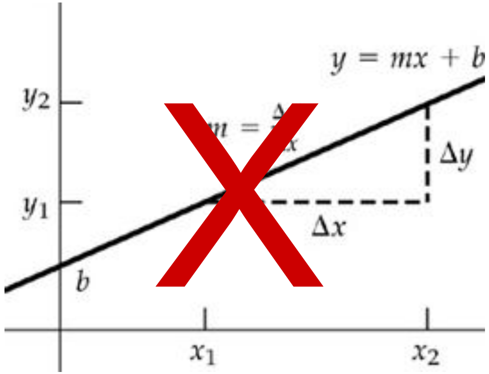

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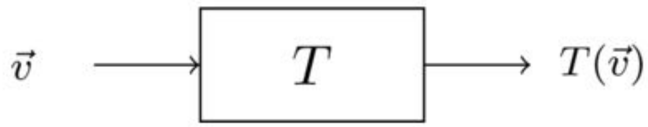

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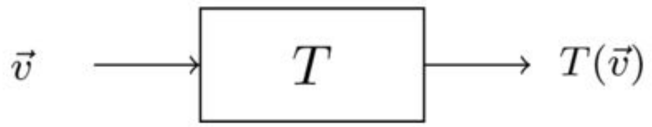

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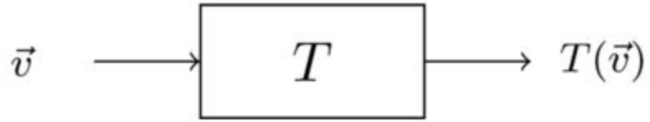

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preserves scaling

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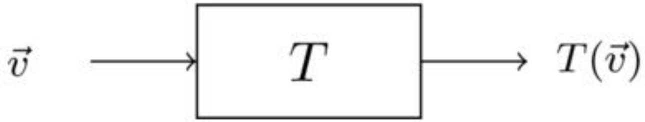

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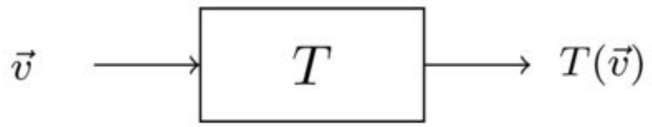

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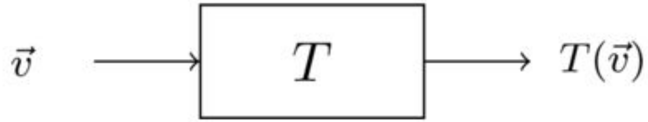

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$$T\left(C_1\vec{v} + C_2\vec{w}\right) = C_1T\left(\vec{v}\right) + C_2T\left(\vec{w}\right)$$ => preserves linear combinations

=> the “superposition” principle applies

Suppose: $f(x) = 3x + 5$

True or False: The function f represents a linear transformation on x.

1. True
2. False

Check yourself:

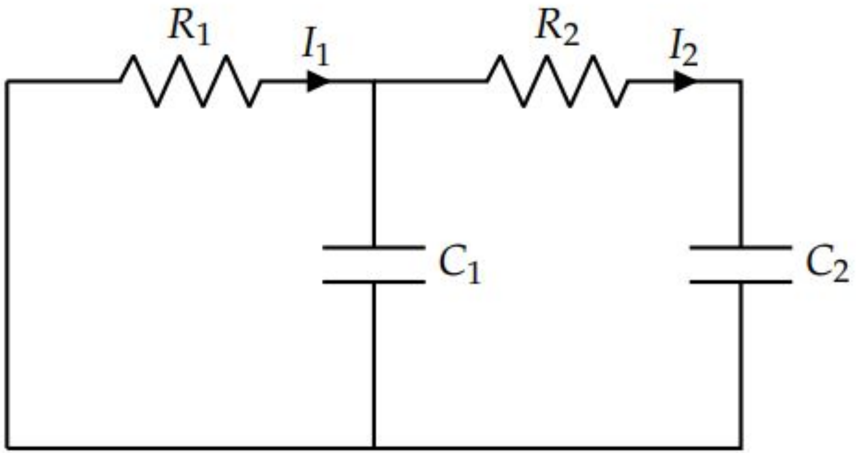
Suppose: $f(x) = 3x + 5$

True or False: The function f represents a linear transformation on x.

1. True
2. False

False. This represents a linear transformation plus a translation.
Mathematicians call this an “affine” transformation.

State Space Representation of Systems



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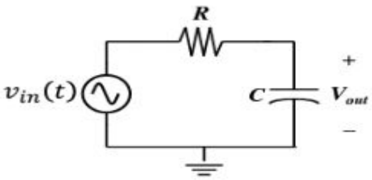

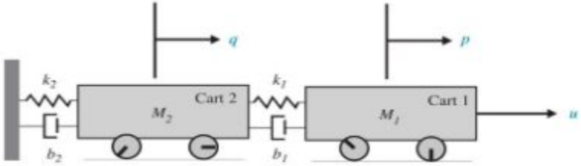

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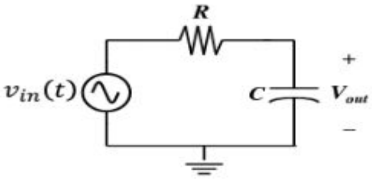

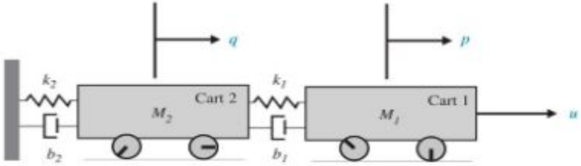

Process starts with developing a model:

1. Define inputs, outputs, state variables.

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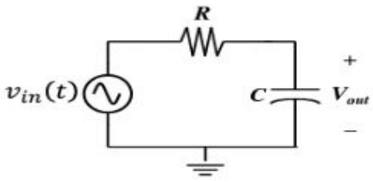

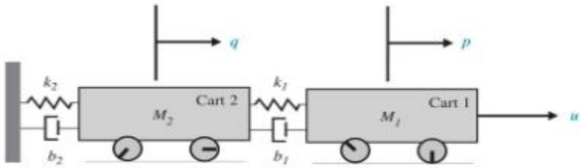

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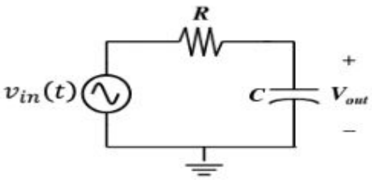

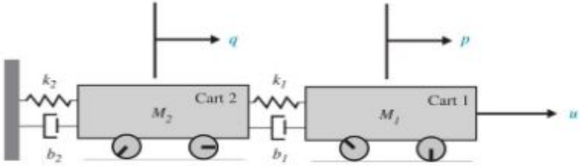

Process starts with developing a model:

1. Define inputs, outputs, state variables.
2. Write differential equations to capture system behavior.
3. Test, refine model as necessary.

State Space Representation of Systems

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State Variables:

- position (x,y,z)

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State Space Representation of Systems

State Variables:

- position (x,y,z)
- orientation (roll, pitch, yaw)
- velocities
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- etc

Inputs:

- thrust
- control surface positions

State Space Representation of Systems

State Variables:

- position (x,y,z)
- orientation (roll, pitch, yaw)
- velocities
- control surface positions
- etc

Inputs:

- thrust
- control surface positions

Outputs:

- position
- orientation
- velocities

State Space Representation / VDEs

$$\frac{d\vec{x}}{dt} = A\vec{x} + B\vec{u}$$

$$\vec{y} = C\vec{x}$$

This is what we call a vector differential equation.

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Compare to: $\frac{dx}{dt} = \lambda x + bu(t)$

State Space Representation / VDEs

$\vec{x} \in \mathbb{R}^n$

$$\begin{aligned} \frac{d\vec{x}}{dt} &= A\vec{x} + B\vec{u} \\ \vec{y} &= C\vec{x} \end{aligned}$$

This is what we call a vector differential equation.

Compare to: $\frac{dx}{dt} = \lambda x + bu(t)$

State Space Representation / VDEs

$\vec{x} \in \mathbb{R}^n$

$A \in \mathbb{R}^{n \times n}$

$$\frac{d\vec{x}}{dt} = A\vec{x} + B\vec{u}$$

$$\vec{y} = C\vec{x}$$

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Compare to: $\frac{dx}{dt} = \lambda x + bu(t)$

State Space Representation / VDEs

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$A \in \mathbb{R}^{n \times n}$

$\vec{u} \in \mathbb{R}^m$

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This is what we call a vector differential equation.

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- We often ignore the output equation and treat the states as the outputs.

State Space Representation / VDEs

Our state-space model is Linear Time Invariant (LTI) => A, B, and C are matrices of *constants*:

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The time-invariance assumption is often a good one. However, almost everything is <u>non-linear</u>.

Why are we focusing on linear systems?

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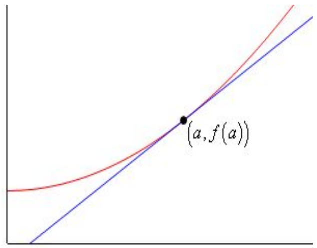

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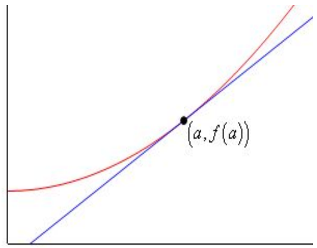
Why are we focusing on linear systems?

- Nonlinear systems can often be approximated using a linear model, especially locally.
- Linear systems are well-understood, and much easier to work with.
  - superposition
  - tools (Fourier, Laplace, Phasors, etc)



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2. **Path Planning:** Given an initial state and a desired final state, find feasible (or optimal) state trajectory.
3. **Control:** Given a desired state trajectory, find inputs to achieve that trajectory.