EECS 16B
Designing Information Devices and Systems II

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Announcements

- MT 2:
  - grades will be released in ~ 1 week
  - redo will be released in ~ 2 days (required to qualify for final exam full clobber)

Today

- review
- applications of the SVD
  - Moore-Penrose pseudoinverse
  - least squares
  - minimum energy control
  - Eckart-Young low rank approximation

Singular Value Decomposition

$$A = U\Sigma V^T$$

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$$AV = U\Sigma$$

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$$A \begin{bmatrix} | & & | \\ v_1 & \cdots & v_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_m \\ | & & | \end{bmatrix} \left[\begin{array}{ccc} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \\ \hline 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{array} \right]$$

Singular Value Decomposition

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$$AV = U\Sigma$$

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$$Av_i = \sigma_i u_i$$

Singular Value Decomposition

To calculate the SVD:

- A^TA => v_i's and σ_i's

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Singular Value Decomposition

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(how do we know A^TA can be diagonalized?)

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Singular Value Decomposition

To calculate the SVD:

- A^TA => v_i's and σ_i's
- use $u_i = Av_i/\sigma_i$ to find u_i's

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To calculate the SVD:

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$$Av_i = \sigma_i u_i$$

Wide A => do the reverse:

- AA^T => u_i's and σ_i's
- use $v_i = A^Tu_i/\sigma_i$ to find u_i's

Singular Value Decomposition - Geometry

$$A \begin{bmatrix} | & & | \\ v_1 & \cdots & v_r \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_r \\ | & & | \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix}$$

Singular Value Decomposition - Geometry

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$$Ax = U\Sigma V^T x$$

Singular Value Decomposition - Geometry

$$A\begin{bmatrix} | & & | \\ v_1 & \cdots & v_r \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_r \\ | & & | \end{bmatrix}\begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix}$$

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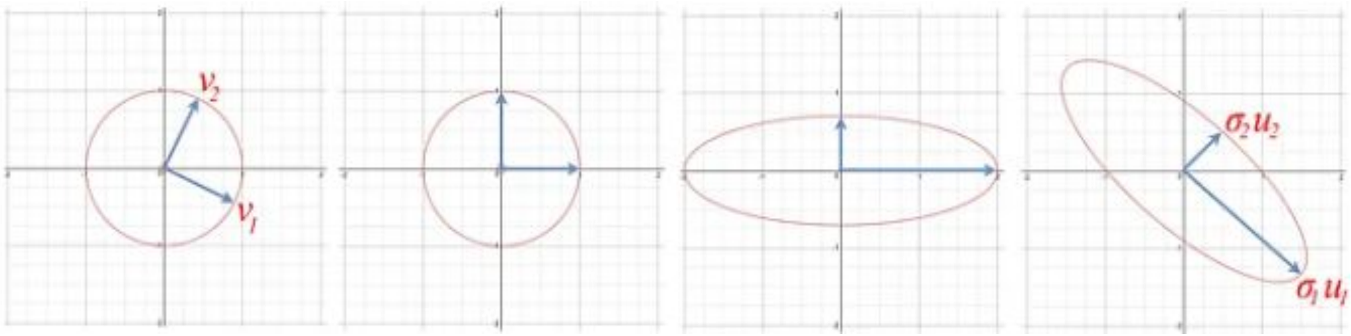

Singular Value Decomposition - Geometry

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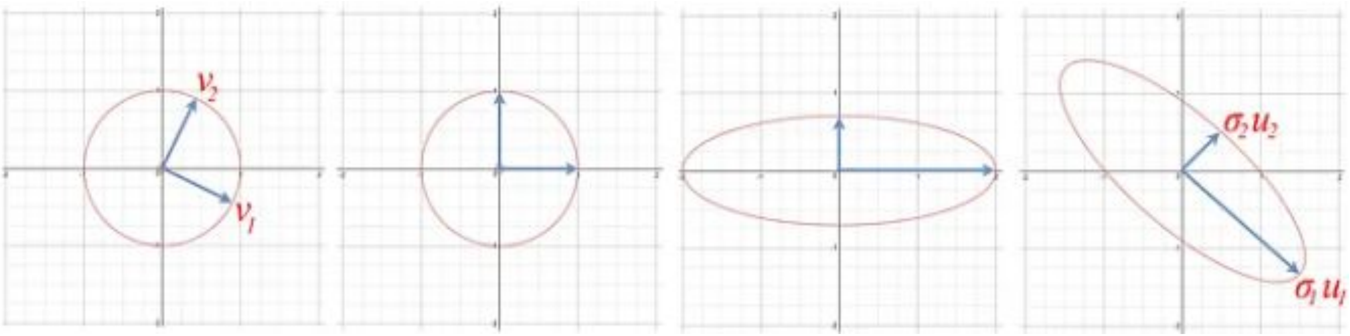
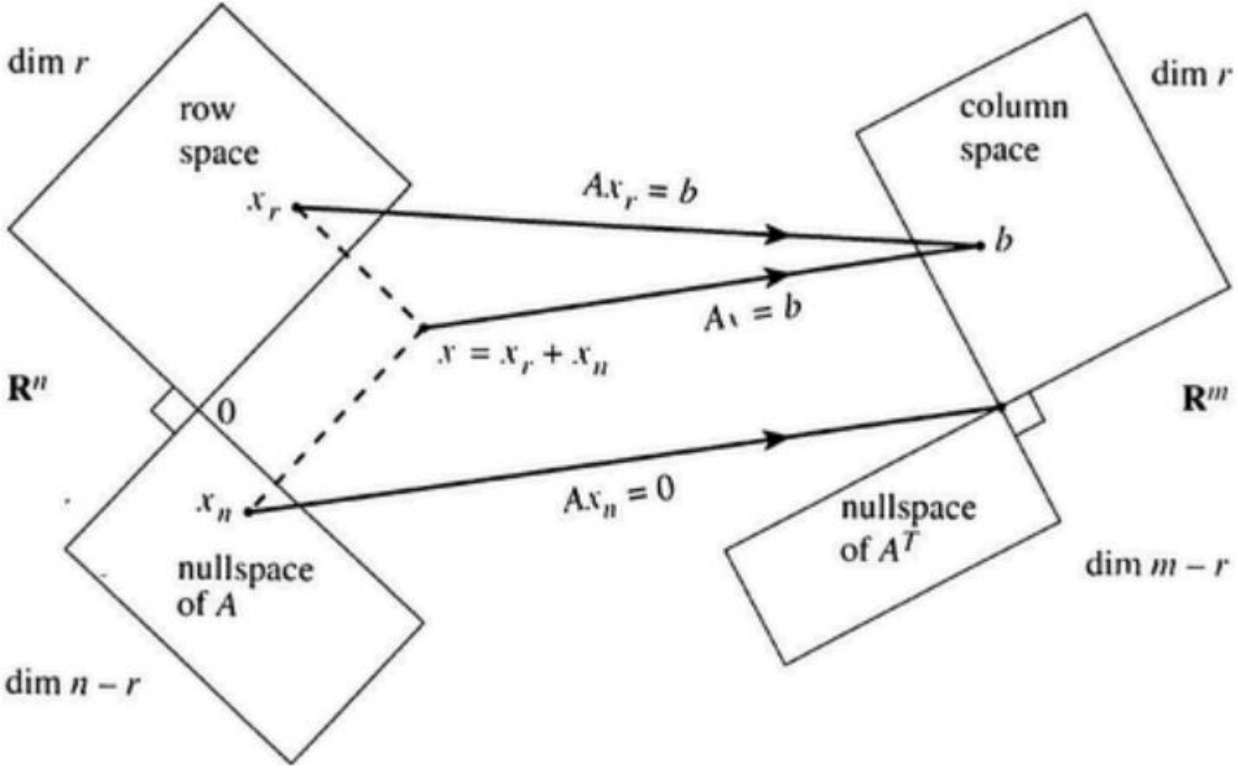


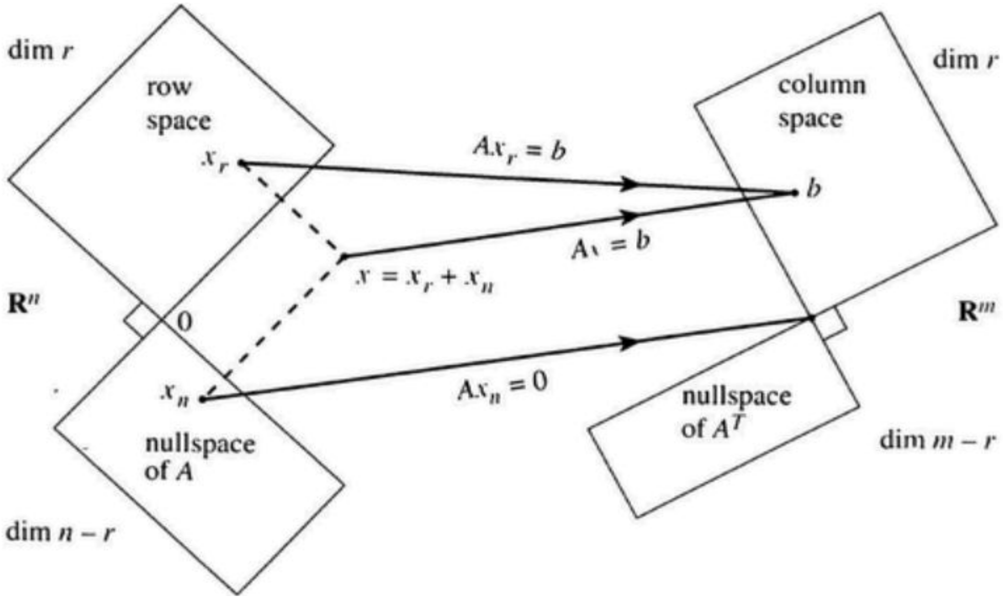
Note that Σ may add or subtract dimensions.

Four Fundamental Spaces of a Matrix



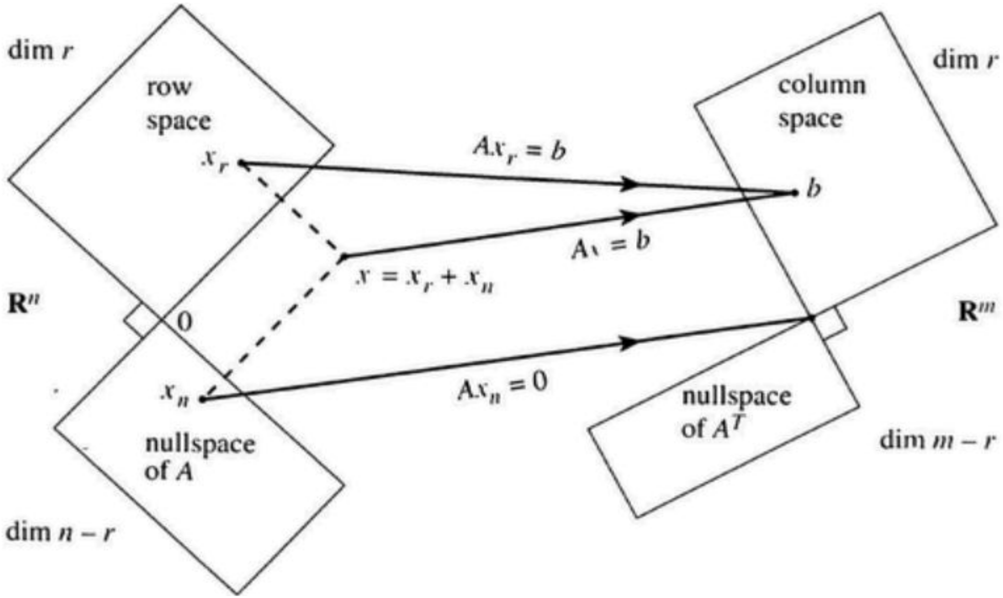
True or False: A 3-dimensional space can be divided into two mutually orthogonal subspaces consisting of perpendicular 2-dimensional planes.

1. True
2. False



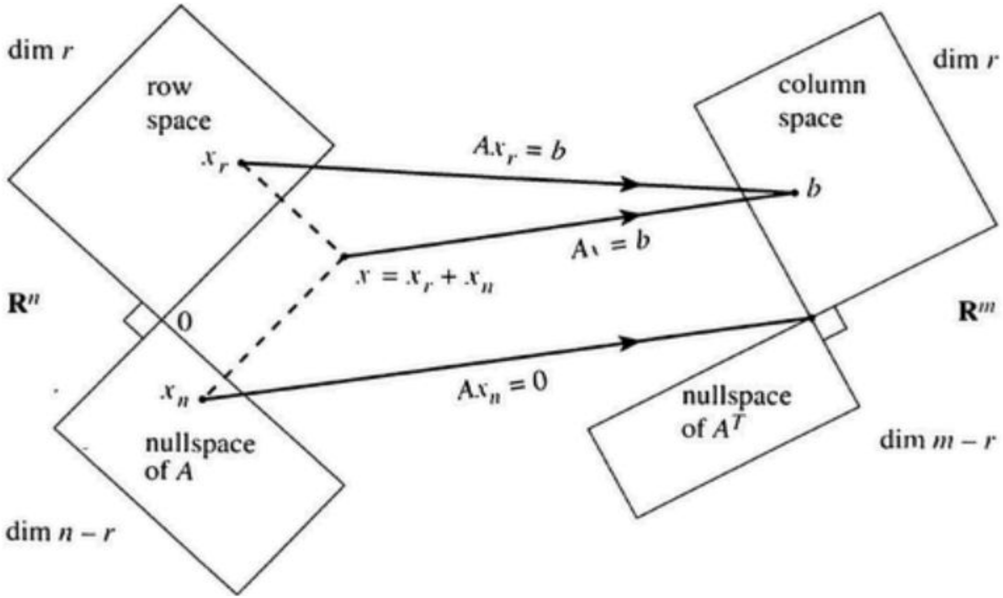
True or False: An invertible matrix can have neither a nullspace nor a left nullspace (other than the zero vector).

1. True
2. False



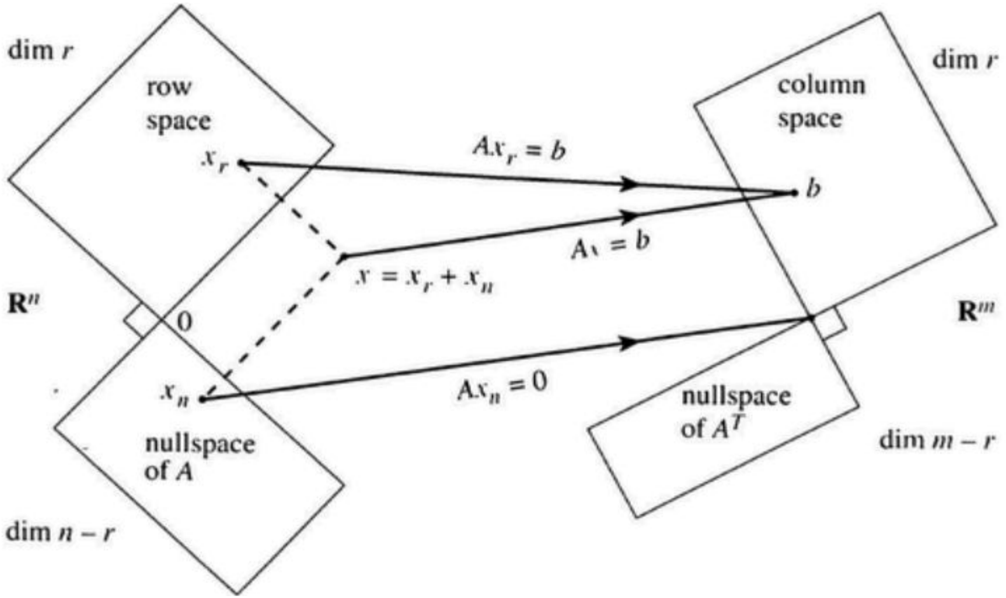
True or False: Ax = b has an infinite number of solutions if and only if A has a nontrivial nullspace.

1. True
2. False

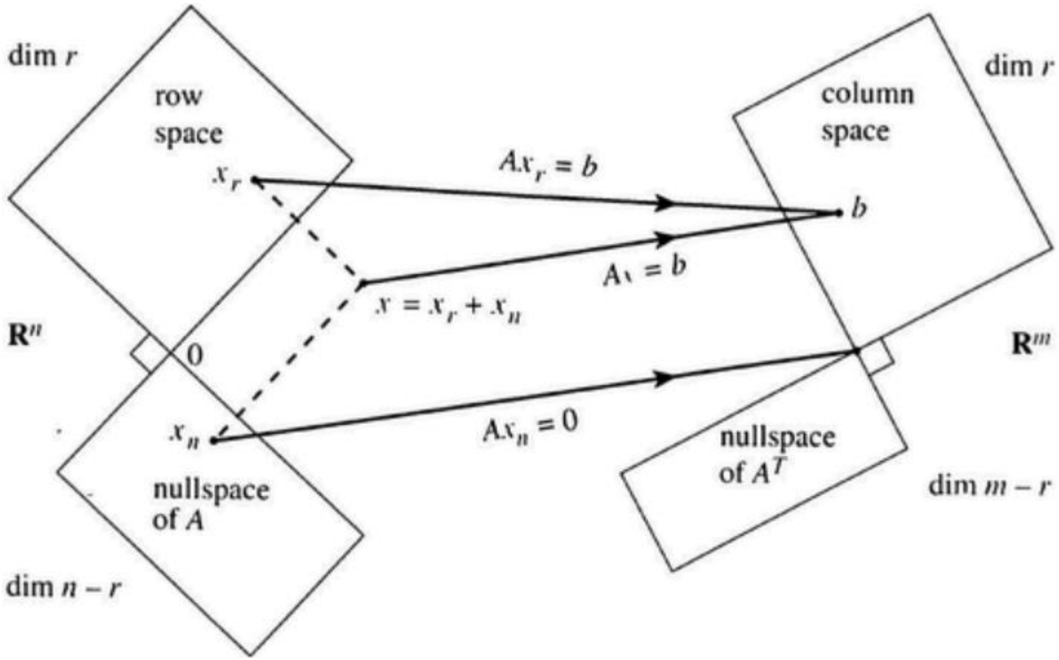


True or False: A matrix with a nontrivial nullspace provides a 1-to-1 mapping between its row space and its column space.

1. True
2. False



Moore-Penrose Matrix Pseudoinverse



Moore-Penrose Matrix Pseudoinverse