EECS 16B
Designing Information Devices and Systems II

Profs. Miki Lustig and JP Tennant
Department of Electrical Engineering and Computer Science

Announcements

- MT 2: Monday 7-9pm
  - covers lectures through Spectral Theorem
  - does not cover minimum energy control, SVD
- Discussion on Monday will be review / Q&A

Today

- review
- more on A^TA and AA^T (positive semidefinite matrices)
- geometric interpretation of SVD
- four fundamental spaces of a matrix / Fundamental Theorem of Linear Algebra (FTLA)

Singular Value Decomposition

$$A = U\Sigma V^T$$

Singular Value Decomposition

$$A = U\Sigma V^T$$

$$A = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_m \\ | & & | \end{bmatrix} \left[\begin{array}{ccc} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \\ \hline 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{array}\right] \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_n^T & - \end{bmatrix}$$

Singular Value Decomposition

$$A = U\Sigma V^T$$

$$A = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_m \\ | & & | \end{bmatrix} \left[\begin{array}{ccc} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \\ \hline 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{array}\right] \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_n^T & - \end{bmatrix}$$

Note: the smallest singular values may be zero

Singular Value Decomposition

$$A = U\Sigma V^T$$

$$A = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_m \\ | & & | \end{bmatrix} \left[\begin{array}{ccc} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \\ \hline 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{array}\right] \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_n^T & - \end{bmatrix}$$

Note: the smallest singular values may be zero

Can we make this representation more compact?

Singular Value Decomposition

$$A = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_m \\ | & & | \end{bmatrix} \left[\begin{array}{ccc} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \\ \hline 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{array}\right] \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_n^T & - \end{bmatrix}$$

Singular Value Decomposition

$$A = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_m \\ | & & | \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \\ \hline 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_n^T & - \end{bmatrix}$$

$$A = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_r \\ | & & | \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix} \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_r^T & - \end{bmatrix}$$

Singular Value Decomposition

$$A = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_r \\ | & & | \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix} \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_r^T & - \end{bmatrix}$$

Singular Value Decomposition

$$A = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_r \\ | & & | \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix} \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_r^T & - \end{bmatrix}$$

$$A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_r u_r v_r^T$$

Singular Value Decomposition

$$A = \begin{bmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ u_1 & \cdots & u_r \\ | & & | \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix} \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_r^T & - \end{bmatrix}$$

$$\begin{aligned} A &= \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_r u_r v_r^T \\ &= \sum_{i=1}^{r} \sigma_i u_i v_i^T \end{aligned}$$