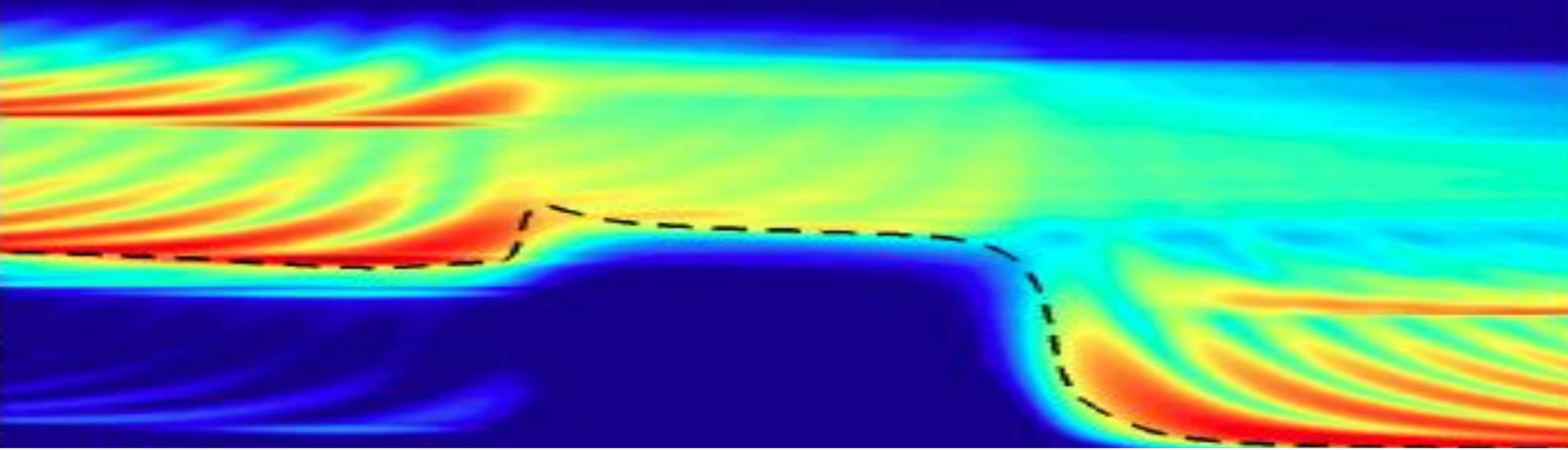


EECS 16B
Designing Information Devices and Systems II
Lecture 5

Prof. Sayeef Salahuddin
Department of Electrical Engineering and Computer Sciences, UC Berkeley,
sayeef@eecs.berkeley.edu

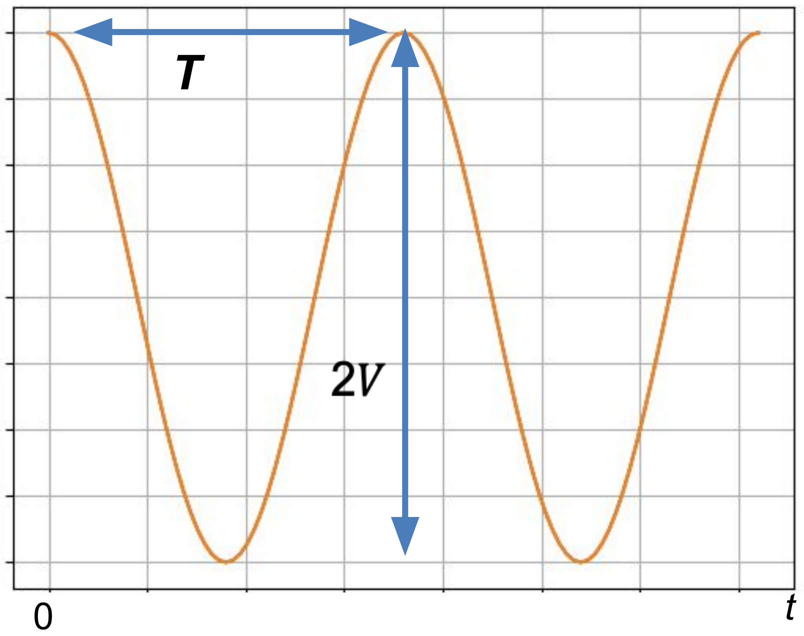
Transient Response

- Outline
  - Phasors
  - Complex Impedances
  - Solution of circuits using complex impedances

- Reading- Hambley text sections 5.2, 5.3,5.4, 5.6, 5.5, slides

Recap: Sinusoidal voltages

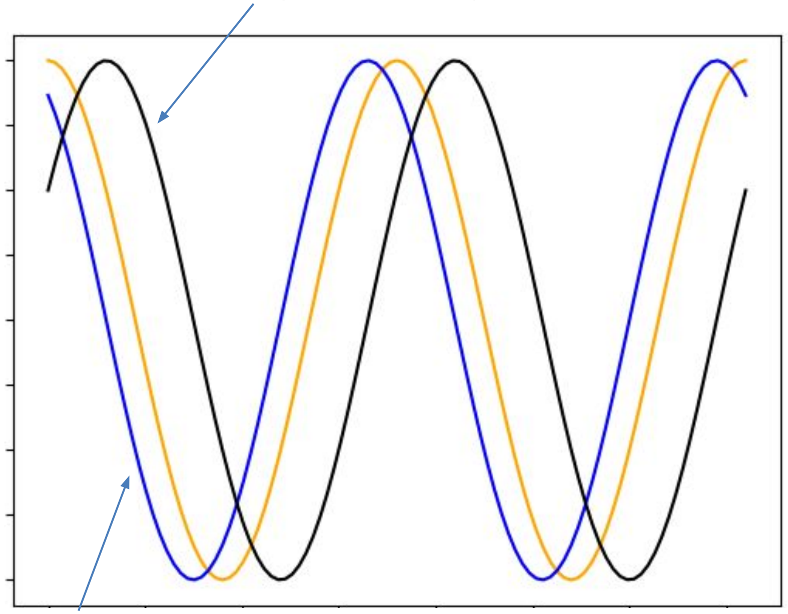
$v(t) = V cos(\omega t)$



T : Period

$$\omega = \frac{2\pi}{T}$$

$v(t) = V cos(\omega t - 60^0)$



$v(t) = V cos(\omega t + 30^0)$

Recap: How do we add arbitrary sinusoids?

$$v(t) = 10cos\omega t + 5sin\omega t - 5\cos(\omega t - 30^0)$$

$$v(t) = 10cos\omega t + 5\cos(\omega t - 90) - 5\cos(\omega t - 30^0)$$

Remember? cos (a + b) = cos a cos b - sin a sin b

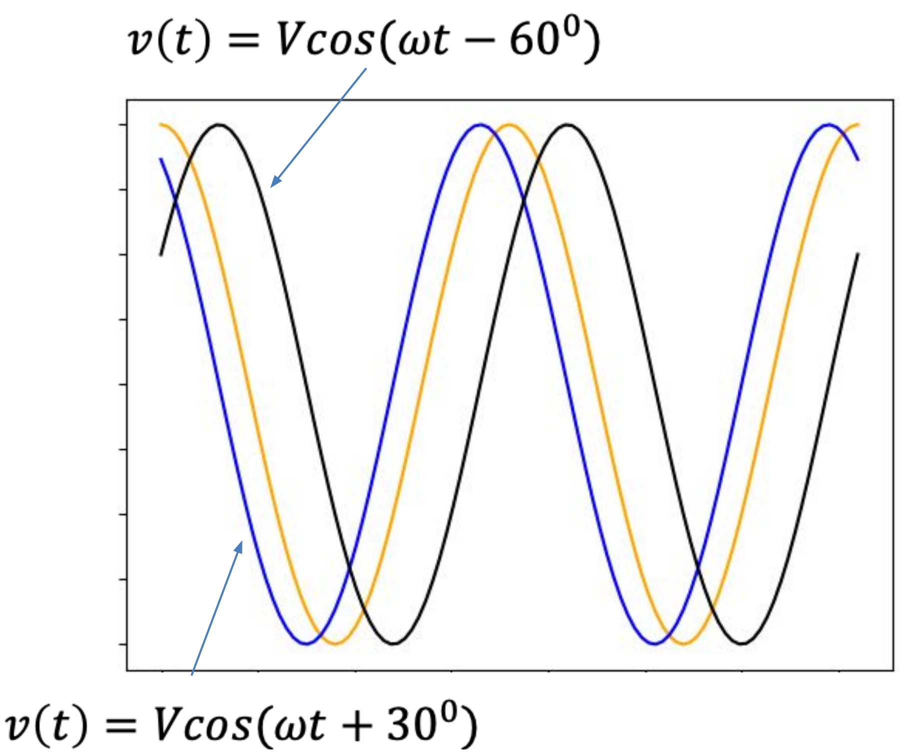
Lets do it it differently $e^{j\theta} = cos\theta + jsin\theta$

$$e^{-j\theta} = cos\theta - jsin\theta$$

Then

$$cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$sin\theta = \frac{1}{2}\left(e^{j\theta} - e^{-j\theta}\right)$$



Recap: How do we add arbitrary sinusoids?

$$v(t) = 10cos\omega t + 5\cos(\omega t - 90^0) - 5\cos(\omega t - 30^0)$$

$$= \frac{1}{2}e^{j\omega t}[10] + \frac{1}{2}e^{j(\omega t-90^0)}[5] - \frac{1}{2}e^{j(\omega t-30^0)}[5]$$

$$+ \frac{1}{2}e^{-j\omega t}[10] + \frac{1}{2}e^{-j(\omega t-90^0)}[5] - \frac{1}{2}e^{-j(\omega t-30^0)}$$

$$cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

$$sin\theta = \frac{1}{2}(e^{j\theta} - e^{-j\theta})$$

$$= \frac{1}{2}e^{j\omega t}[10 + 5e^{-j90} - 5e^{-j30}] + \frac{1}{2}e^{-j\omega t}[10 + 5e^{+j90} - 5e^{+j30}]$$

$$= \frac{1}{2}e^{j\omega t}[10 + 5cos90 - j5sin90 - 5cos30 + j5sin30] + cc$$

$$= \frac{1}{2}e^{j\omega t}\left[10 + 0 - j5 - 5\frac{\sqrt{3}}{2} + \frac{j5}{2}\right] + cc$$

$$= \frac{1}{2}e^{j\omega t}[5.66 - j2.5] + cc$$

$$= \frac{1}{2}6.18e^{j(\omega t-23^0)} + cc$$

$$= 6.18\cos(\omega t - 23^0)$$

$$Ae^{-j\theta} = 5.66 - j2.5$$

$$Ae^{j\theta} = 5.66 + j2.5$$

$$A^2 = 5.66^2 - j^2 2.5^2$$

$$A^2 = 5.66^2 + 2.5^2$$

$$A = \sqrt{5.66^2 + 2.5^2} = 6.18$$

$$cos\theta = 5.66/6.18; sin\theta = 2.5/6.18$$

$$tan\theta = \frac{2.5}{5.66} = 0.44$$

$$\theta = 0.41 = 23^0$$

Recap: Some Observations

$$v(t) = \frac{1}{2}e^{j\omega t}[5.66 - j2.5] + cc$$

$$= \frac{1}{2}6.18e^{j(\omega t - 23^0)} + \frac{1}{2}6.18e^{-j(\omega t - 23^0)}$$

$$= \frac{1}{2}6.18[\cos(\omega \mathrm{t} - 23^0) + jsin(\omega t - 23^0)\cos(\omega \mathrm{t} - 23^0) - jsin(\omega t - 23^0)]$$

$$= Real\ [6.18e^{j(\omega t - 23^0)}]$$

Phasors

In short hand, it is represented as $6.18\ \angle - 23^0$

Some Observations

$$A(t) = 5\cos(\omega t) = Real[5e^{j(\omega t)}]$$

$$B(t) = 5\cos(\omega t - 90^0) = Real[5e^{j(\omega t - 90^0)}]$$

$$C(t) = 5\cos(\omega t + 90^0) = Real[5e^{j(\omega t + 90^0)}]$$

At any given time *t, B(t)* is trailing or lagging behind *A(t)* by 90^0 while *C(t)* is leading *A(t)* by the same amount

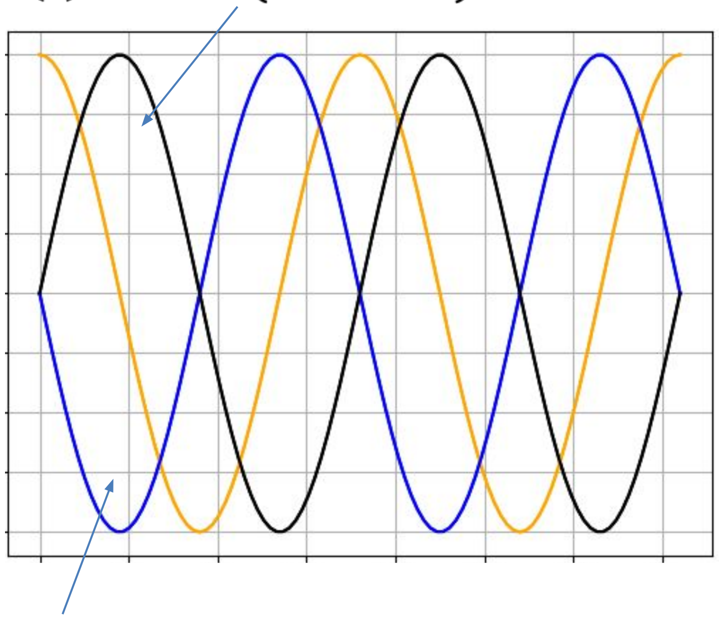
Let us now look at the phasors at *t=0*

$$5e^{j(\omega t - 90^0)} = 5$$

$$5e^{j(\omega t - 90^0)} = 5(cos90^0 - jsin90^0) = -j5$$

$$5e^{j(\omega t + 90^0)} = 5(cos90^0 + jsin90^0) = +j5$$

$v(t) = Vcos(\omega t - 90^0)$

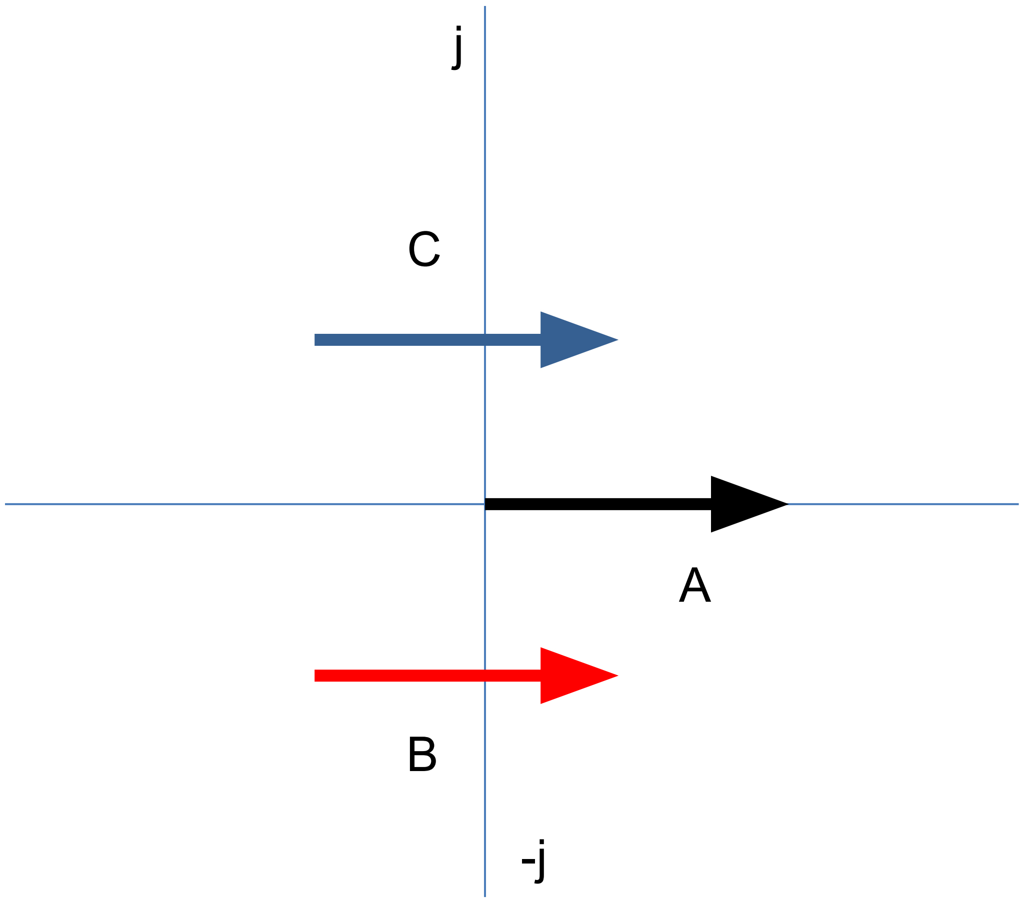


$v(t) = Vcos(\omega t + 90^0)$

Therefore *+j or –j* signifies signals having 90^0 phase lead or lag respectively.

Some Observations

+j or –j signifies signals having 90^0 phase lead or lag respectively.



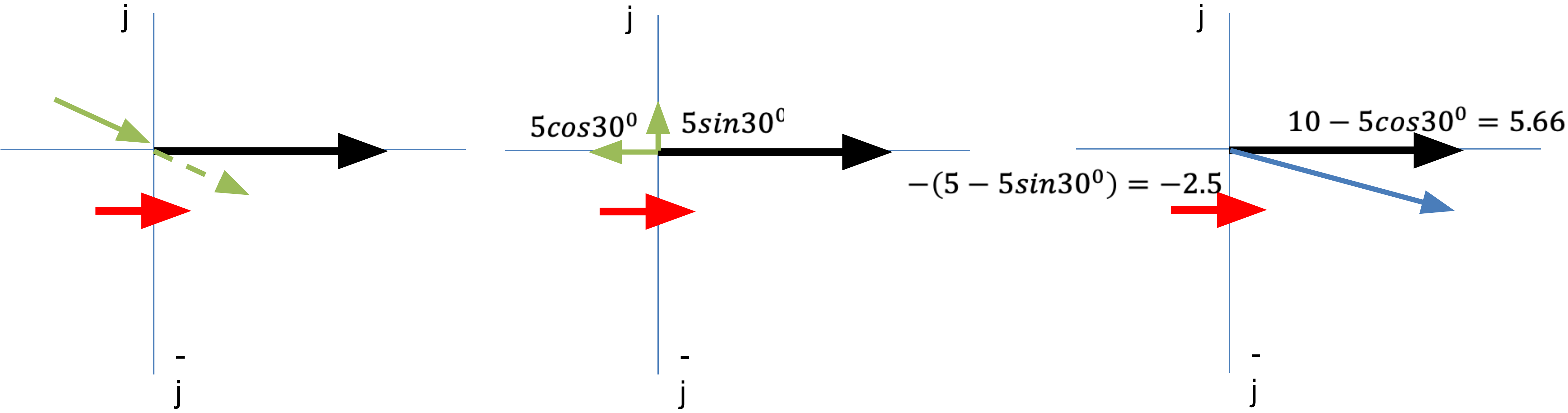
$$5e^{j(\omega t-90^0)} = 5$$
$$5e^{j(\omega t-90^0)} = 5(cos90^0 - jsin90^0) = -j5$$
$$5e^{j(\omega t+90^0)} = 5(cos90^0 + jsin90^0) = +j5$$

Some Observations

$$v(t) = 10cos\omega t + 5\cos(\omega t - 90^0) - 5\cos(\omega t - 30^0)$$

$$= 10 + 5\angle - 90^0 - 5\angle - 30^0$$ In phasor notation

$$= 6.18\angle - 23^0$$



Phasors are like vectors where the phase angle denotes the angle between coordinate axes with *j* representing 90^0

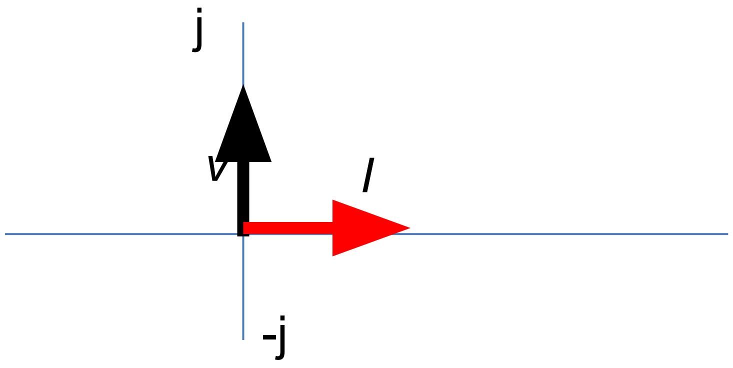
Some Observatons

- What is $\frac{1}{\angle\theta}$?

Complex Impedances

Inductance:

Say a sinusoidal current is flowing in a circuit with inductance



$$i(t) = I_0 \sin(\omega t) = I_0 \cos(\omega t - 90^0)$$

$$v_L(t) = L\frac{di}{dt} = \omega L I_0 cos\omega t$$

Therefore, the current in an inductor lags the voltage by 90^0

In the phasor notation

$$\boldsymbol{V} = \omega L I_0$$
$$\boldsymbol{I} = I_0 \angle -90^0$$

Then, inductive impedance

$$\boldsymbol{Z_L} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{\omega L}{\angle -90^0} = \frac{\omega L}{-j} = j\omega L$$

We could have obtained the same result working directly with exponentials

$$\boldsymbol{V} = L\frac{d}{dt}\left[I_0 e^{j(\omega t - 90^0)}\right]$$

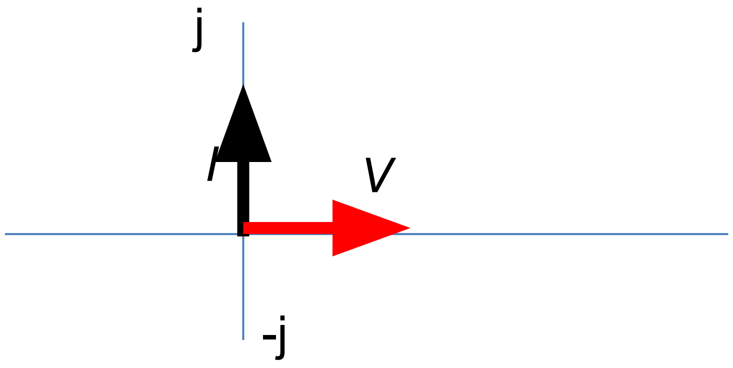
$$\boldsymbol{V} = Lj\omega\left[I_0 e^{j(\omega t - 90^0)}\right]$$

$$\boldsymbol{V} = j\omega L I_0 \angle -90^0$$

$$\boldsymbol{V} = j\omega L \boldsymbol{I}$$

Complex Impedances

Capacitance:



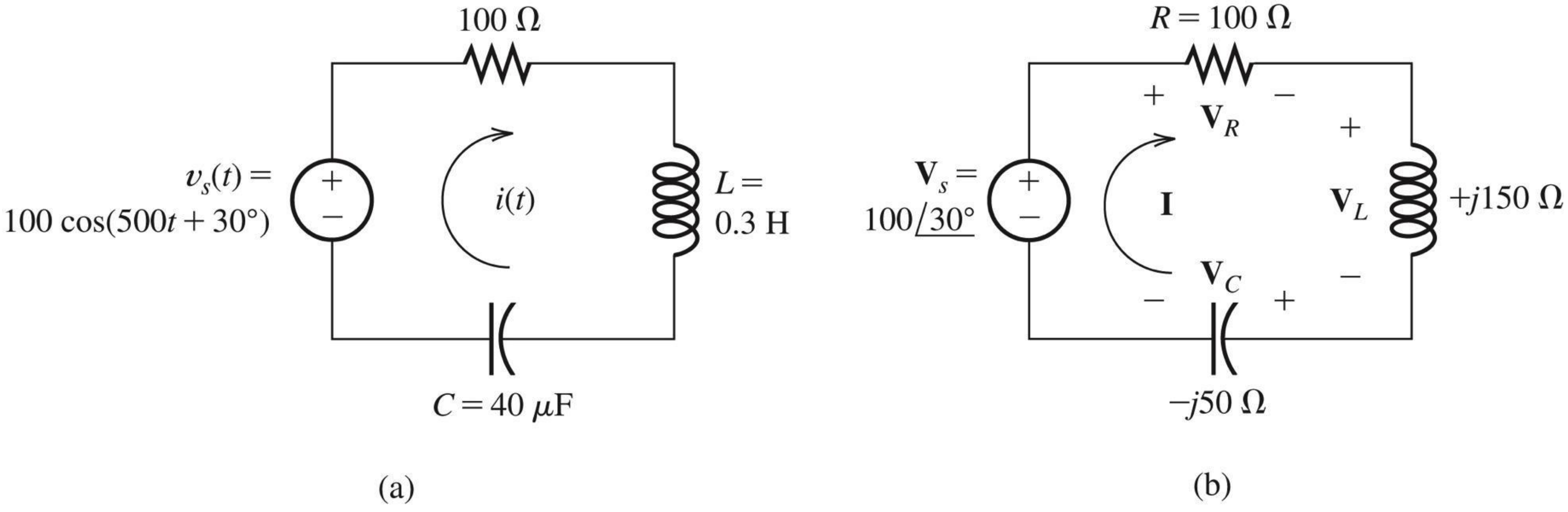
Complex Impedances

- AC steady-state analysis using phasors allows us to express the relationship between current and voltage using a formula that looks likes Ohm's law:

$$\boldsymbol{V} = \boldsymbol{IZ}$$

- Impedance depends on the frequency ω.
- Impedance is a complex number.
- Impedance allows us to use the same solution techniques for AC steady state as

Circuit Solution with sinusoidal sources



Circuit Solution with sinusoidal sources

Power in AC Circuits

Power in AC Circuits