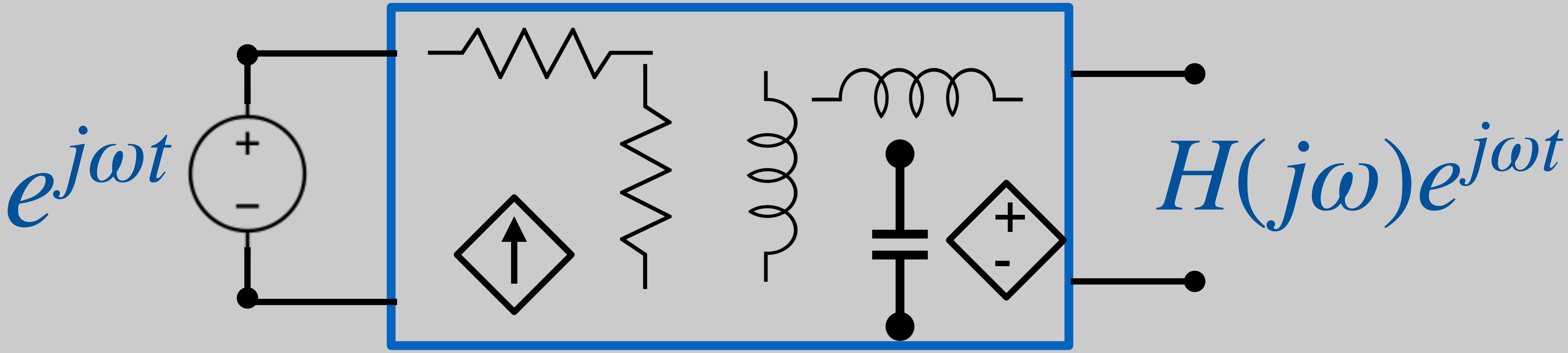
EE16B
Designing Information Devices and Systems II

Lecture 4B
More Phasors, Freq. Response / Transfer function / AC Power And MRI Coils

Announcements

- Last time:
  - Resonant Tank
  - AC Response / Phasors
- Today:
  - Cont. AC response
  - AC Power

Recap



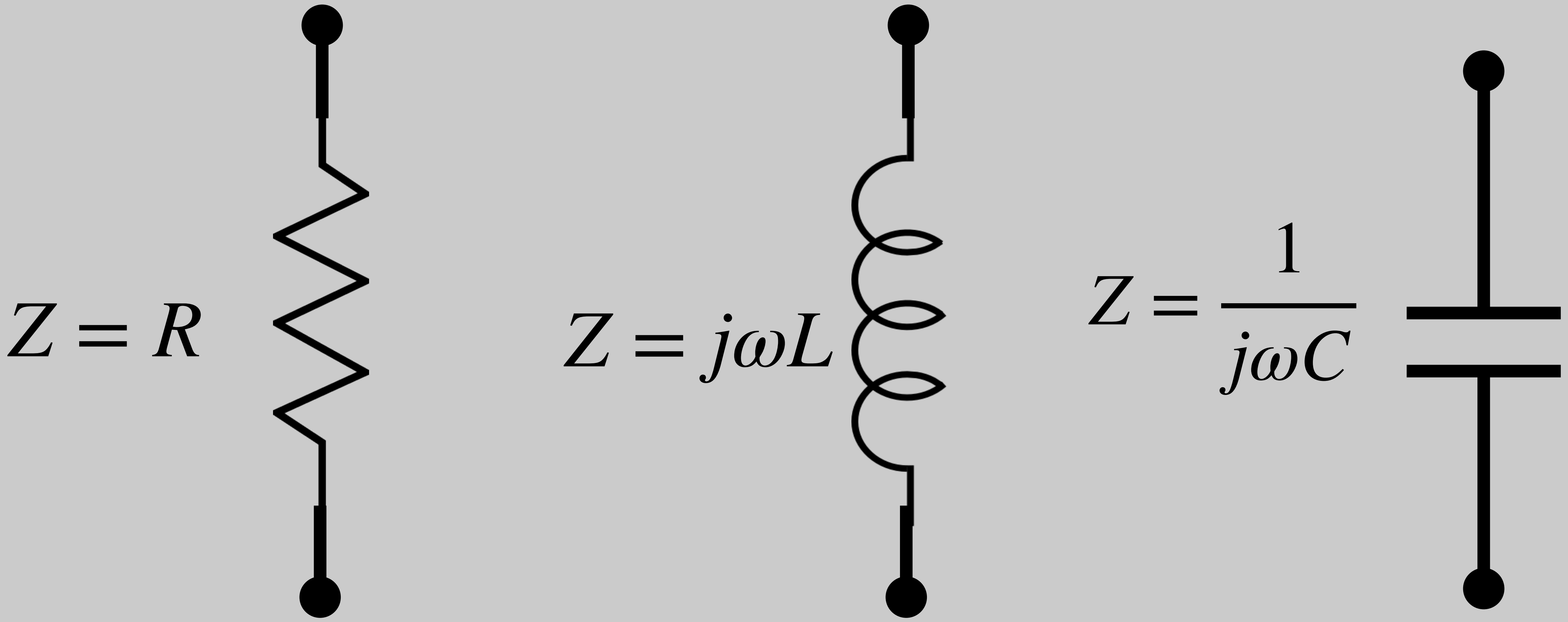
Complex valued “resistance”

$$Z = R + jX$$

Impedance

Resistance

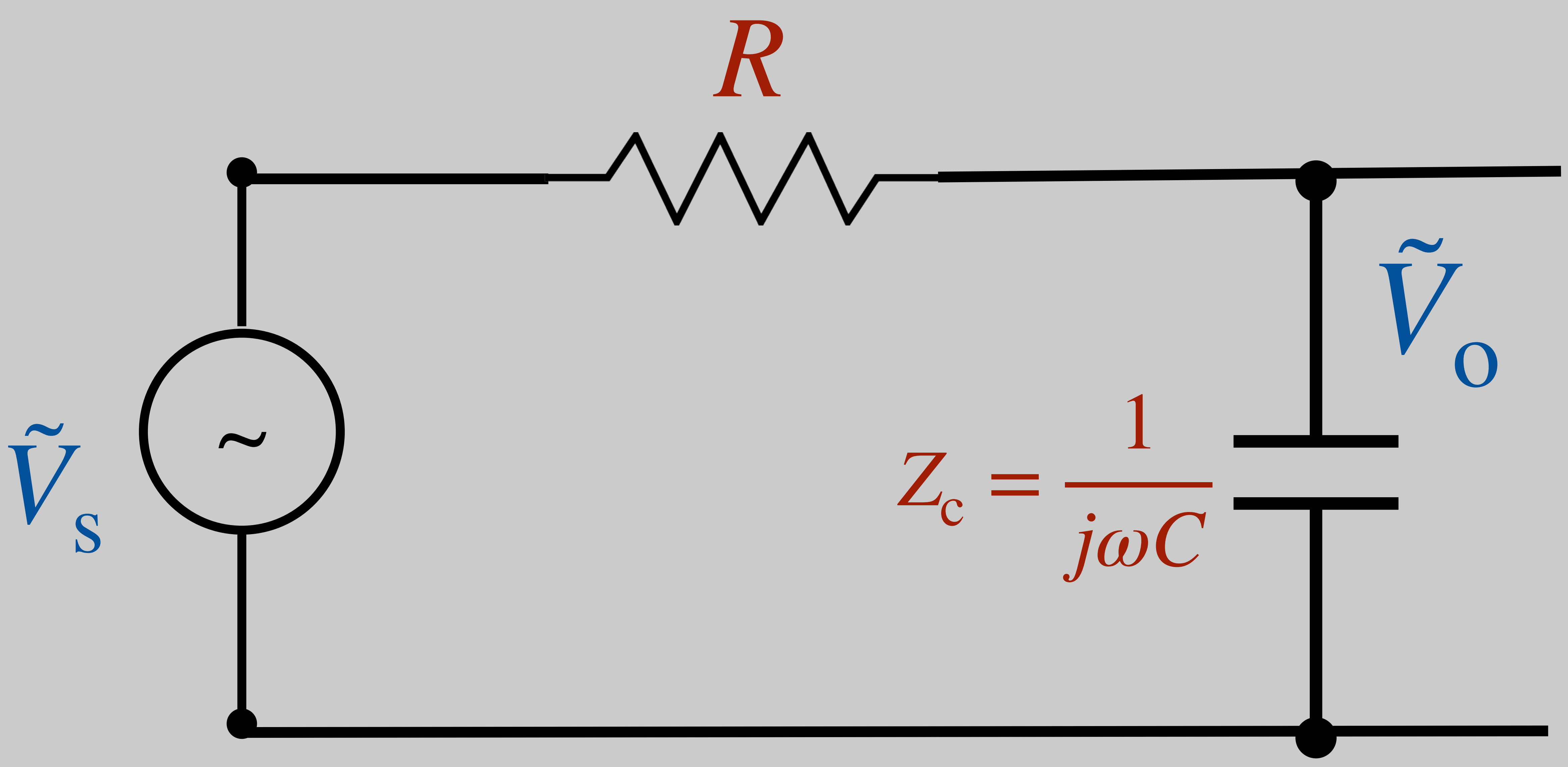
Reactance



Example 1: AC Response of an RC

$$V_s(t) = V_{DD}\cos(\omega t)$$

AC responses using phasors:



Voltage divider:

$$\tilde{V}_o = \frac{Z_c}{R + Z_c}\tilde{V}_s$$

$$\tilde{V}_o = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}\tilde{V}_s$$

$$\tilde{V}_o = \frac{1}{j\omega RC + 1}\tilde{V}_s$$

Example 1: AC Response of an RC

$$\tilde{V}_o = \frac{1}{j\omega RC + 1}\tilde{V}_s$$

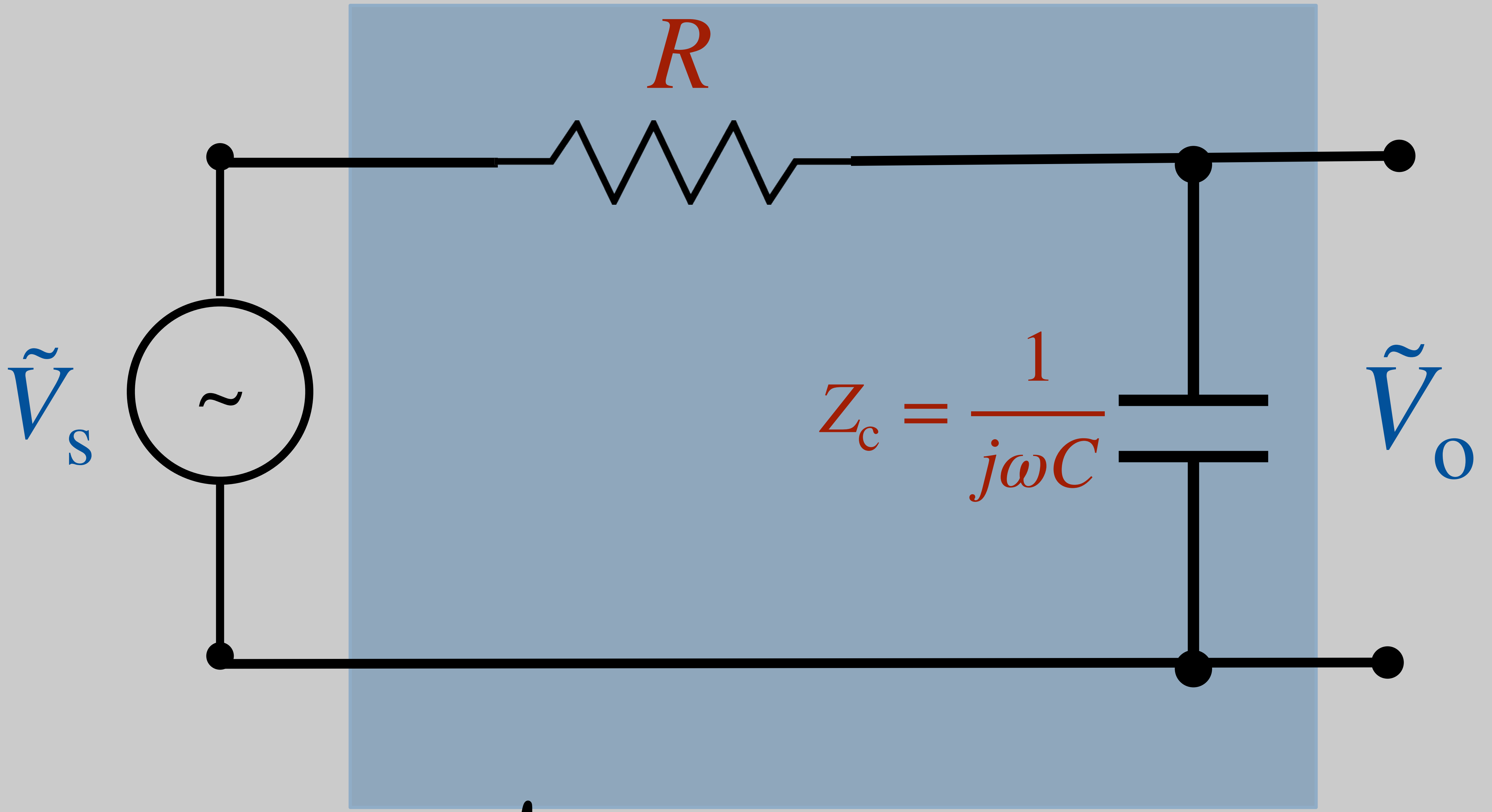
$$H(j\omega) = \frac{\tilde{V}_o}{\tilde{V}_s} = \frac{1}{j\omega RC + 1}$$

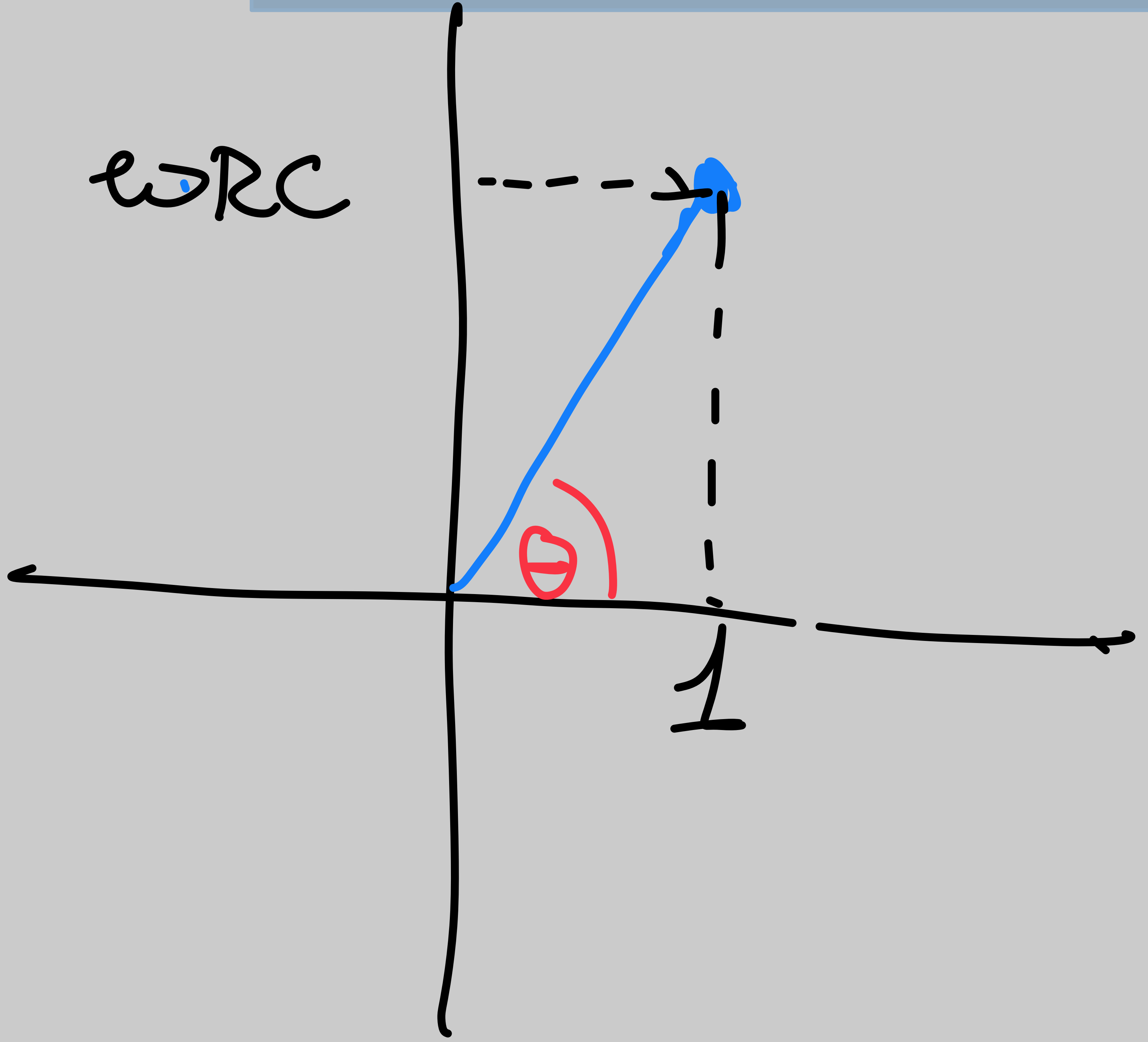
magnitude Resp:

$$|H(j\omega)| = \frac{1}{\sqrt{\omega^2(RC)^2 + 1}}$$

phase Resp:

$$\angle H(j\omega) = -\operatorname{atan}(\omega RC)$$





Example 1: AC Response of an RC

$$|H(j\omega)| = \frac{1}{\sqrt{\omega^2(RC)^2+1}}$$

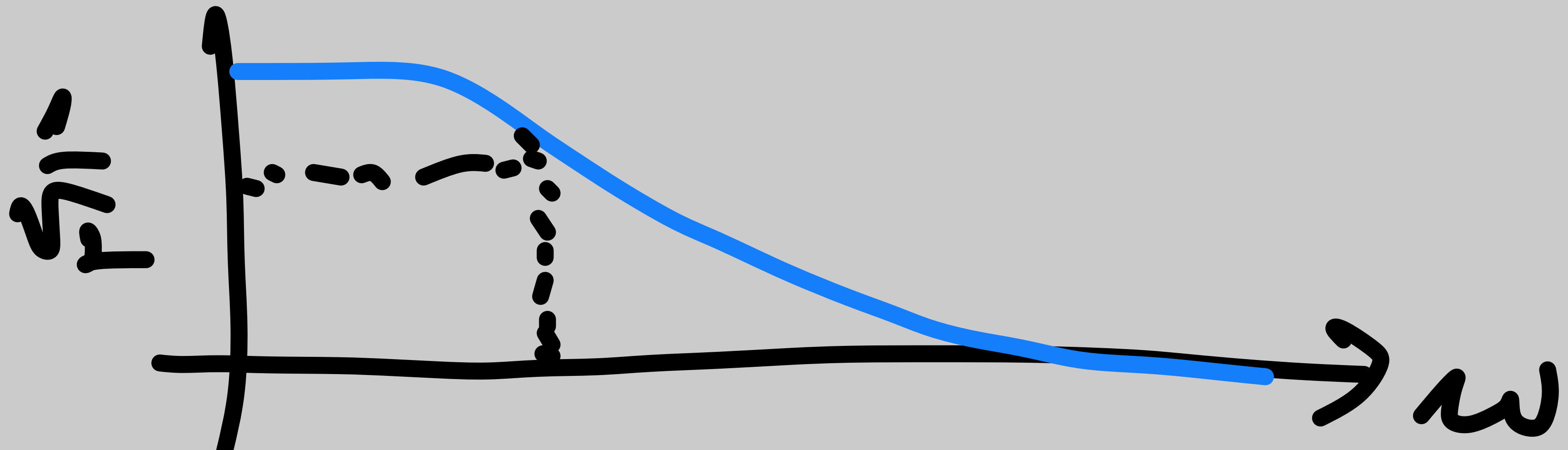
$$\angle H(j\omega) = -\operatorname{atan}(\omega RC)$$

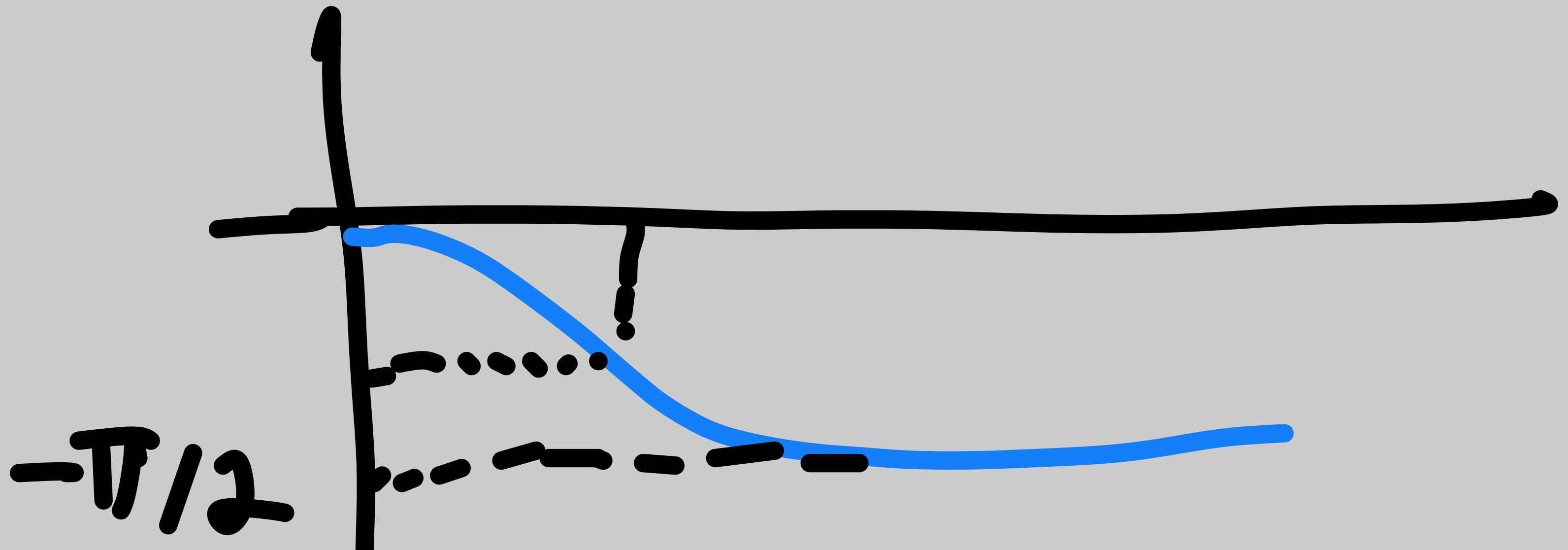
$\omega = 0 \quad |H(j\omega)| = 1 \quad \angle H(j\omega) = 0$

$\omega = \dfrac{1}{RC} \quad |H(j\omega)| = \dfrac{1}{\sqrt{2}} \quad \angle H(j\omega) = -\dfrac{\pi}{4} = -45^\circ$

$\omega \gg \dfrac{1}{RC} \quad |H(j\omega)| \approx \dfrac{1}{\omega(RC)}$

$\omega \to \infty \quad |H(j\omega)| \to 0 \quad \angle H(j\omega) = -\dfrac{\pi}{2} = -90^\circ$

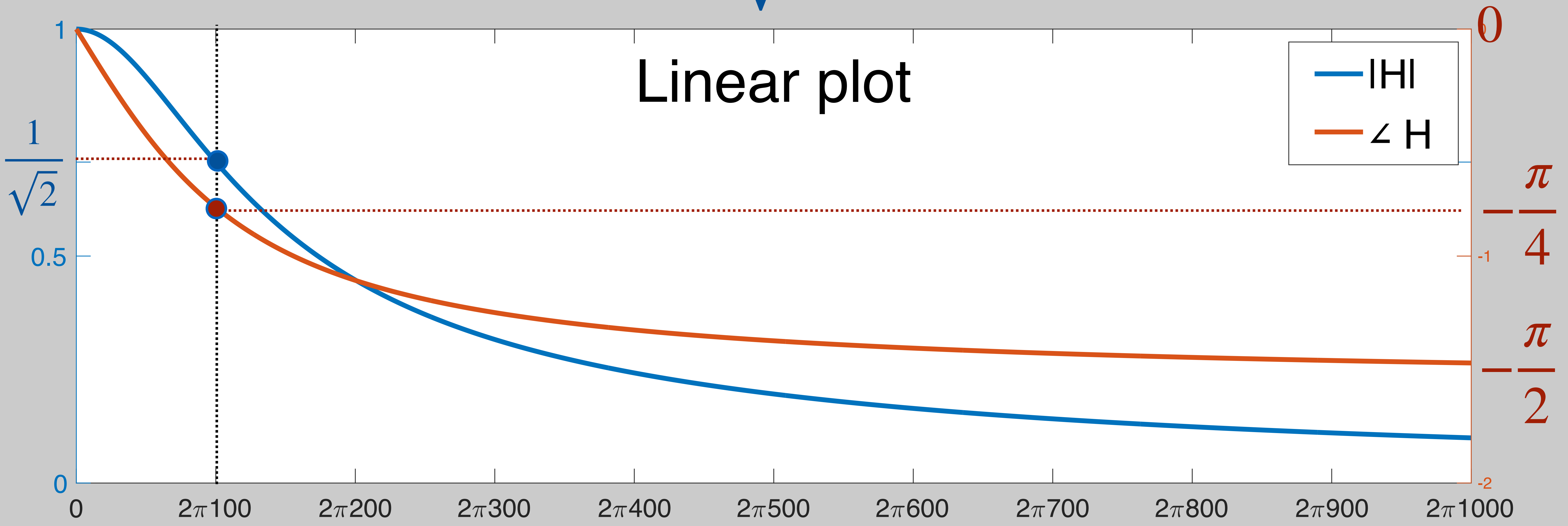




Example 1: AC Response of an RC

$$|H(j\omega)| = \frac{1}{\sqrt{\omega^2(RC)^2+1}} \qquad \angle H(j\omega) = -\operatorname{atan}(\omega RC)$$

$$\omega = \frac{1}{RC} = 2\pi 100 \qquad |H(j\omega)| = \frac{1}{\sqrt{2}} \qquad \angle H(j\omega) = -\frac{\pi}{4} = -45°$$



Input:

$$V_s(t) = V_{DD}\cos(2\pi\cdot 100t)$$

Output:

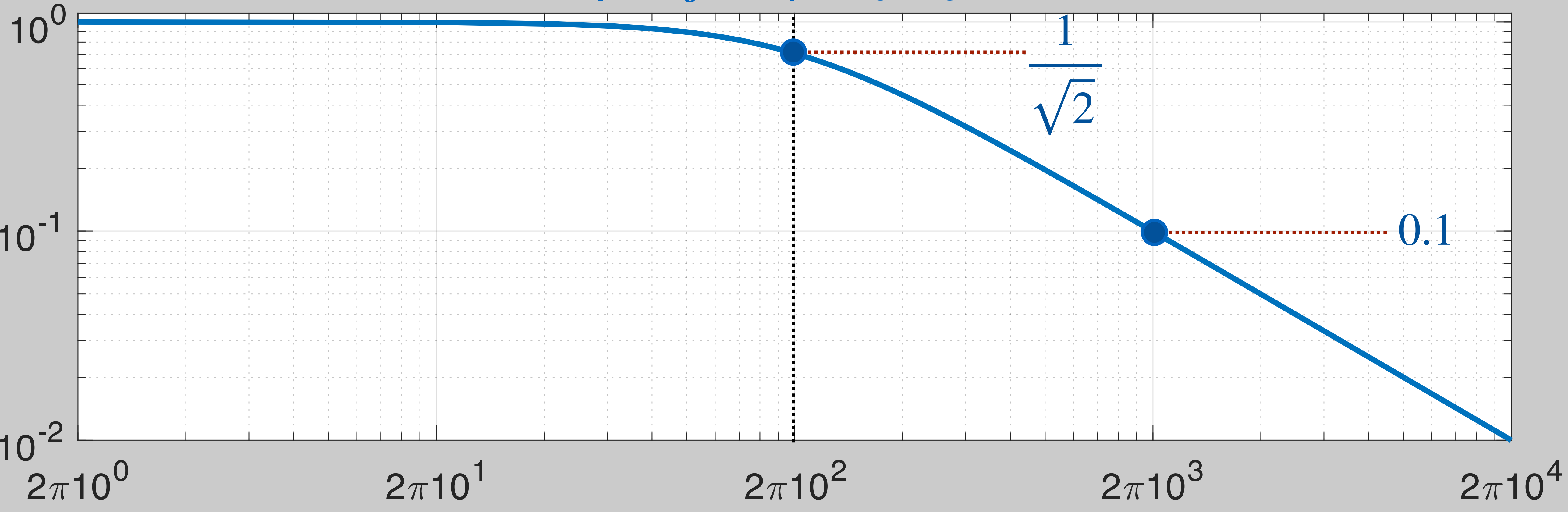
$$V_s(t) = \frac{1}{\sqrt{2}}V_{DD}\cos\left(2\pi\cdot 100t - \frac{\pi}{4}\right)$$

Example 1: AC Response of an RC

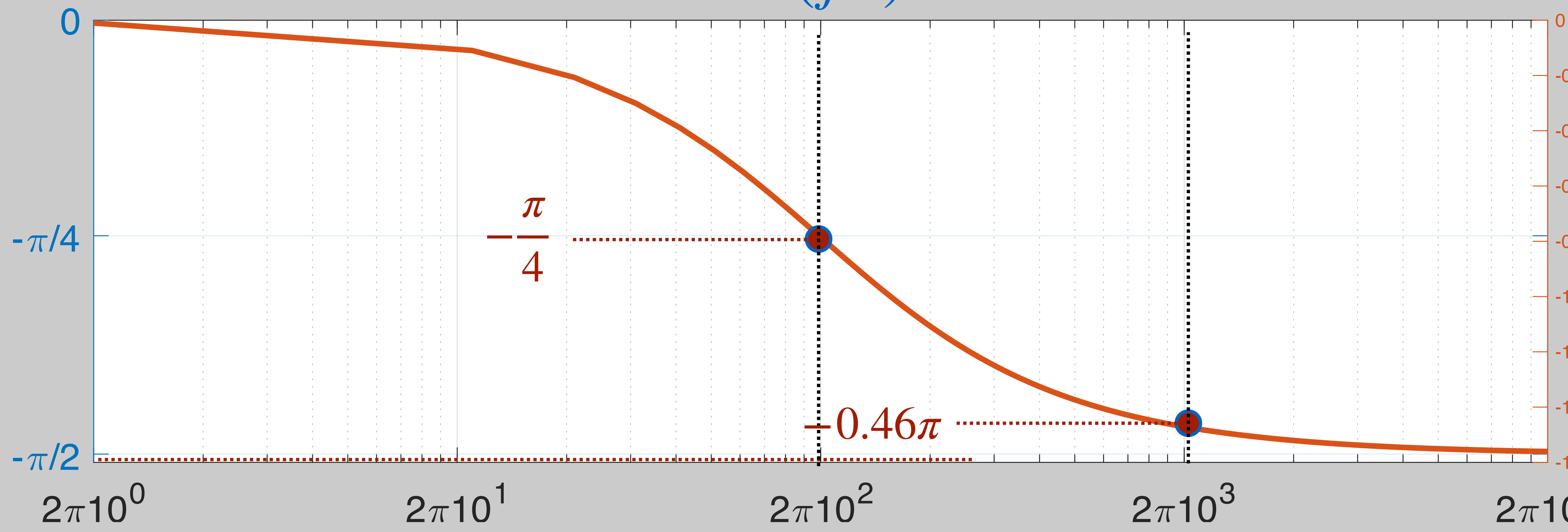
$$|H(j\omega)| = \frac{1}{\sqrt{\omega^2(RC)^2 + 1}} \qquad \angle H(j\omega) = -\operatorname{atan}(\omega RC)$$

$$\omega = \frac{1}{RC} = 2\pi 100 \qquad |H(j\omega)| = \frac{1}{\sqrt{2}} \qquad \angle H(j\omega) = -\frac{\pi}{4} = -45°$$

$|H(j\omega)|$ Loglog plot



$\angle H(j\omega)$



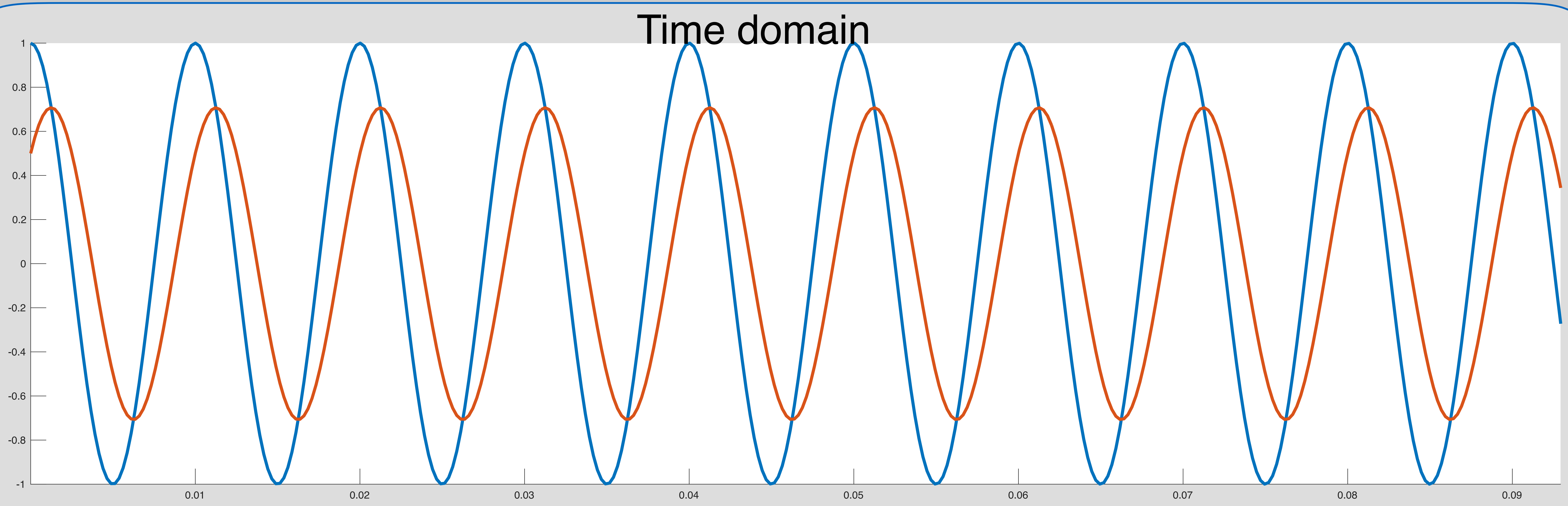
Input:

$$V_s(t) = V_{DD}\cos(2\pi \cdot 100t)$$

Output:

$$V_s(t) = \frac{1}{\sqrt{2}} V_{DD}\cos\left(2\pi \cdot 100t - \frac{\pi}{4}\right)$$

Example 1: AC Response of an RC



Input:

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Output:

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Example 1: AC Response of an RC

Input:

$$V_s(t) = V_{DD} \cos(2\pi \cdot 1000t)$$

Output:

$$V_s(t) \approx 0.1 V_{DD} \cos\left(2\pi \cdot 1000t - 0.46\pi\right)$$

Example 1: AC Response of an RC

$$|H(j\omega)| = \frac{1}{\quad} \qquad \angle H(j\omega) = -\operatorname{atan}(\omega RC)$$

Input:

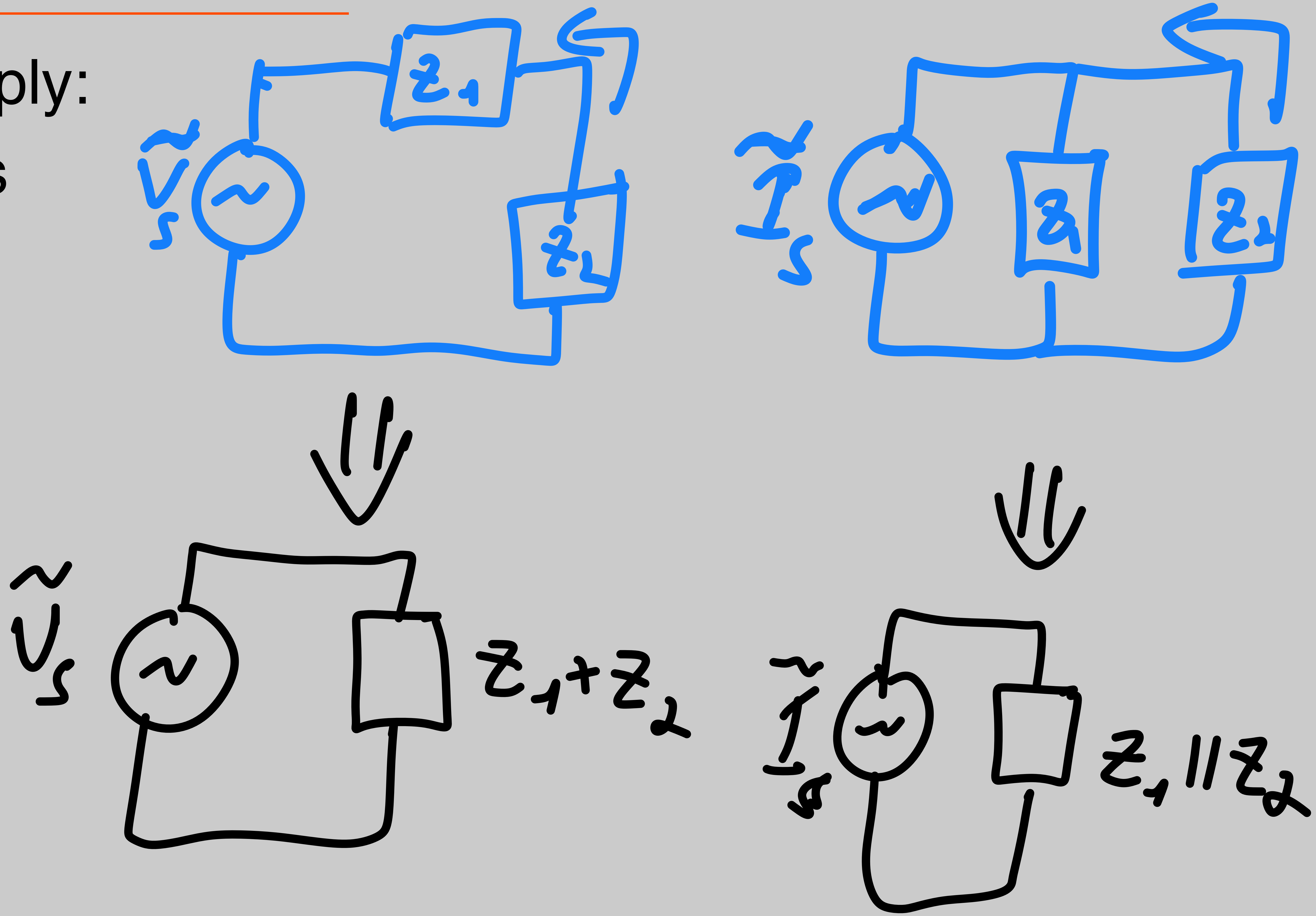
$$V_s(t) = V_{DD}\cos(2\pi \cdot 100t)$$

Output:

$$V_s(t) \approx 0.1V_{DD}\cos\left(2\pi \cdot 1000t - 0.46\pi\right)$$

AC circuits in steady state

- Network theorems that apply:
  - Voltage / current dividers
  - Source superposition
  - Thevenin/Norton



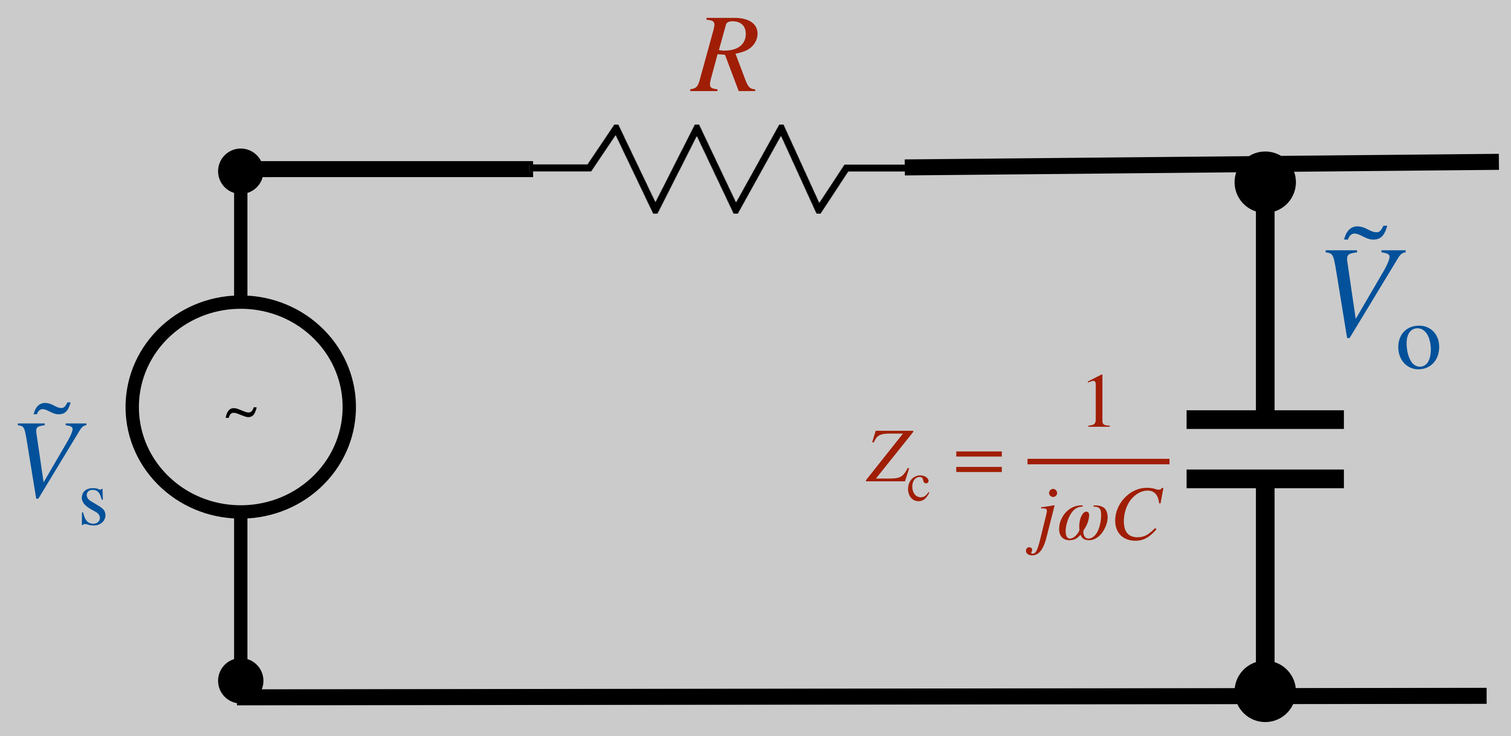
AC circuits in steady state

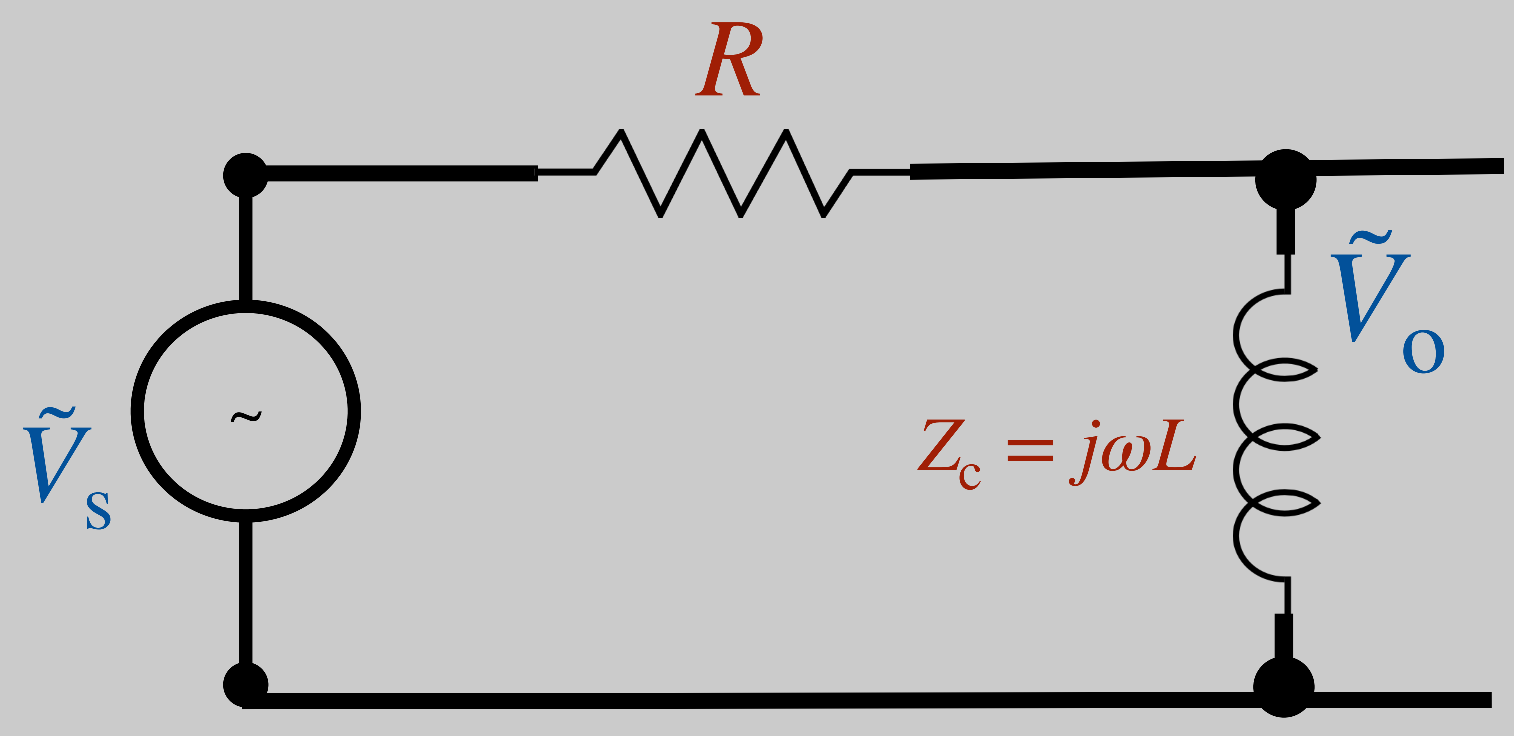
Rules of 👍 for L and C:

For DC:

$$\lim_{\omega \to 0} Z_c = \frac{1}{j\omega C} \quad = \infty$$

$$\lim_{\omega \to 0} Z_L = j\omega L \quad = 0$$





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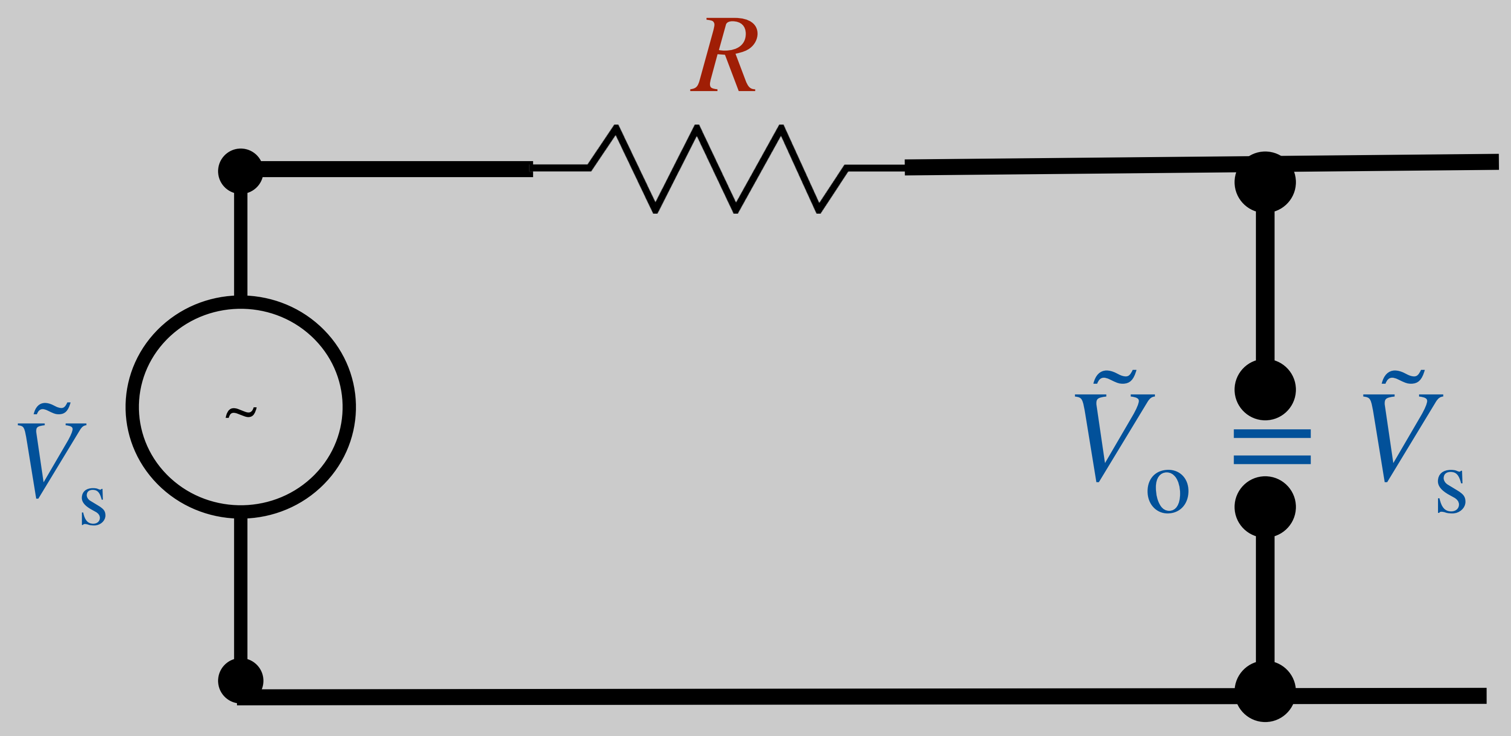

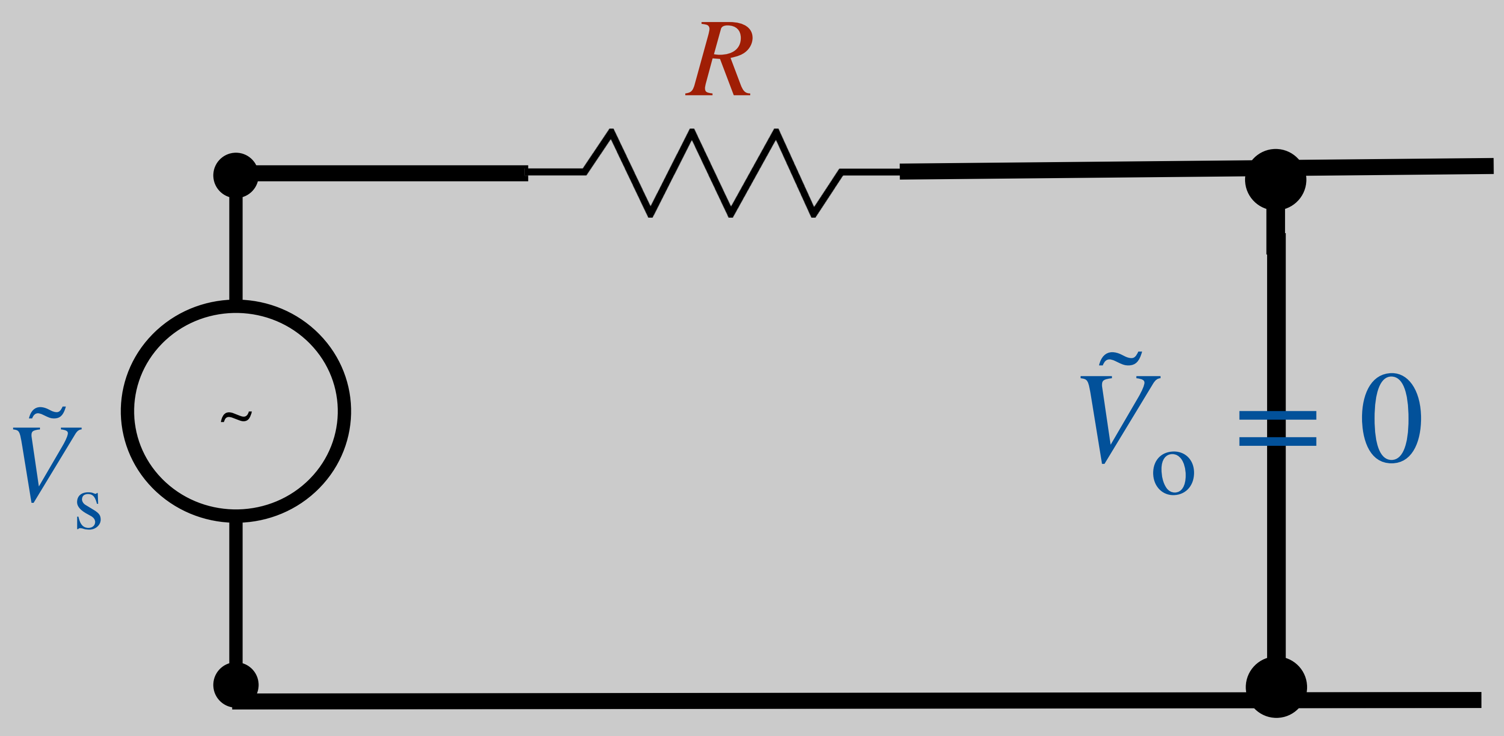

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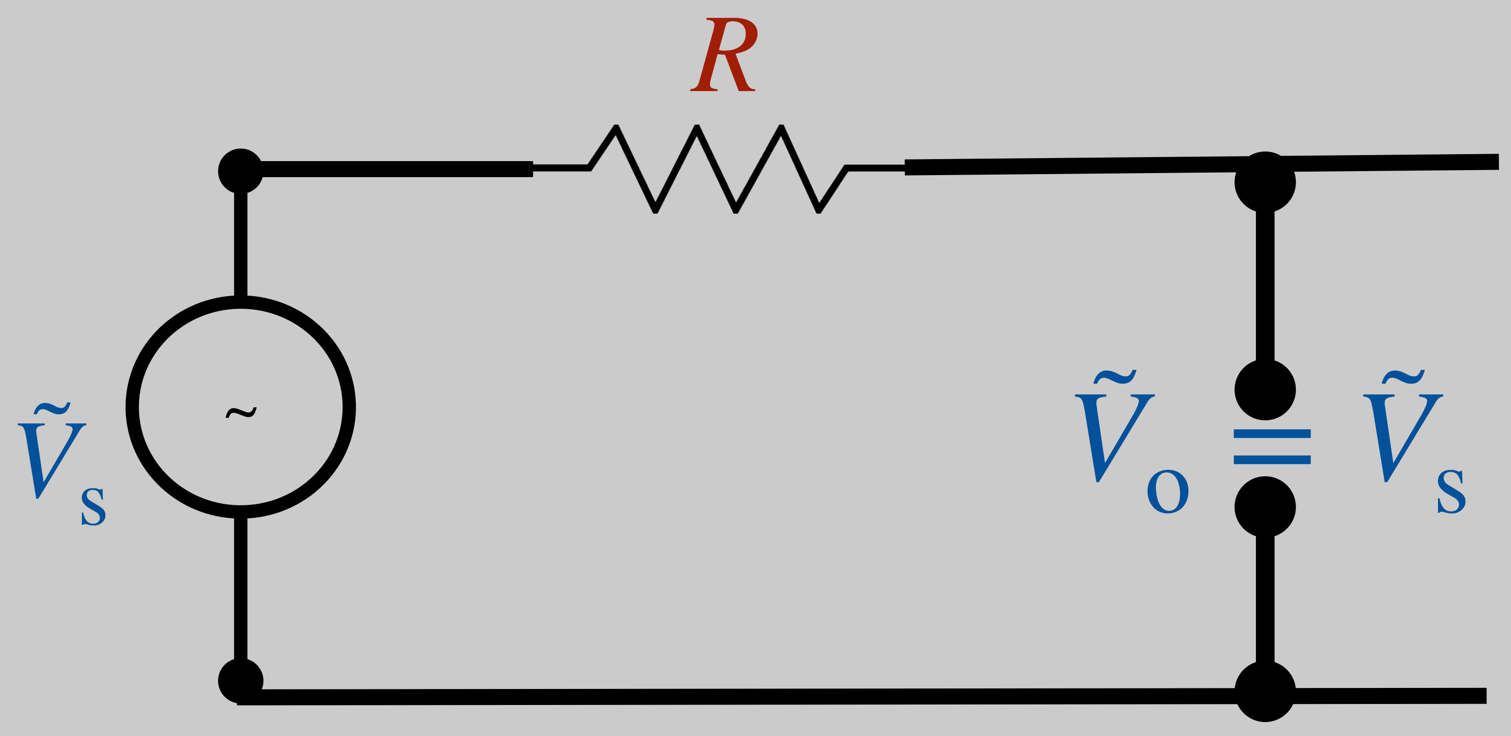
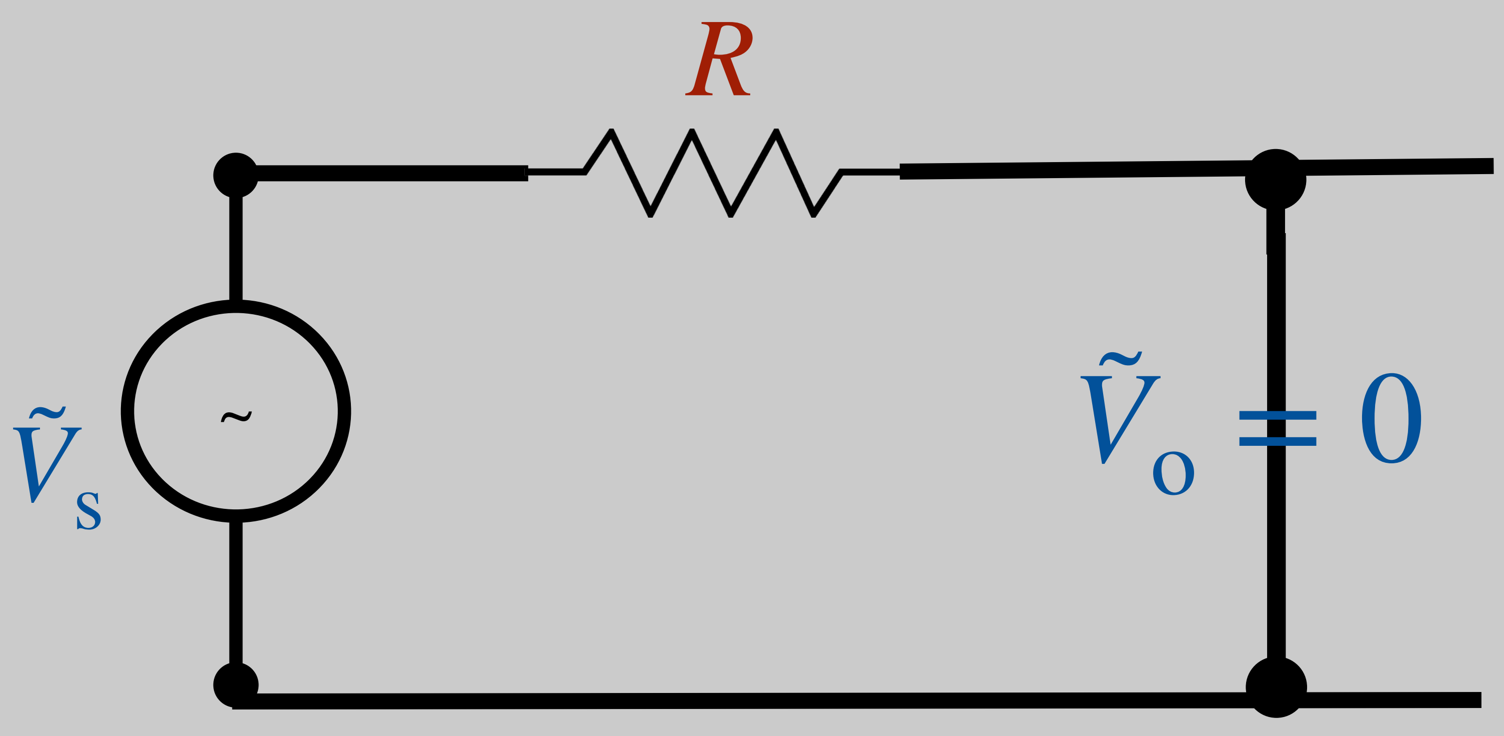
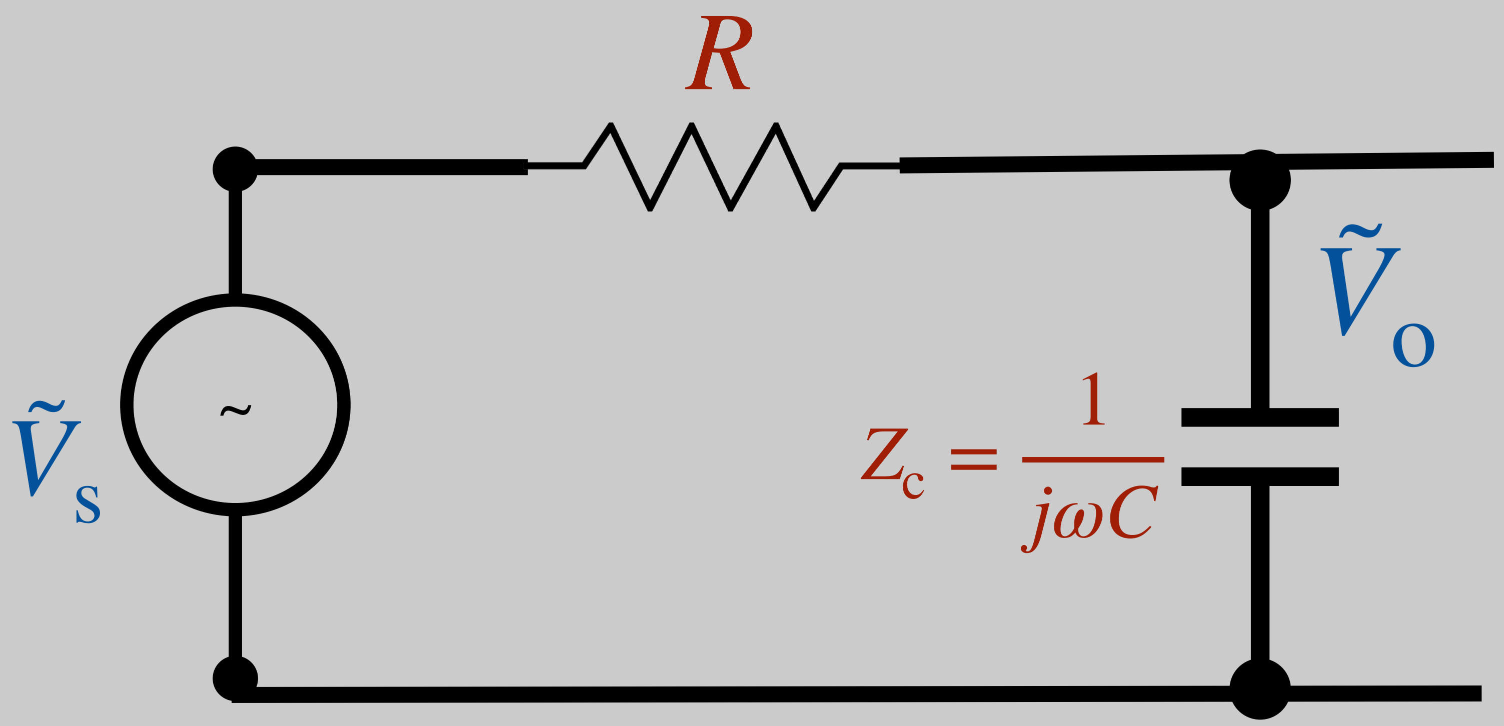


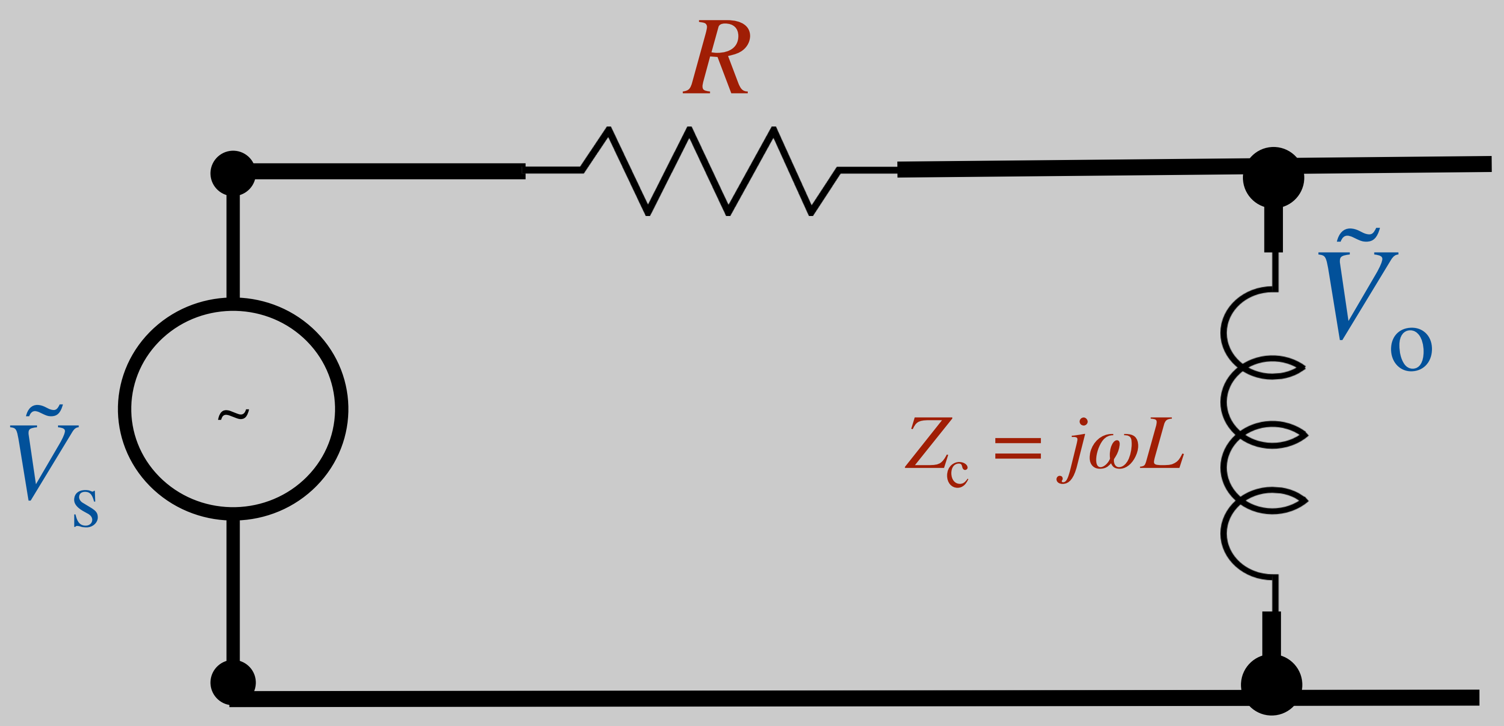


For very high frequency:

$$\lim_{\omega \to \infty} Z_c = \frac{1}{j\omega C} = 0$$

$$\lim_{\omega \to \infty} Z_L = j\omega L = \infty$$





AC circuits in steady state

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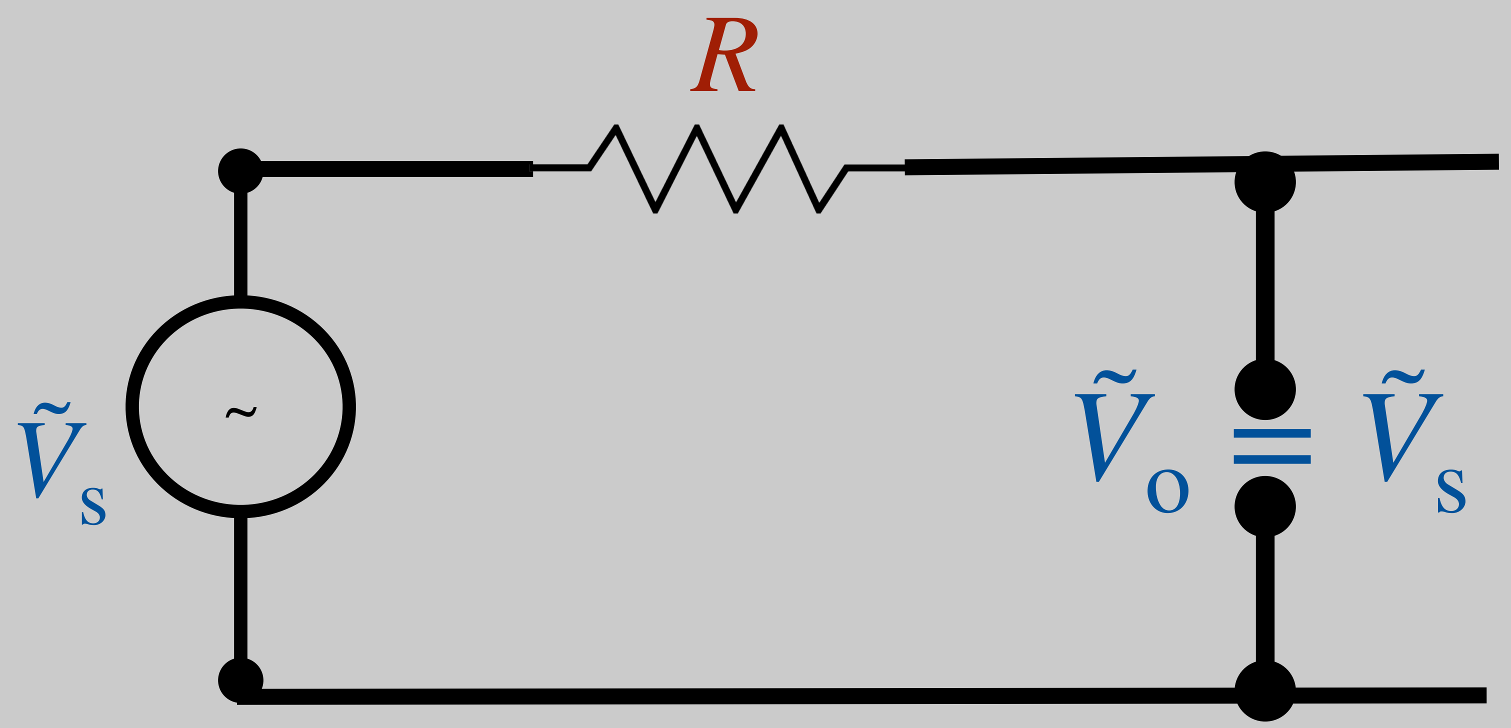

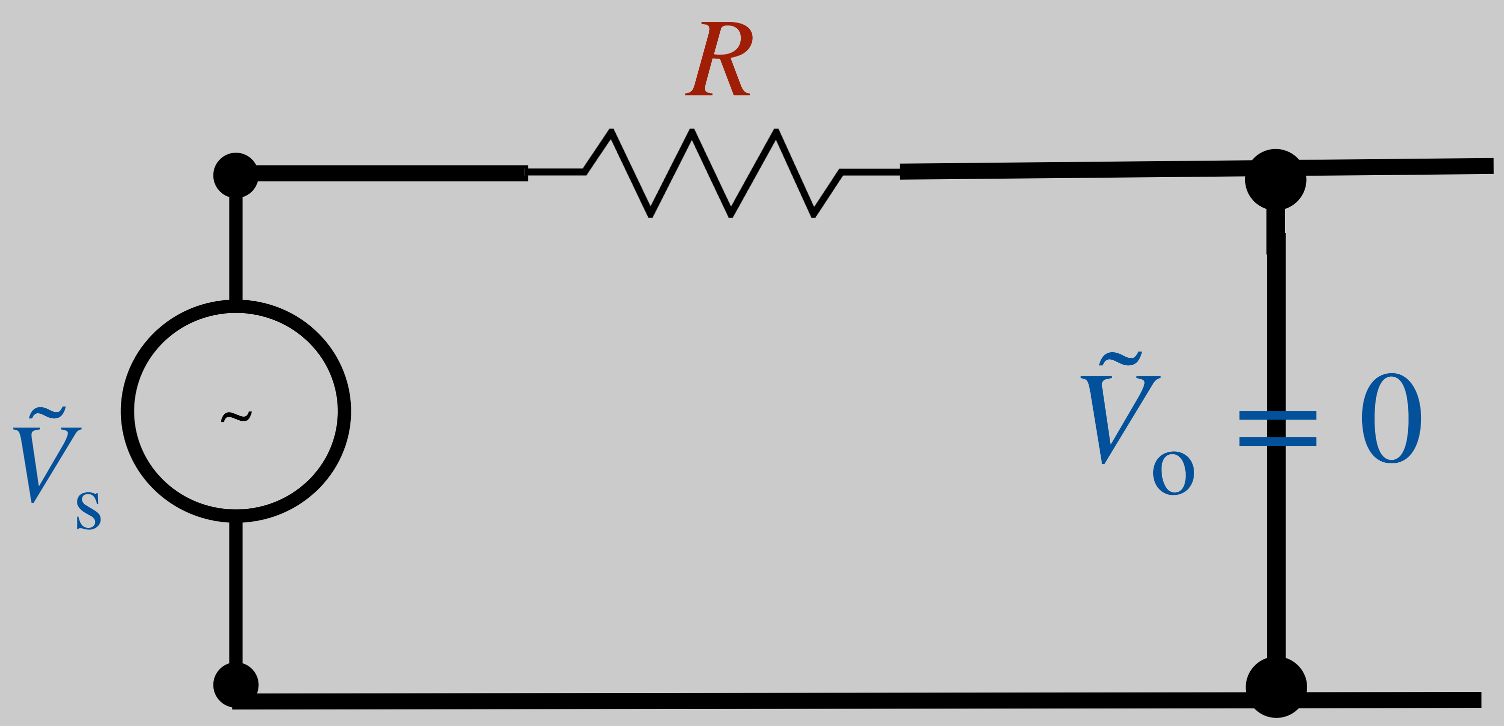

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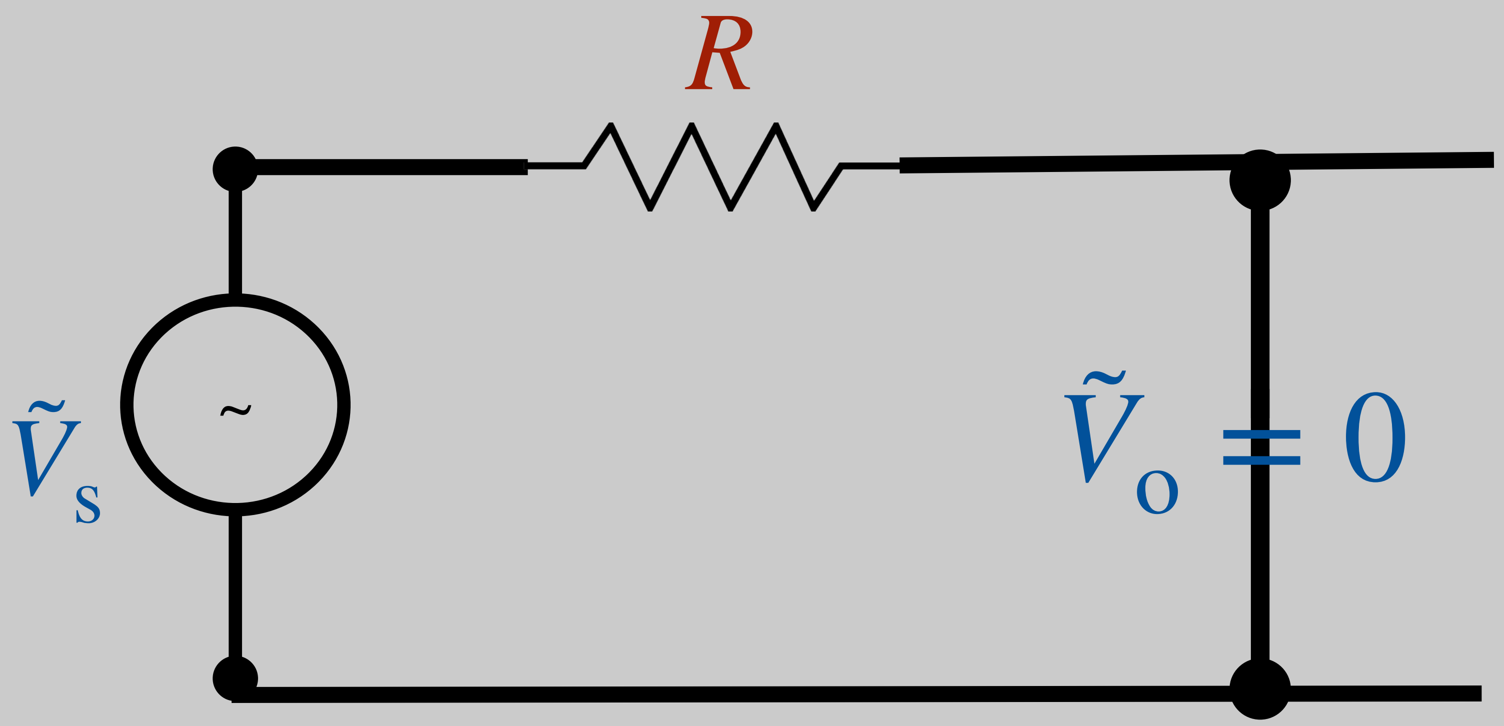
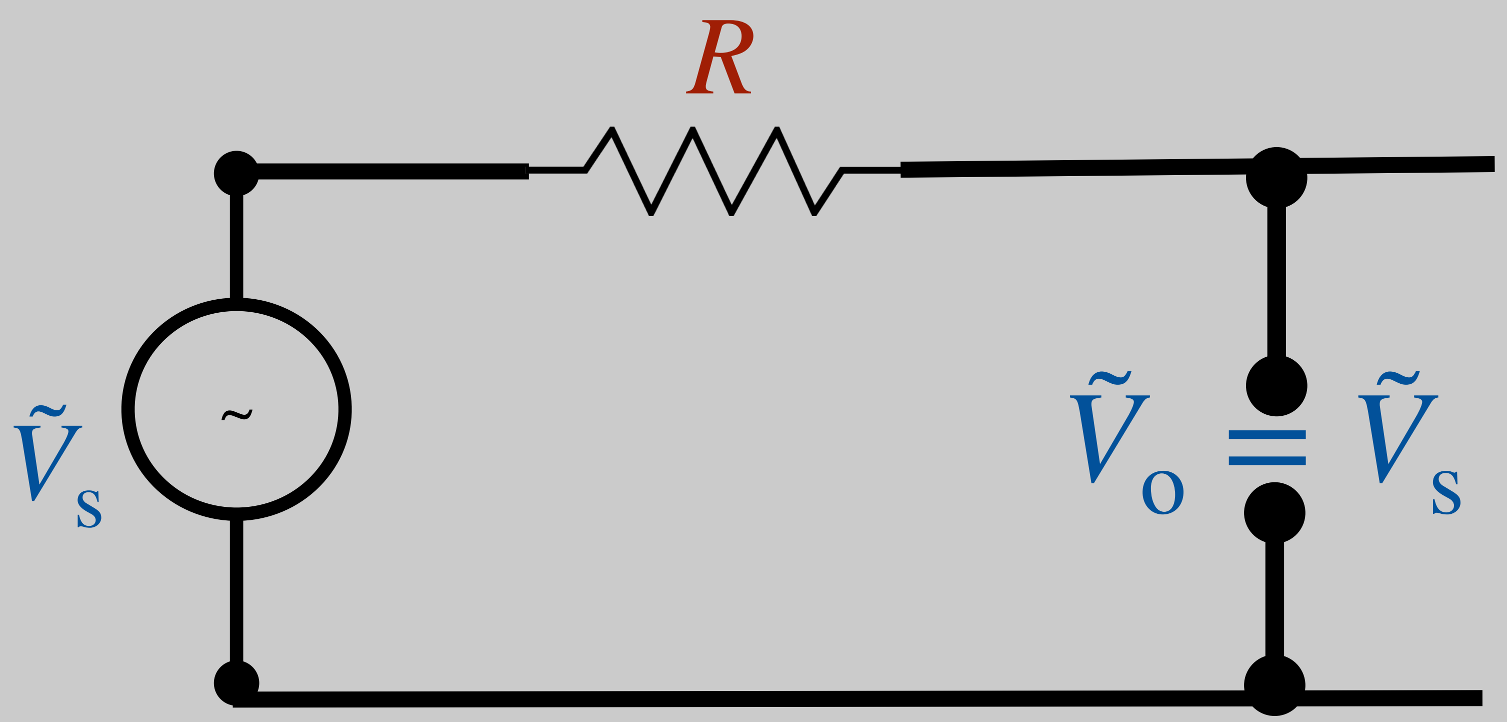
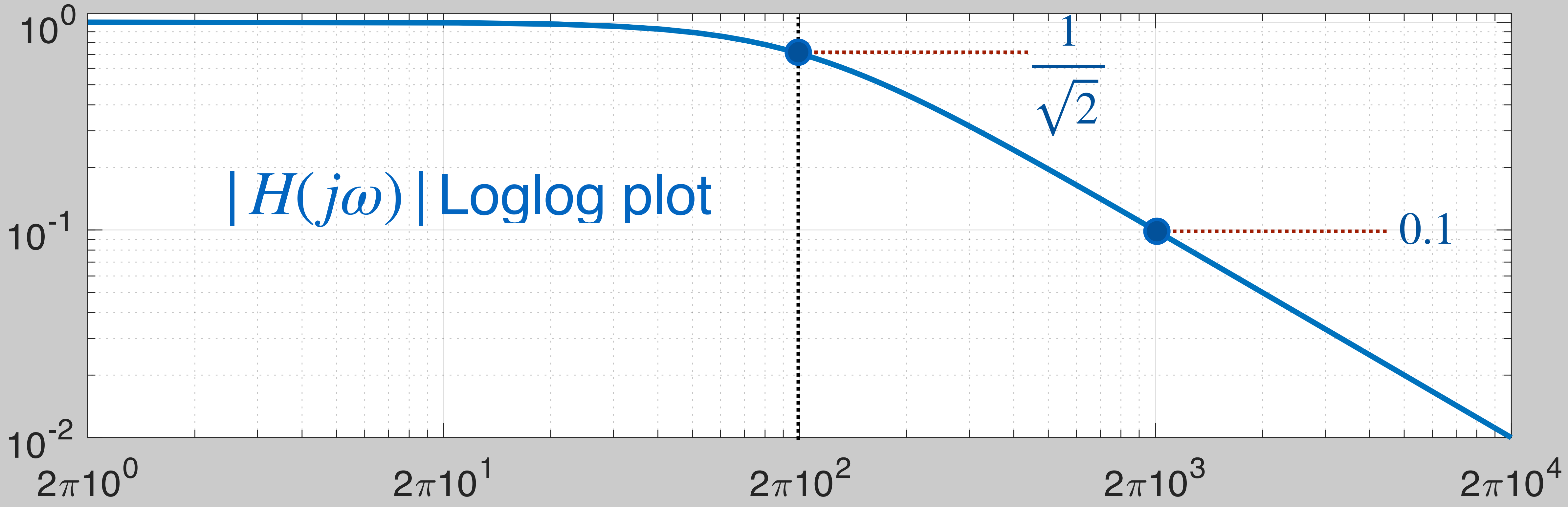
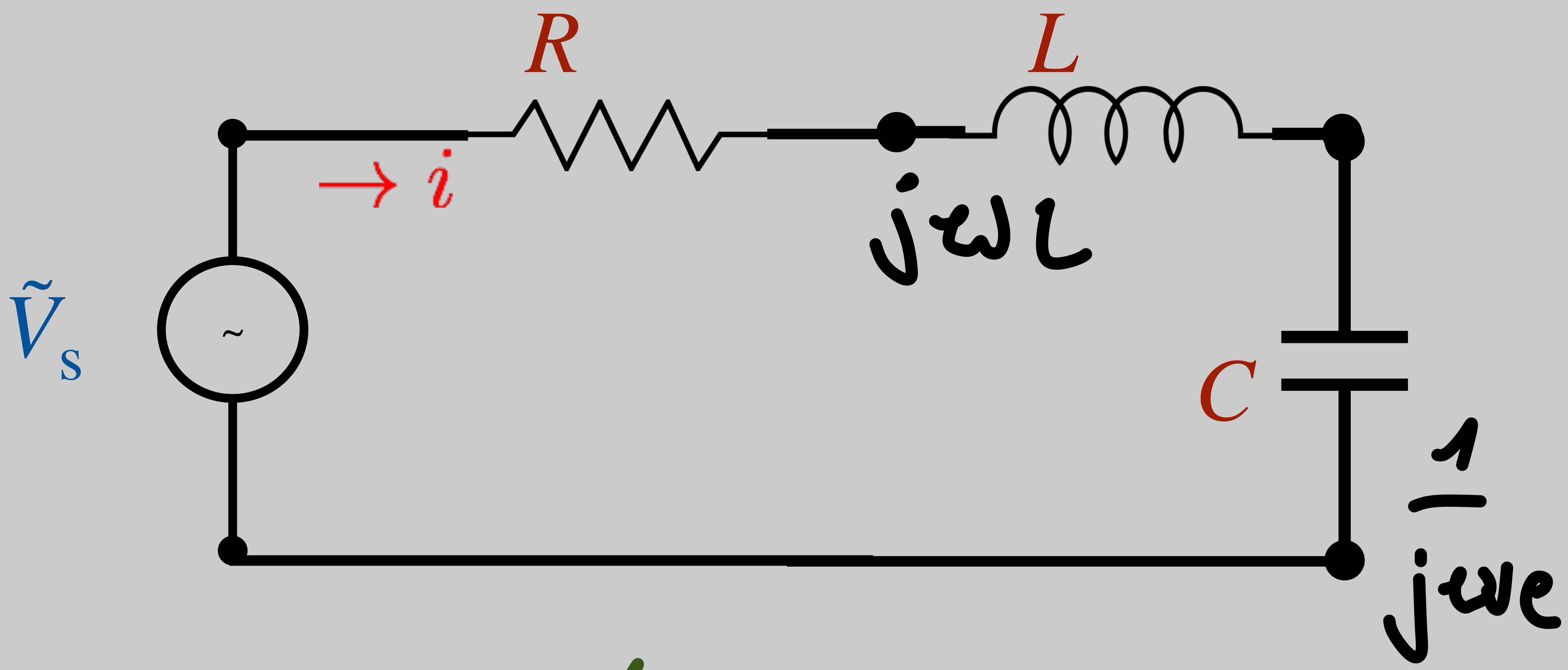






Series RLC AC analysis



$$\tilde{I} = \frac{V_s}{R + Z_L + Z_C} = \frac{V_s}{R + j\omega L + \frac{1}{j\omega C}}$$

Q: when $\tilde{I}_s$ is maximized?

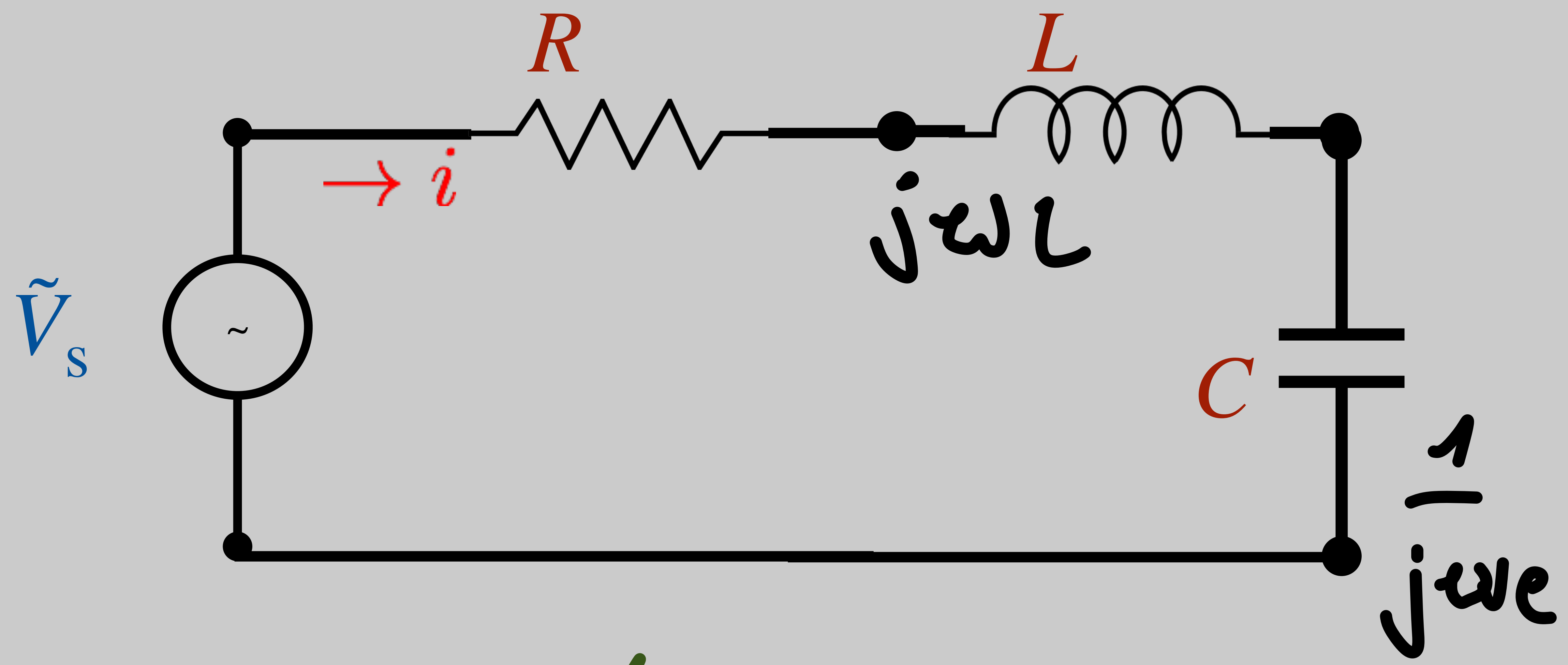
A: when $j\omega L = \frac{1}{j\omega C} \Longrightarrow \omega = \frac{1}{\sqrt{LC}}$ Resonance!

$\hookrightarrow I = \frac{\tilde{V}_s}{R}$

Q: what is $\tilde{V}_c$?

A: $\tilde{V}_C = \tilde{I} \cdot \frac{1}{j\omega C} = \frac{\tilde{V}_s}{jR\frac{1}{\sqrt{LC}} \cdot C} = \frac{\tilde{V}_s}{jR}\sqrt{\frac{L}{C}}$

Series RLC AC analysis



$$\tilde{I} = \frac{V_s}{R + Z_L + Z_C} = \frac{V_s}{R + j\omega L + \frac{1}{j\omega C}}$$

Q: when $\tilde{I}_s$ is maximized?

A: when $j\omega L = \frac{1}{j\omega C} \Longrightarrow \omega = \frac{1}{\sqrt{LC}}$ Resonance!

$\hookrightarrow I = \frac{\tilde{V}_s}{R}$

Q: what is $\tilde{V}_c$?

$$\text{A: } \tilde{V}_c = \tilde{I} \cdot \frac{1}{j\omega C} = \frac{\tilde{V}_s}{jR\frac{1}{\sqrt{LC}} \cdot C} = \frac{\tilde{V}_s}{jR}\underbrace{\sqrt{\frac{L}{C}}}_{= Q} = \frac{\tilde{V}_s}{j} Q$$

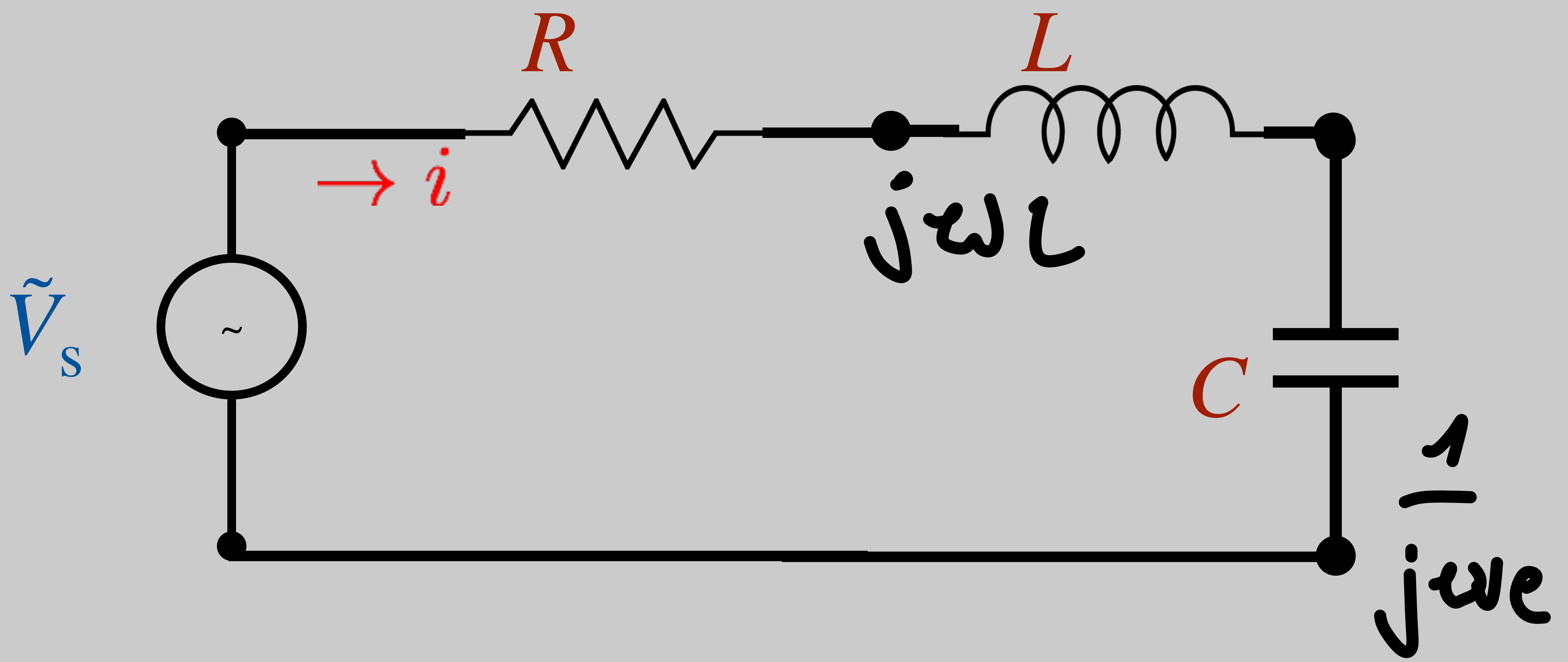
Series RLC Resonance

At: $\omega = \dfrac{1}{\sqrt{LC}} \quad \Rightarrow Z_c + Z_L = 0$

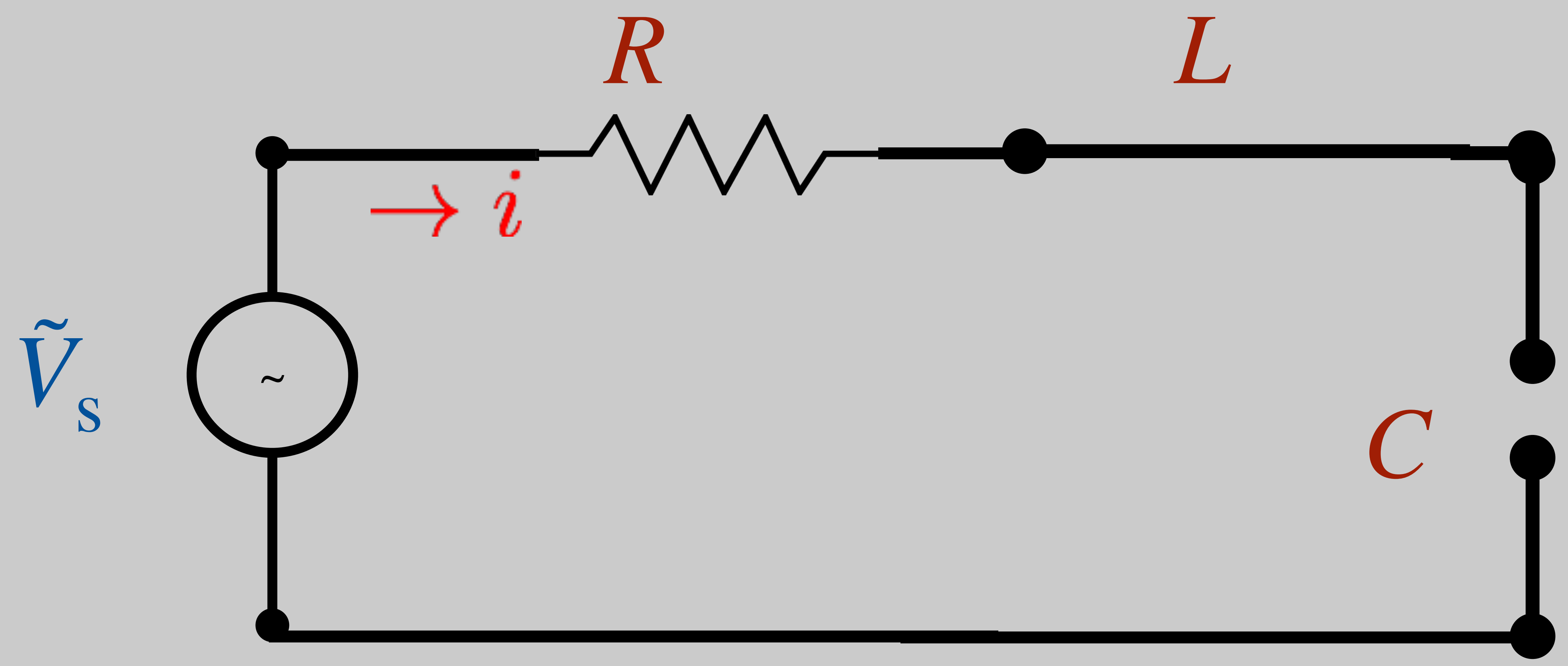
But.... $\tilde{V}_c = -j\tilde{V}_s Q$

If $Q = 400 \quad |\tilde{V}_c| = 400|\tilde{V}_s|$!

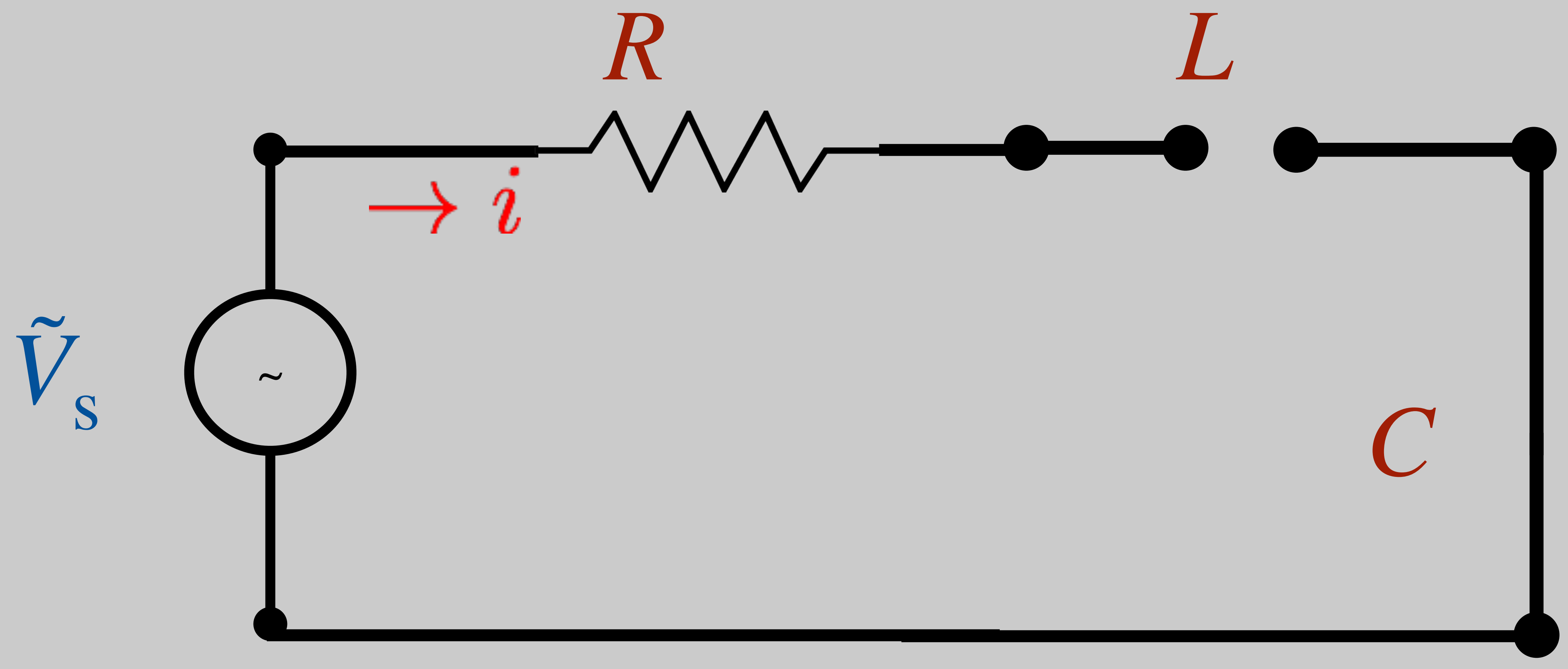
Passive voltage gain!



Circuit at DC ($\omega = 0$):



Circuit at high-freq ($\omega \to \infty$):



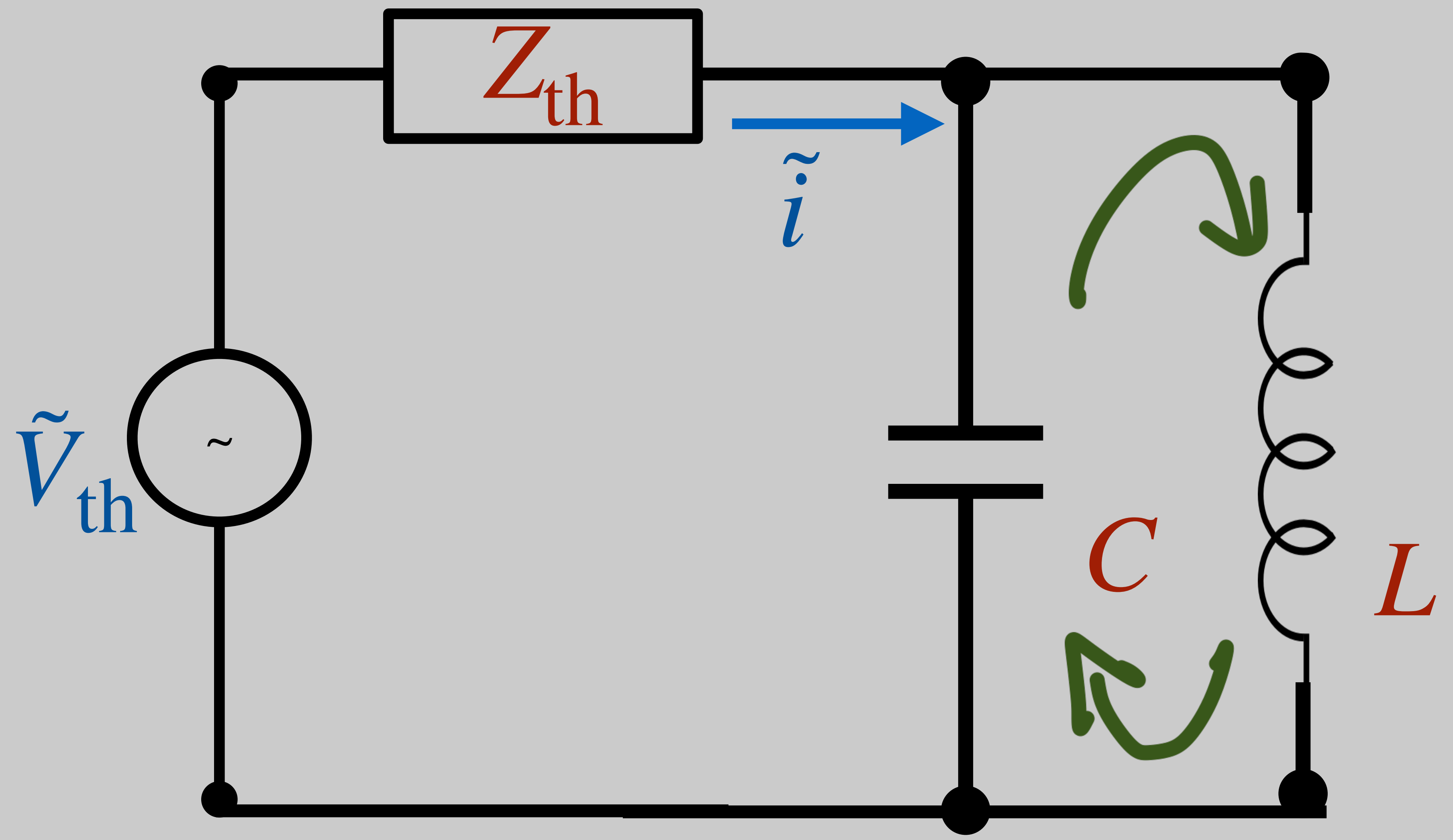
Parallel RLC Resonance

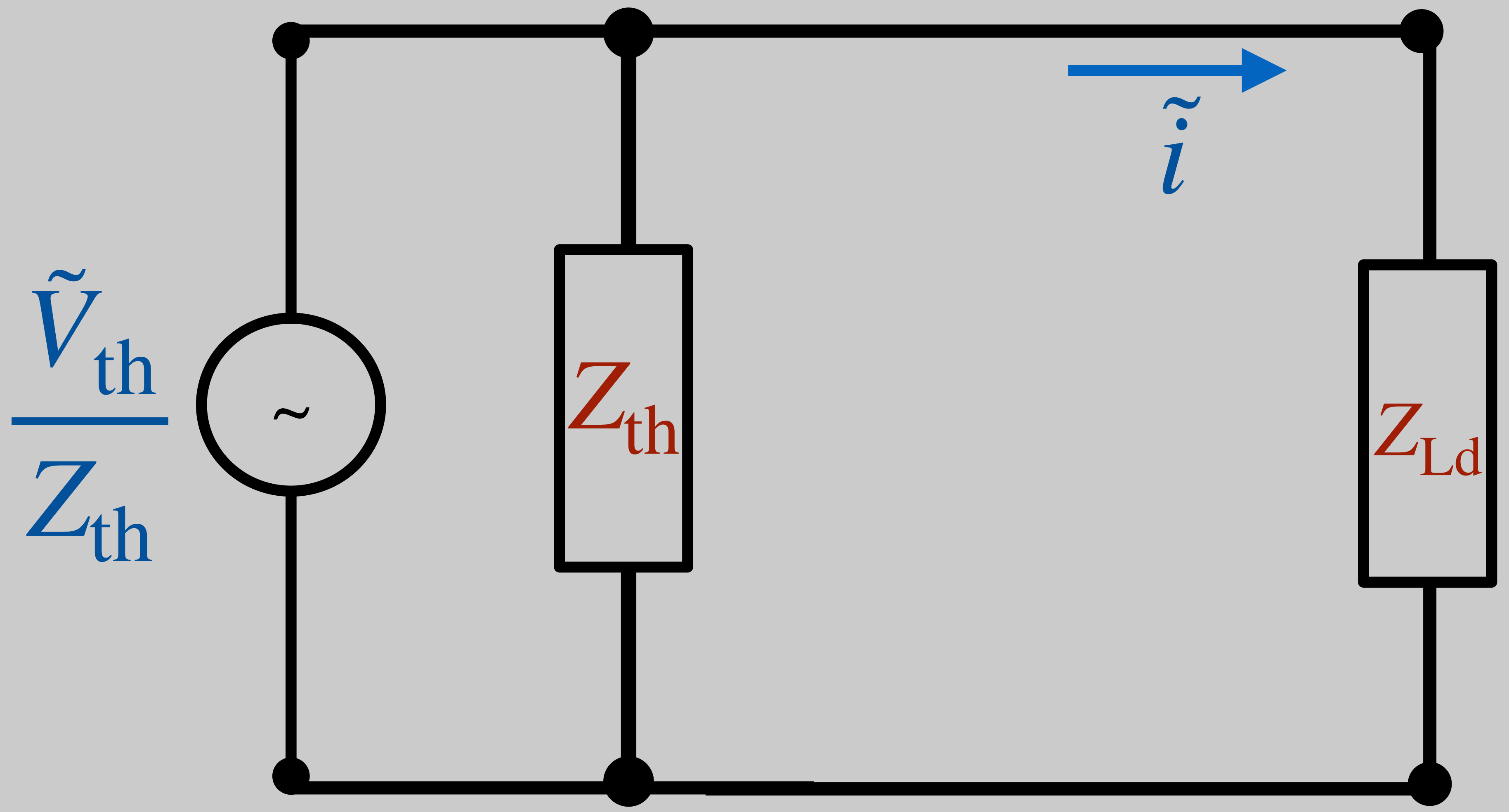
$$Z_{\mathrm{Ld}} = Z_{\mathrm{C}} || Z_{\mathrm{L}}$$

$$Z_{\mathrm{Ld}} = \frac{1}{j\omega C + \frac{1}{j\omega L}} = \frac{1}{1 - \omega^2 LC}$$

$$\lim_{\omega \to \frac{1}{\sqrt{LC}}} Z_{\mathrm{LD}} = \infty \qquad \Rightarrow \tilde{i} \to 0$$

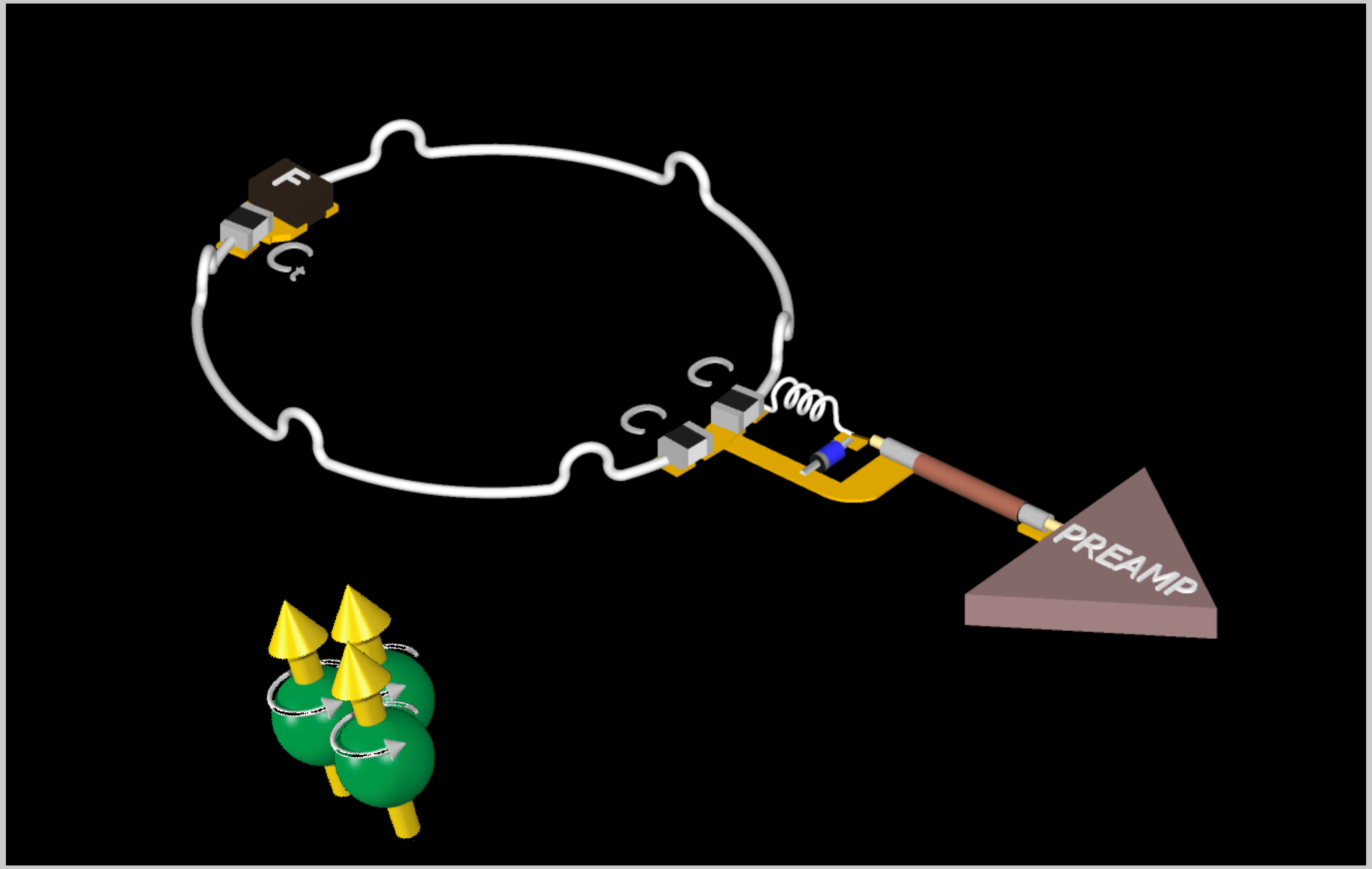
$$\Rightarrow \tilde{V}_{Z_{\mathrm{Ld}}} \to \tilde{V}_{\mathrm{th}}$$





MRI Receiver Coils

- Near-field antennas
- Receive the MRI signal
- Designed to maximize signal at the MRI frequency
  - Arrays are key for SNR & acceleration



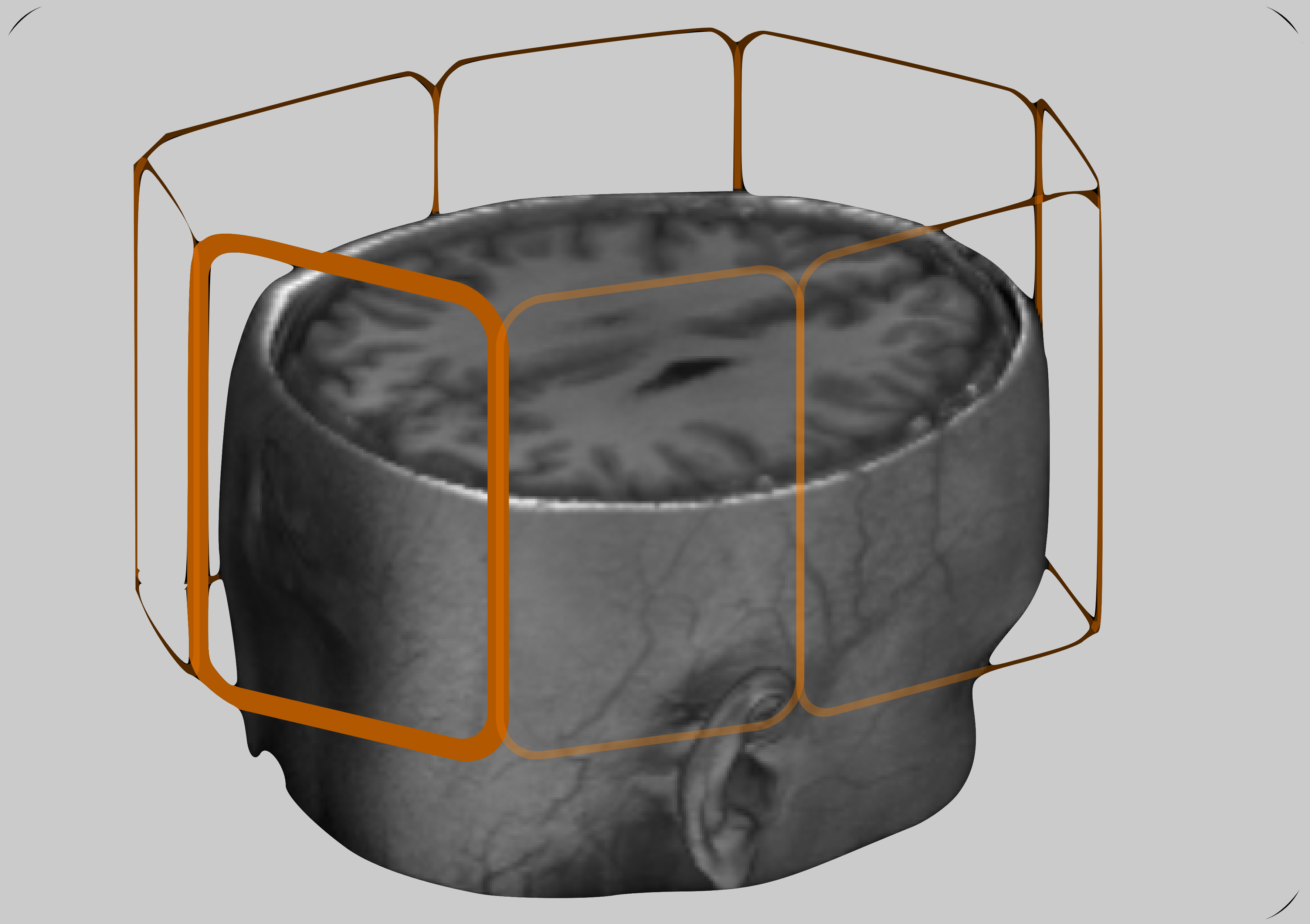
courtesy Boris Keil, Larry Wald, MGH

Proximity is the Key

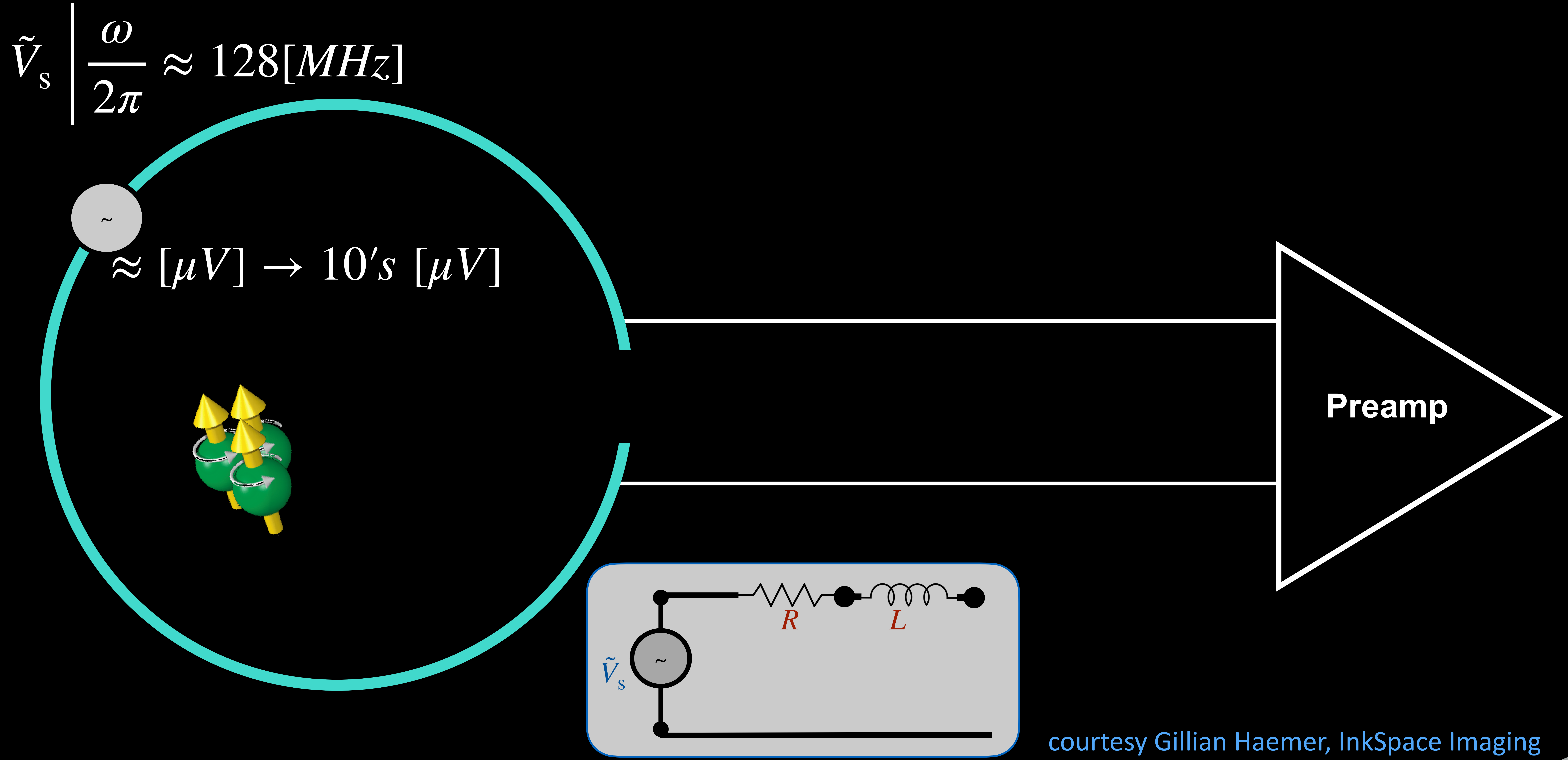
*Corea et al, Nat Commun. 2016;7:10839

Coil Arrays

- Use multiple small elements to
  - Gain SNR and coverage
  - Gain Speed (parallel imaging)

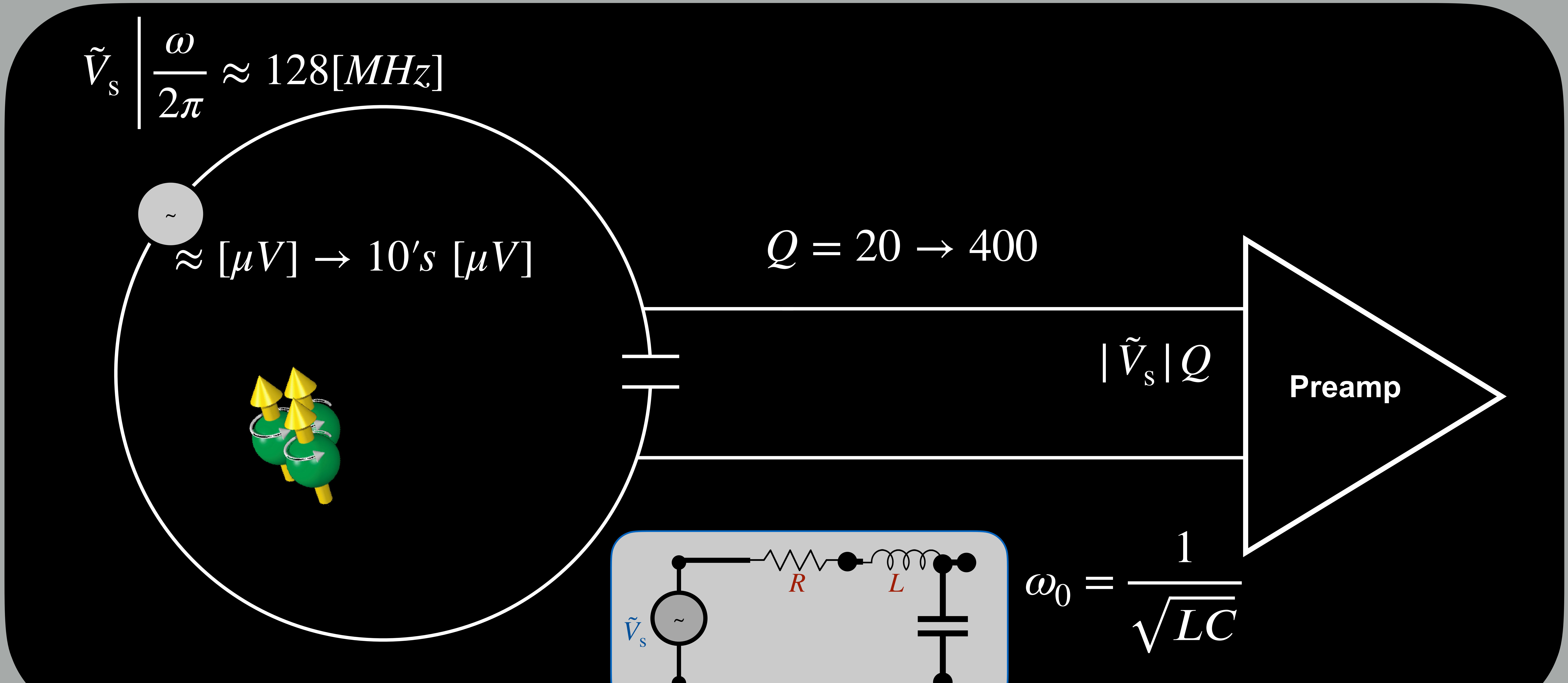


Receive Coil Components



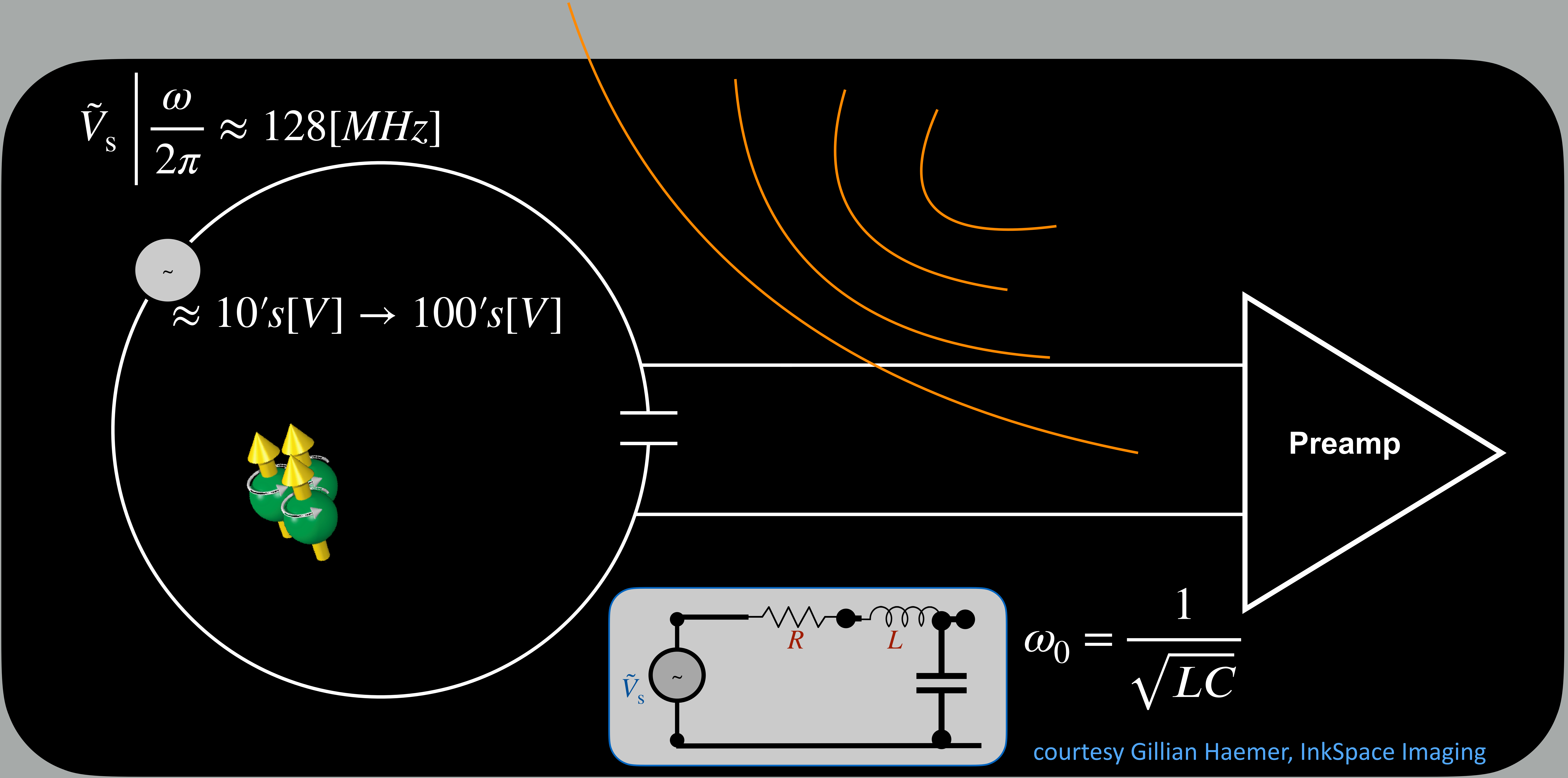
courtesy Gillian Haemer, InkSpace Imaging

Receive Coil Components: Capacitors



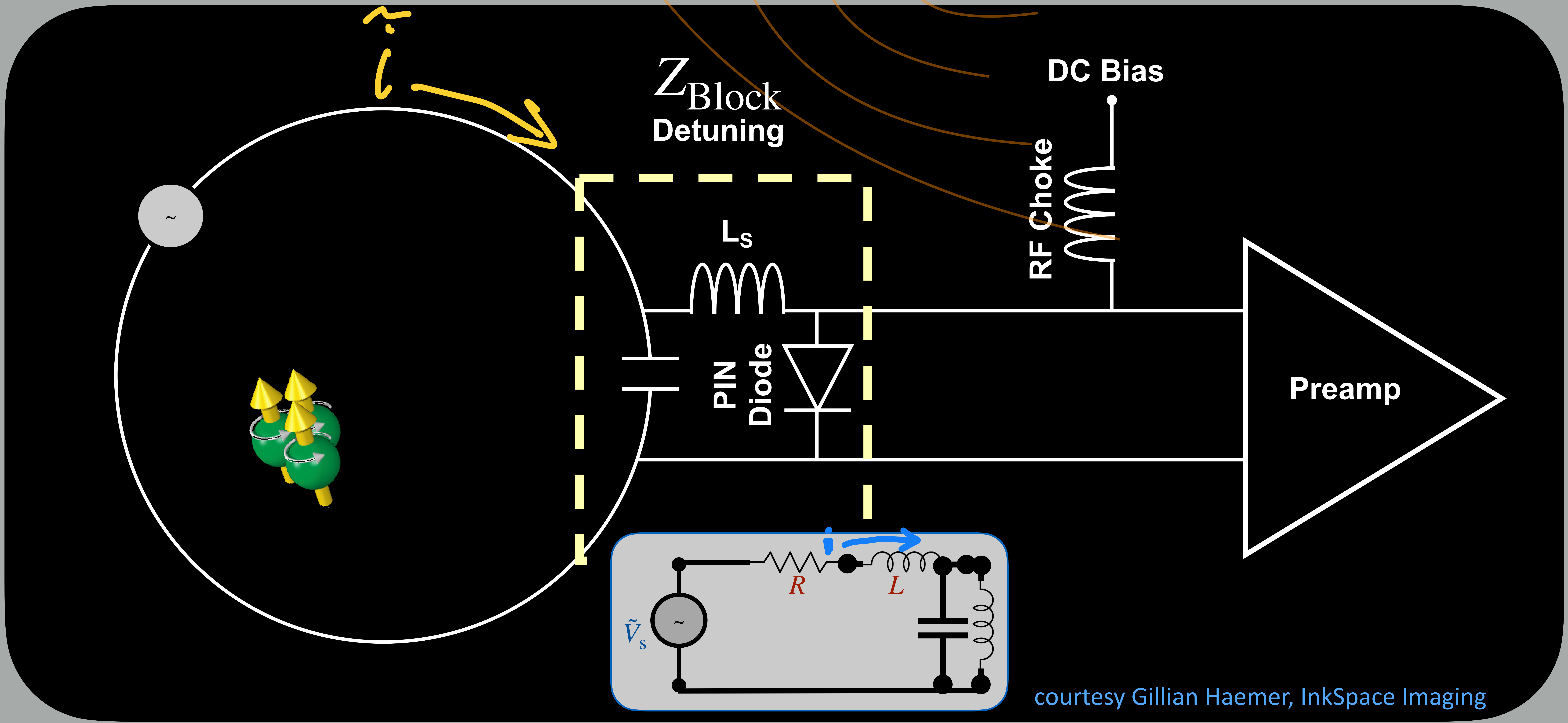
courtesy Gillian Haemer, InkSpace Imaging

Receive Coil Components: Capacitors



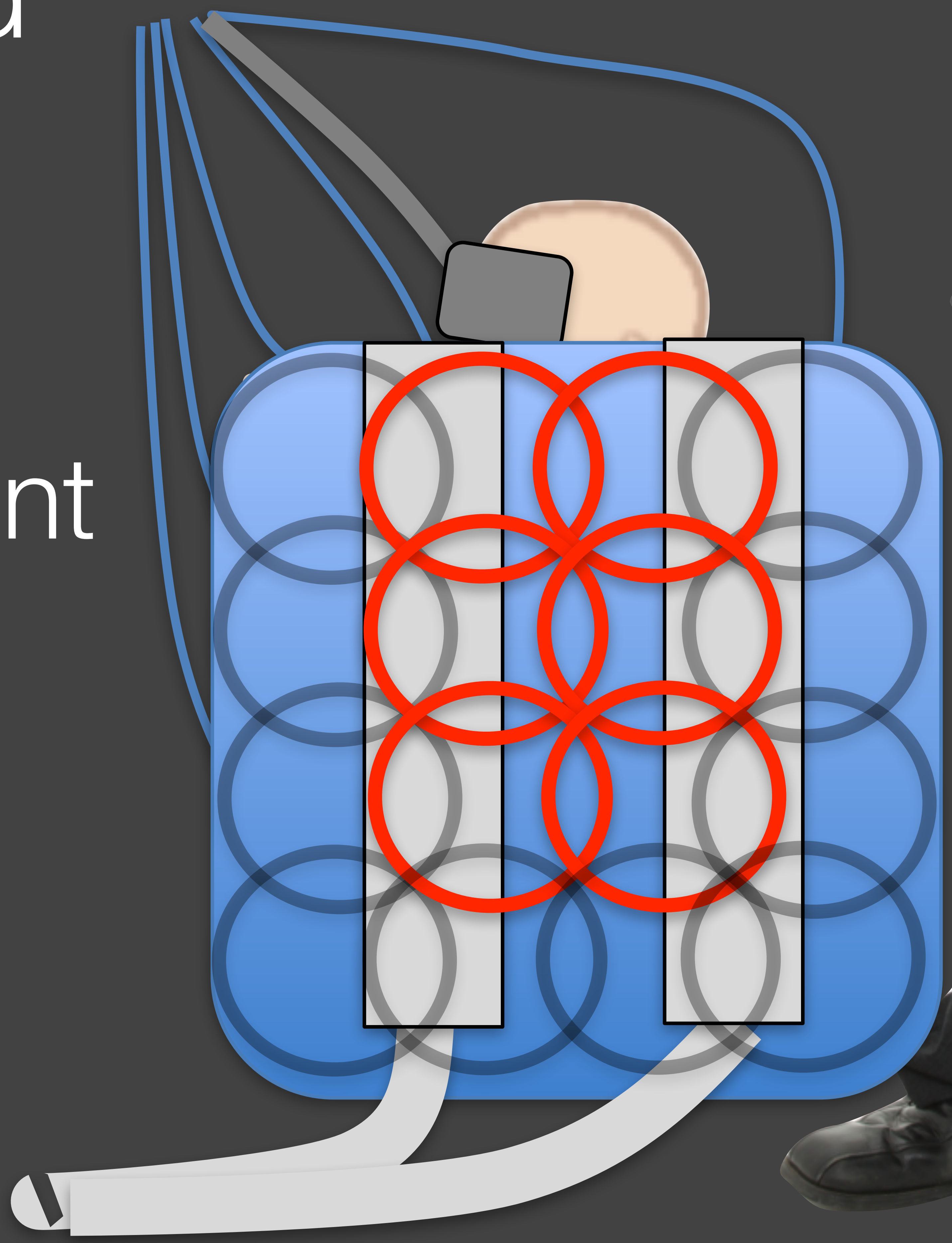
courtesy Gillian Haemer, InkSpace Imaging

Receive Coil Components: Detuning



Pediatric Coil Arrays

- Array density, coverage and fit are key
- Often adult coils are used
  - Limits SNR
  - Limits acceleration
  - Limits patients management



Origin Story (2011)….

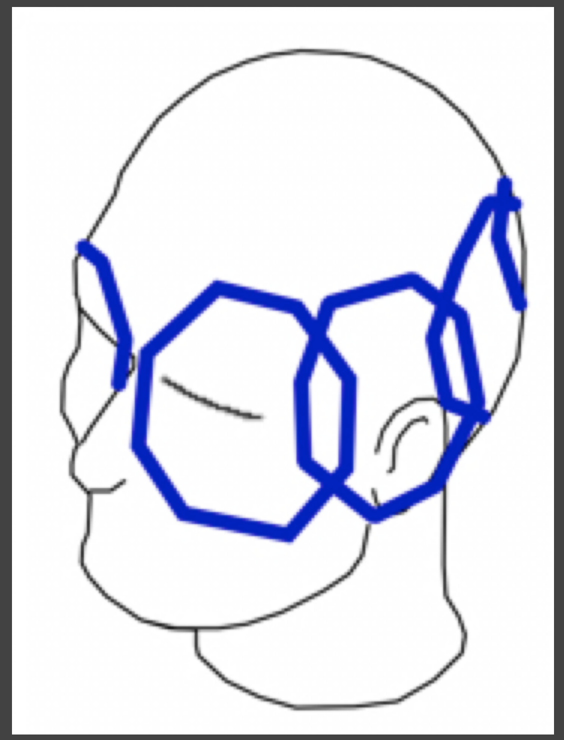
AC Arias

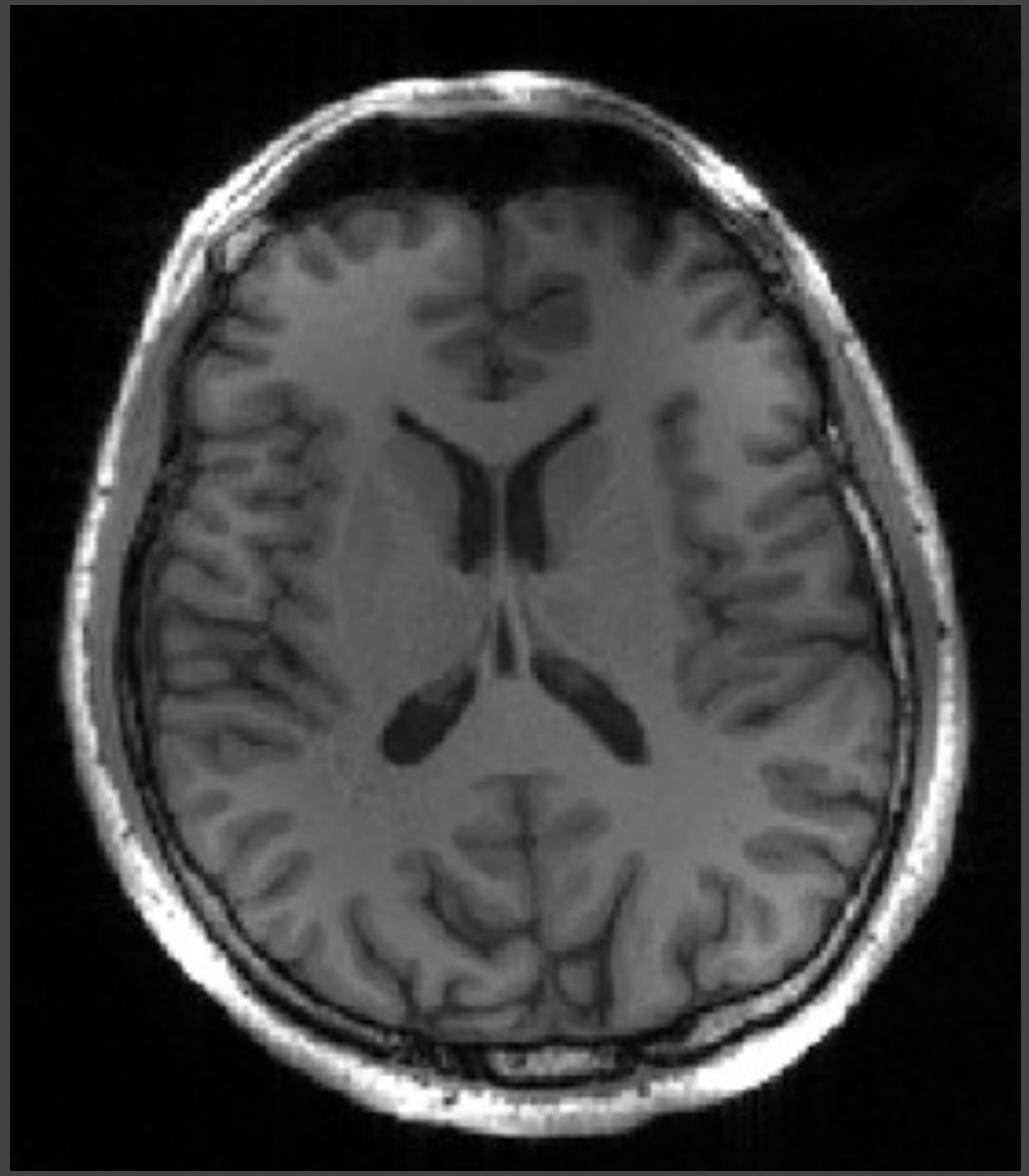
The Dream

Substrate loaded into screen printer

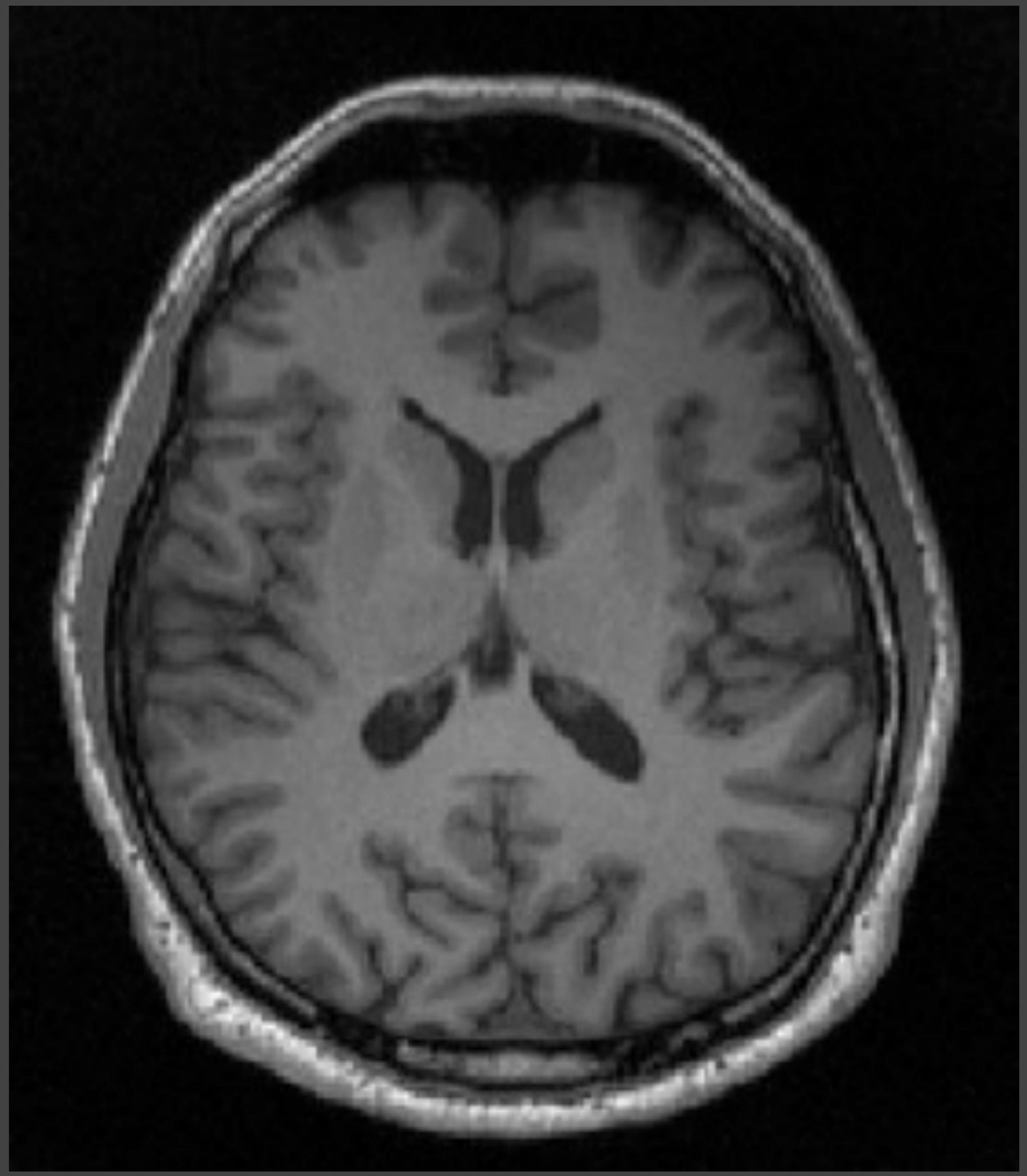
J. Corea, AC Arias et al. Nature comm. 2016;7:10839. UC Berkeley

1st Volunteer Imaging 2013





8 Channel Printed Array

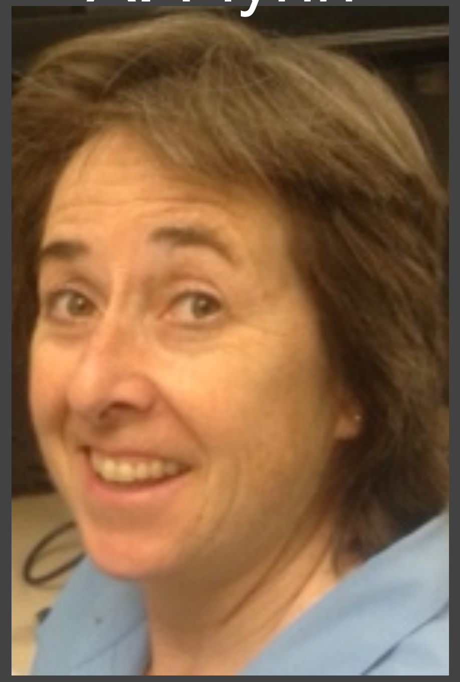


12 Channel Coil

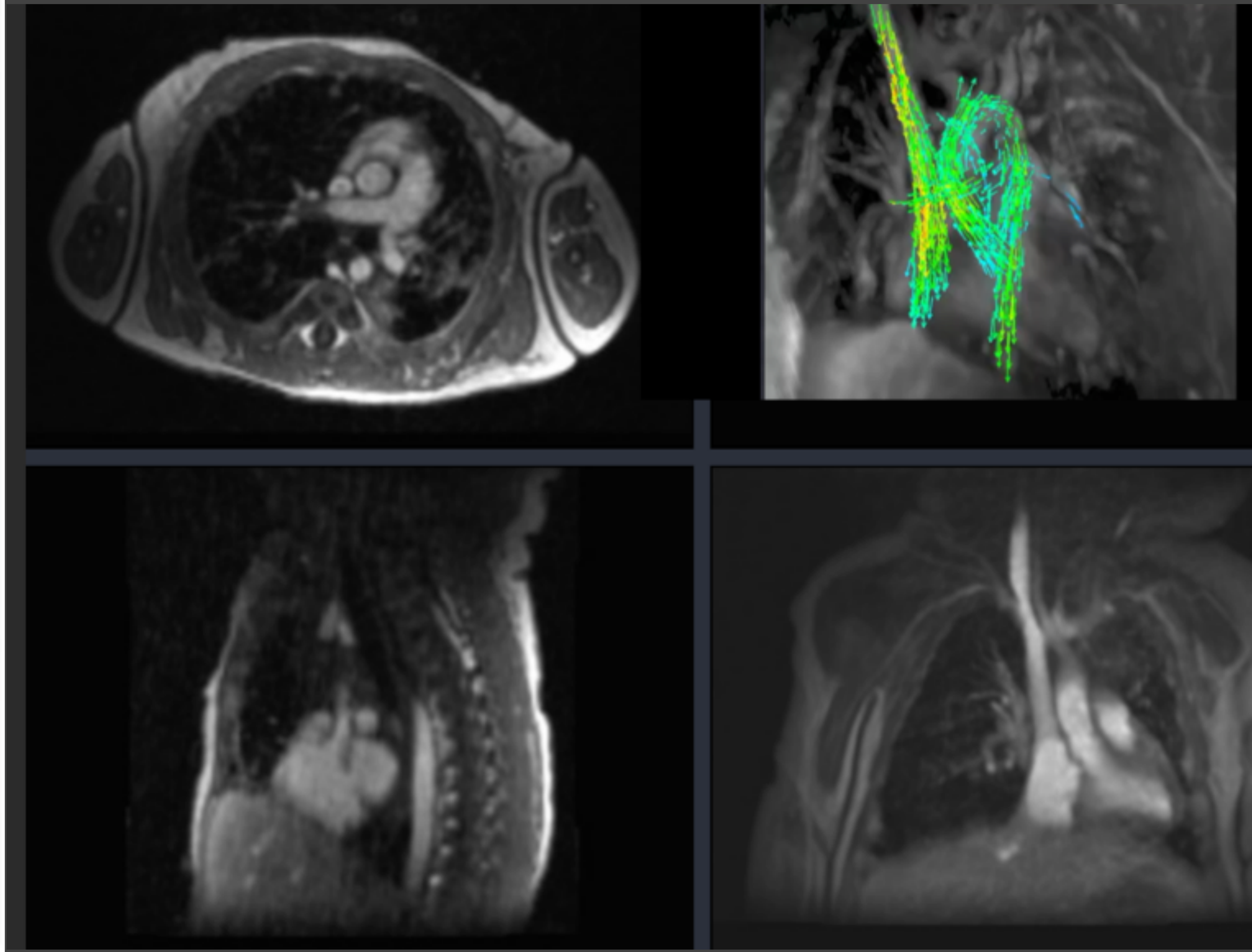
J. Corea

Baby Hope – preparing for patient imaging 2014

A. Flynn



1st Baby – 3kg, 10 weeks old patient



S. Vasanawala

“The Berkeley 12” - 2016

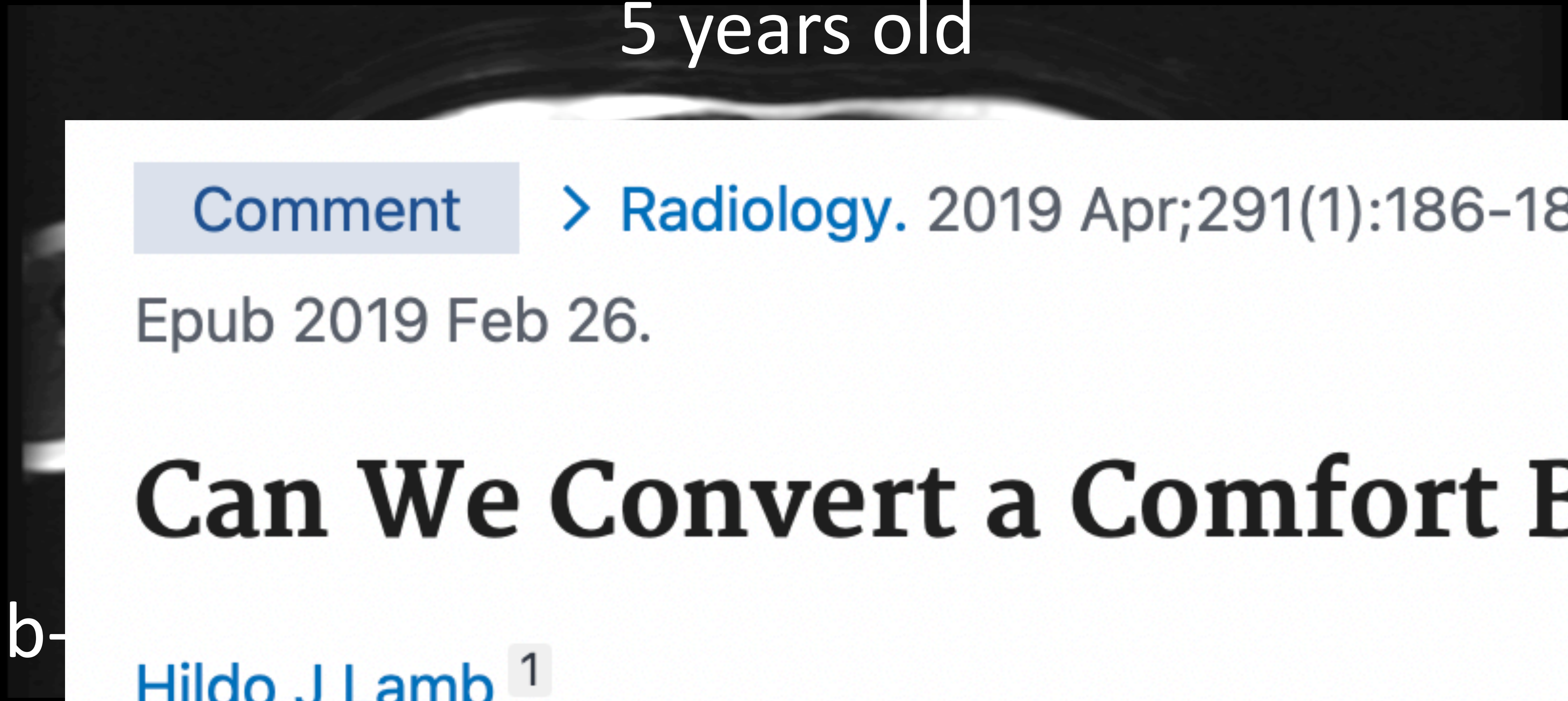
- 12ch
- Printed Antennas
- Remote Detuning
- Unique Packaging
- Weight 360gr

J. Corea

J. Corea, Lustig, Arias, *et al.* Nature comm. 2016;7:10839. UC Berkeley

J. Corea, Lustig, Arias, *et al.* MRM. 2017;78(2):775. UC Berkeley

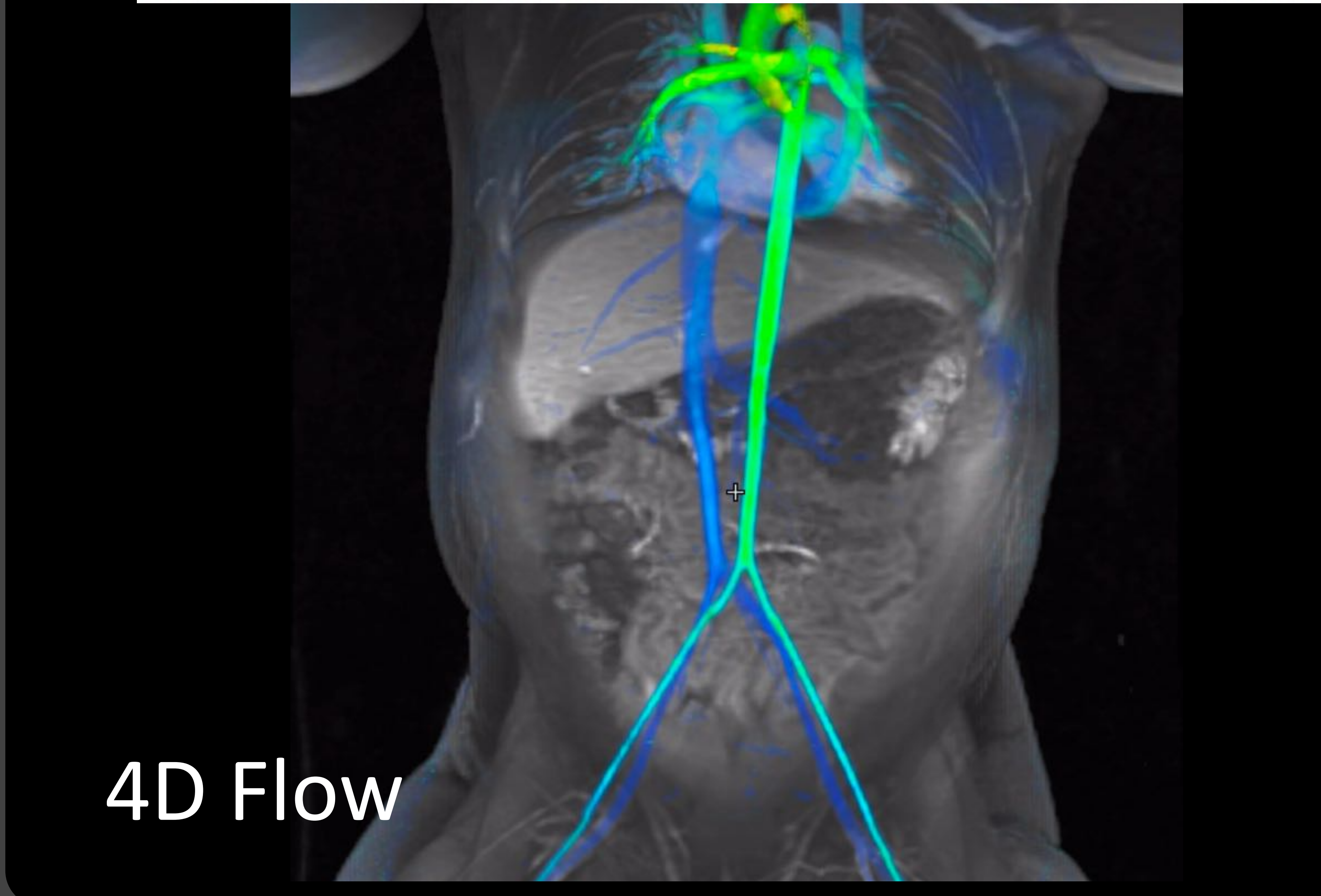
The Berkeley 12ch Array

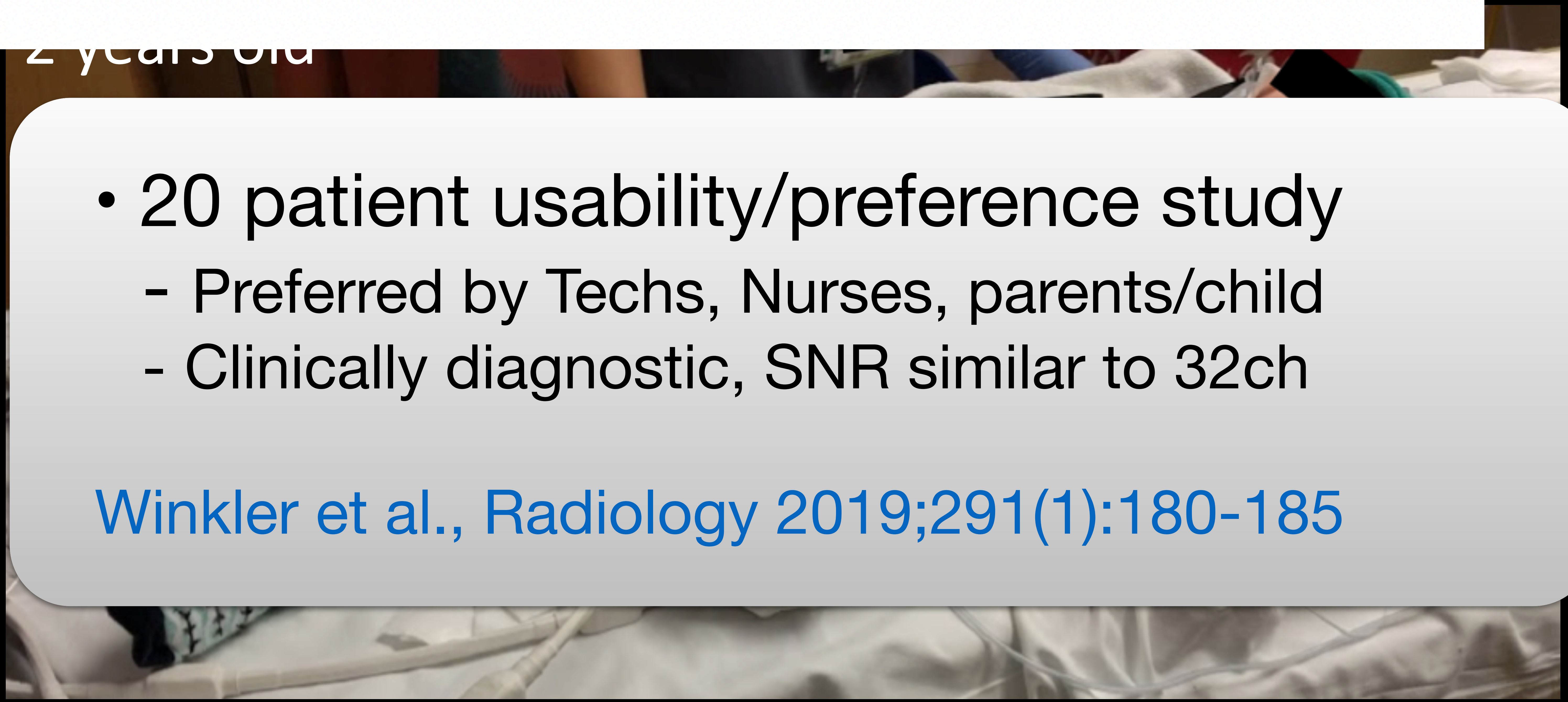


Comment > Radiology. 2019 Apr;291(1):186-187. doi: 10.1148/radiol.2019190209. Epub 2019 Feb 26.

Can We Convert a Comfort Blanket to an MRI Coil?

Hildo J Lamb [1]





- 20 patient usability/preference study
  - Preferred by Techs, Nurses, parents/child
  - Clinically diagnostic, SNR similar to 32ch

Winkler et al., Radiology 2019;291(1):180-185

Founded InkSpace Imaging 2017

INKSPACE
IMAGING

Improving the *comfort*, *reliability*, and *performance* of MRI exams

M. Lustig

AC Arias

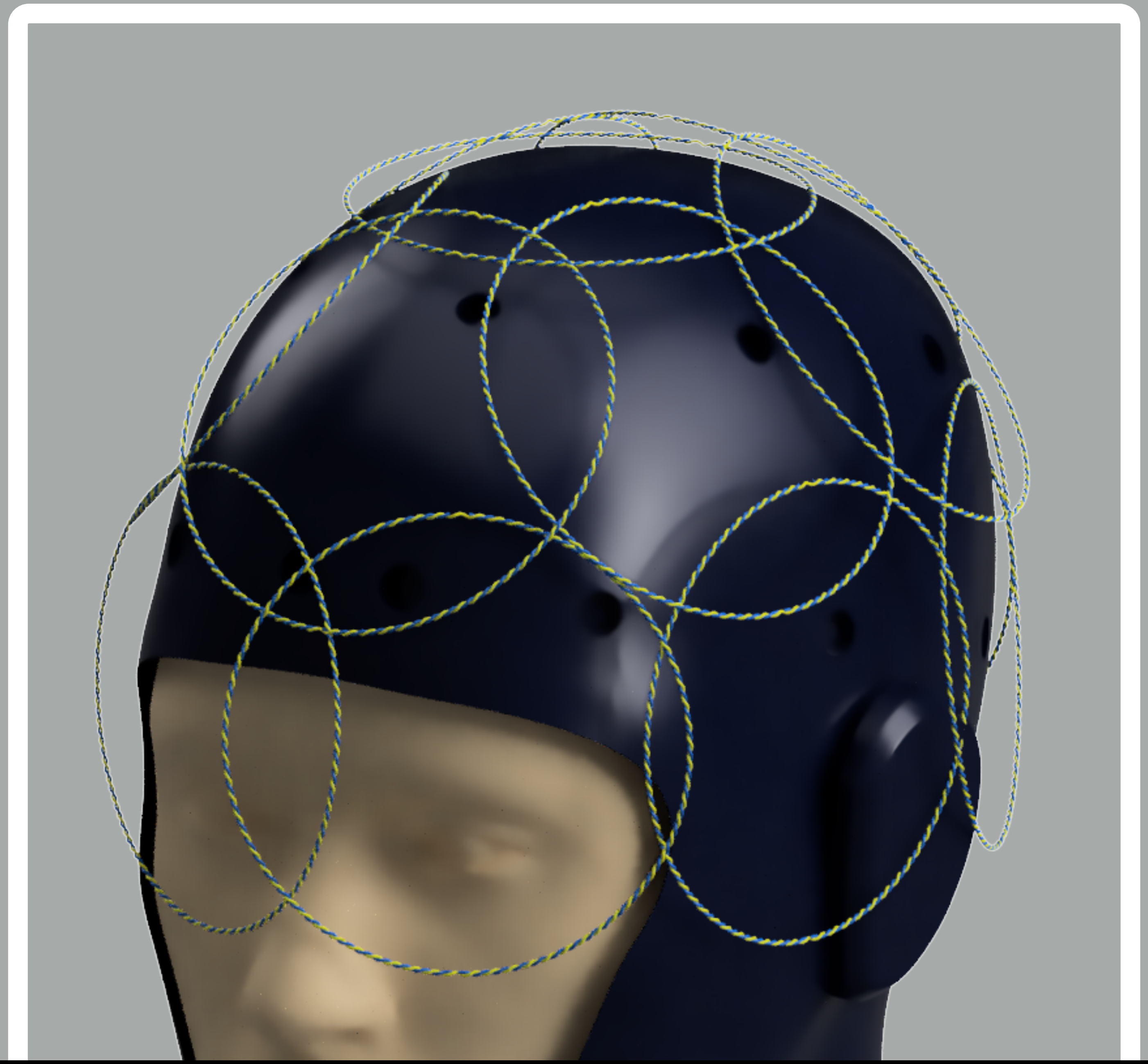
B. Lechêne

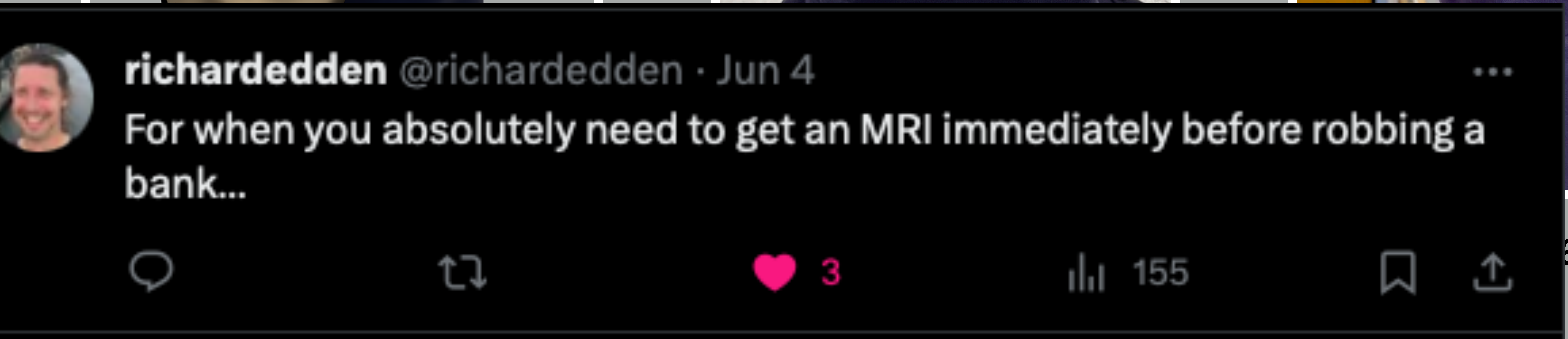
J. Corea

Pediatric 24ch printed array

- Technology commercialized by InkSpace Imaging
  - FDA 510K Approved as of 12/2021

15-Channel Twisted Wire Head Array





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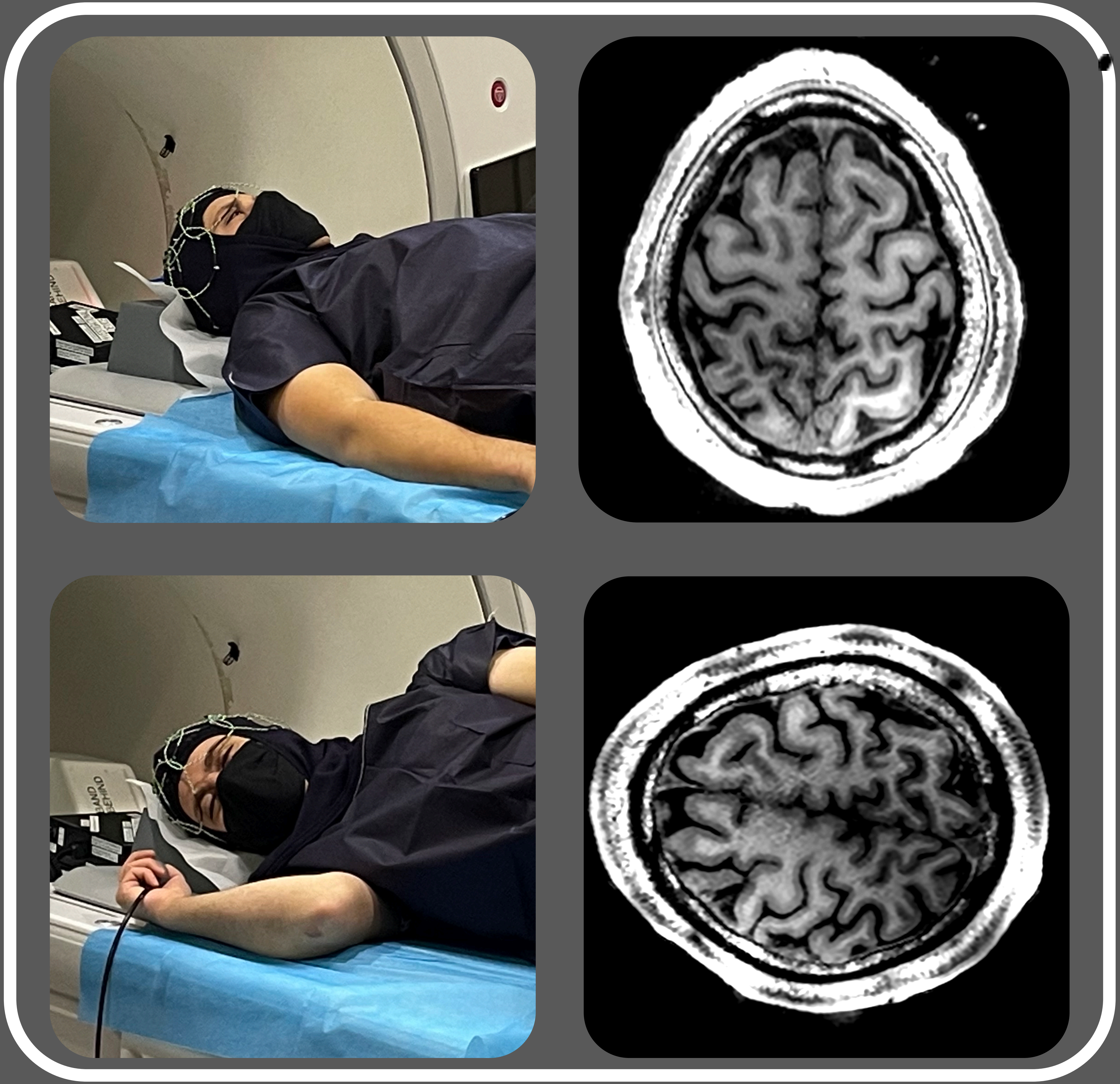
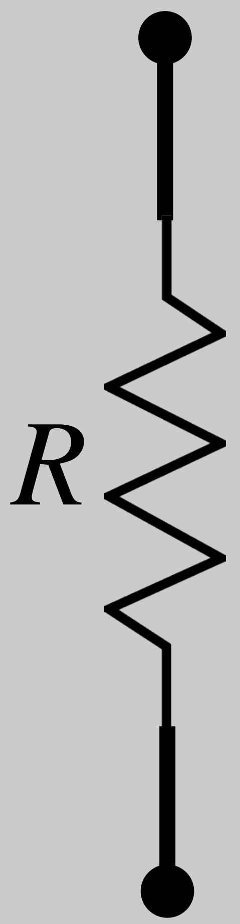
Multiple Positions, compatible with EEG



What now?

AC Voltage/Current of a Resistive Load



$$V_{\mathrm{R}}(t) = V_0 \cos(\omega t)$$

Phasor form:

$$V_{\mathrm{R}}(t) = \tilde{V}_{\mathrm{R}} e^{j\omega t}$$

Amplitude: $|\tilde{V}_{\mathrm{R}}| = V_0$

Phase: $\angle \tilde{V}_{\mathrm{R}} = 0$

[Hz]

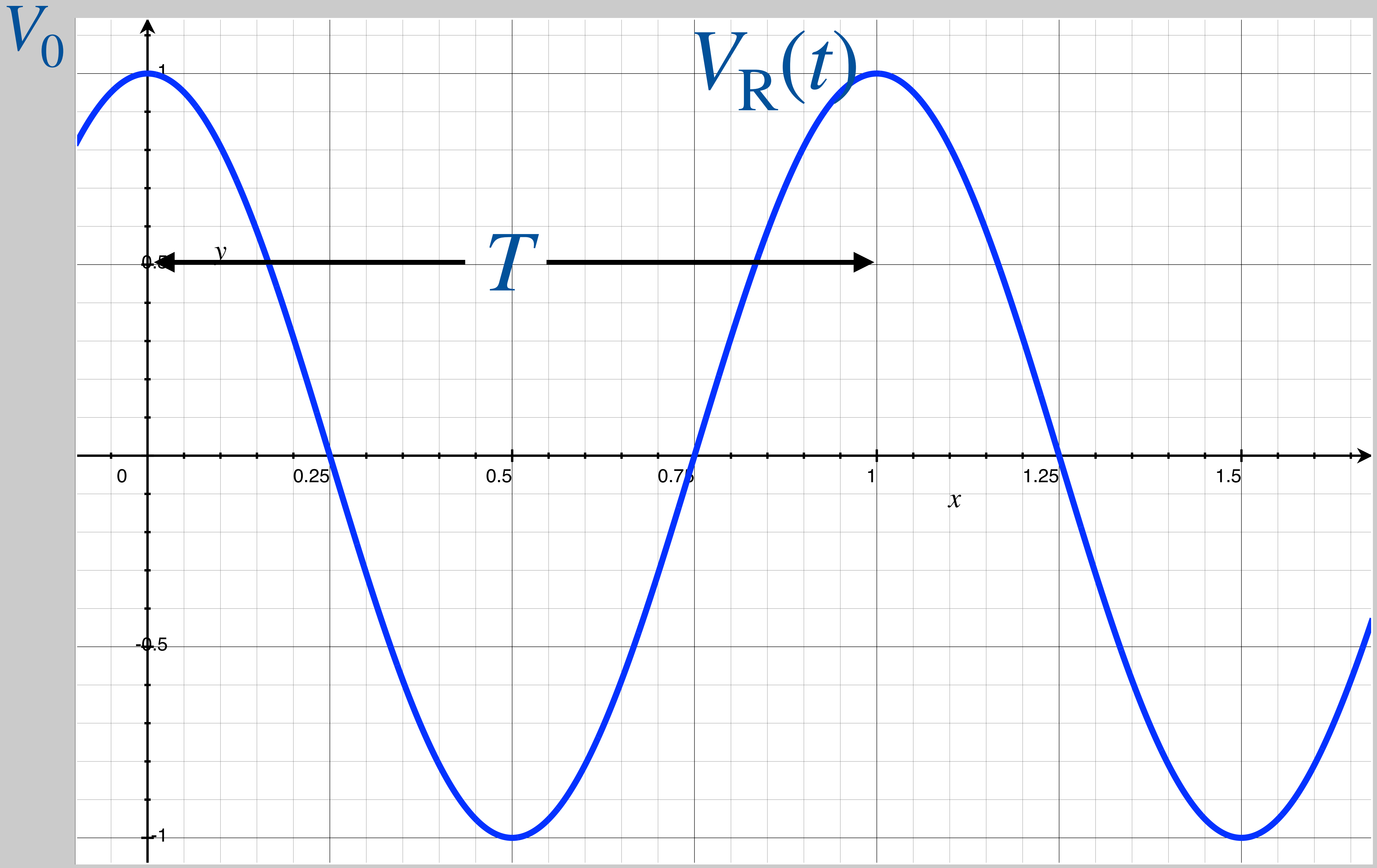
Frequency: $\omega = \dfrac{2\pi}{T} = 2\pi f \text{ [rad/s]}$

[s]

Resistive Load:

$$i_{\mathrm{R}}(t) = \frac{V_{\mathrm{R}}(t)}{R} = \frac{\tilde{V}_{\mathrm{R}} e^{j\omega t}}{R}$$

$$\tilde{I}_{\mathrm{R}} = \frac{\tilde{V}_{\mathrm{R}}}{R} \quad \Rightarrow i_{\mathrm{R}}(t) = \frac{V_0}{R}\cos(\omega t)$$



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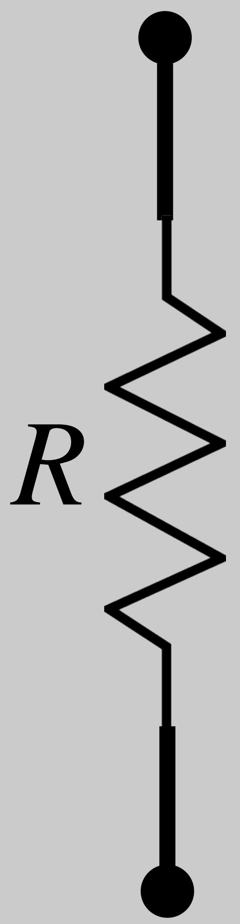

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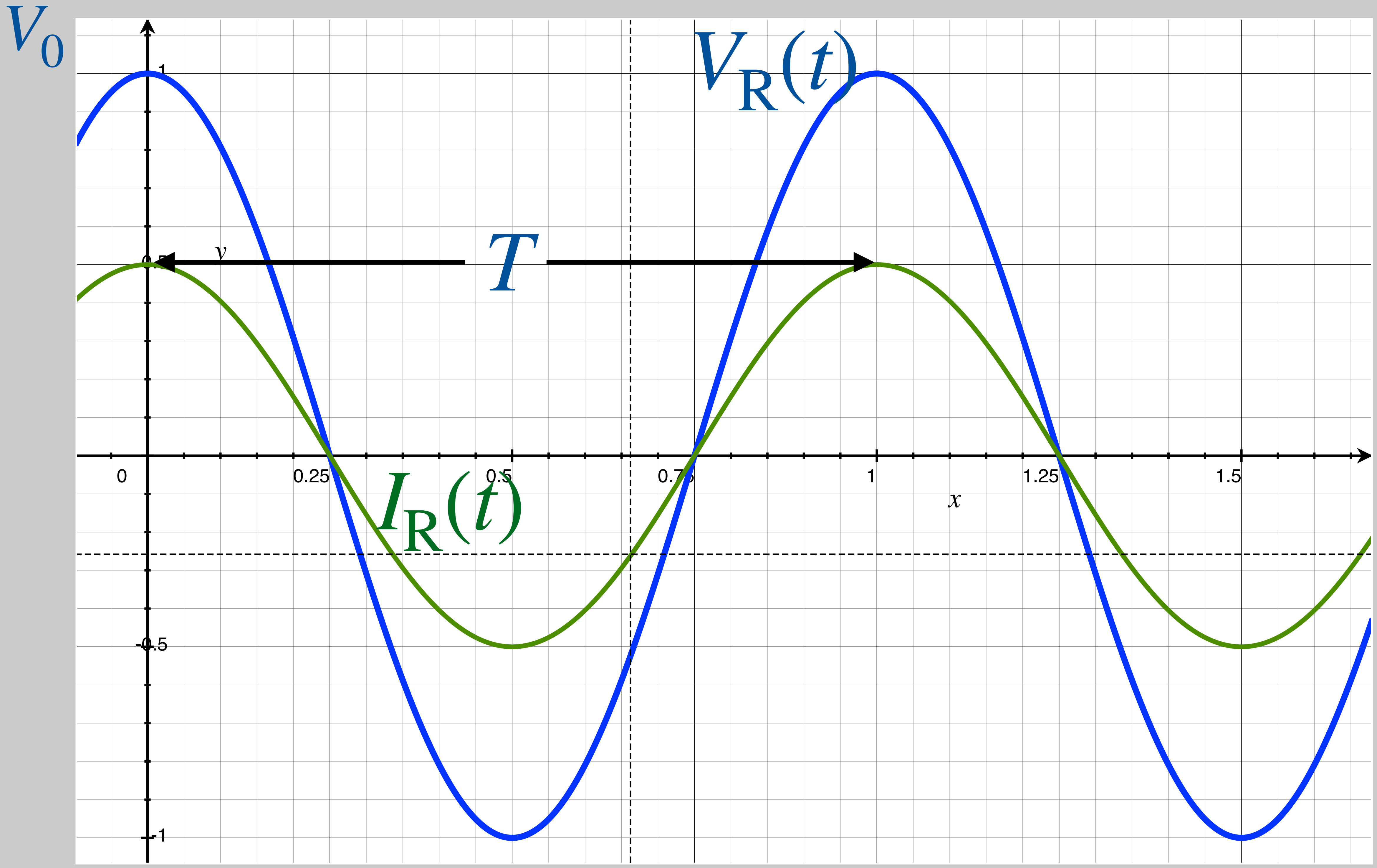
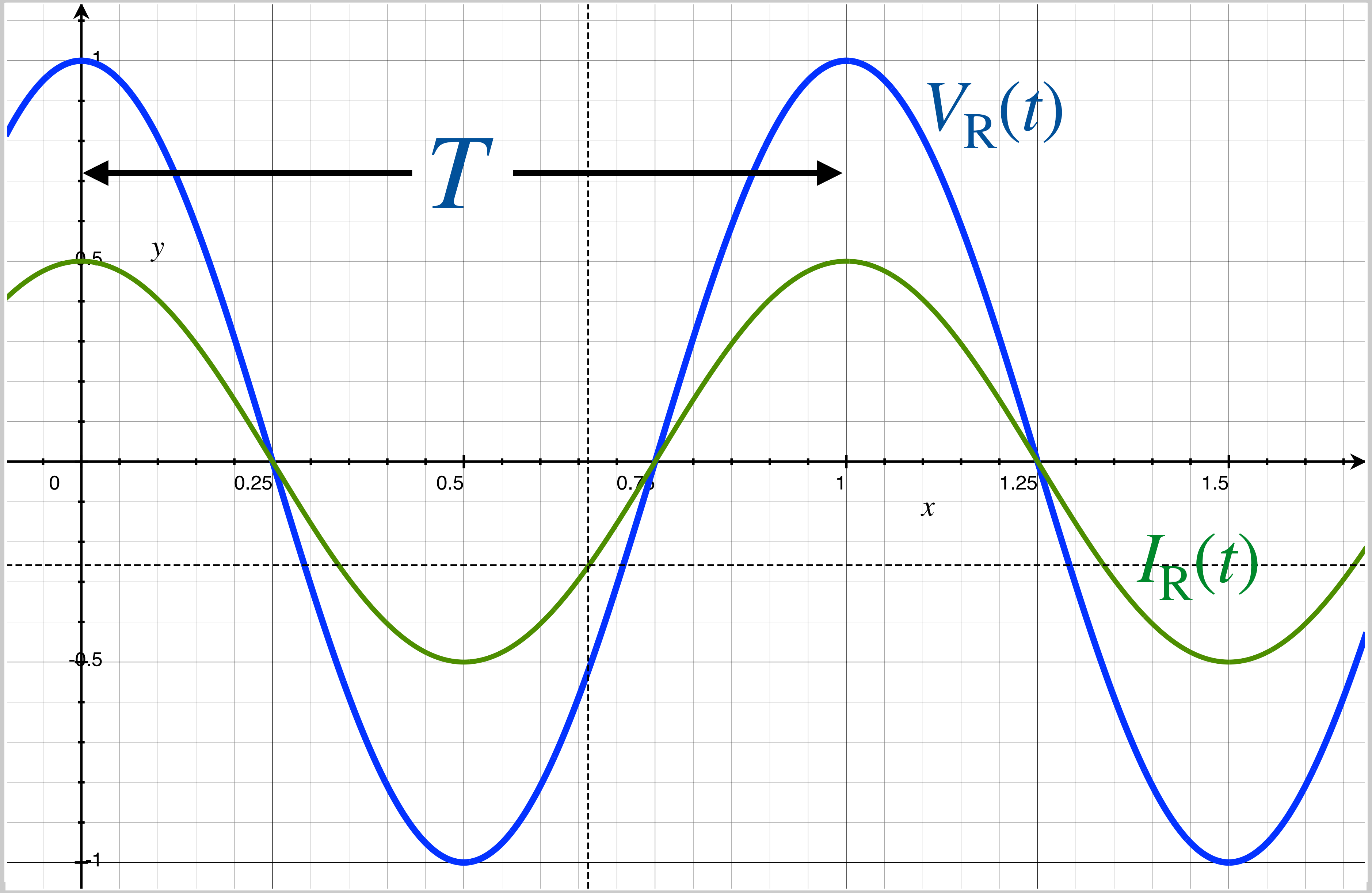
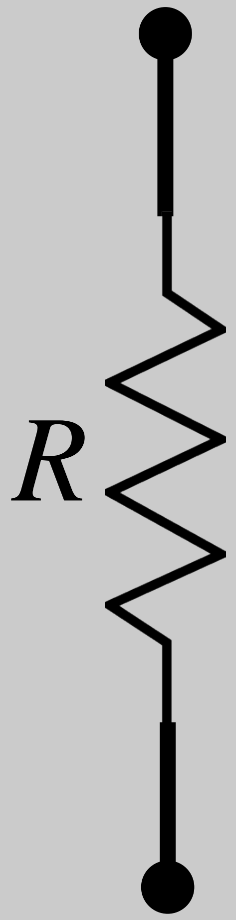


AC Power of a Resistive Load

$$V_R(t) = V_0 \cos(\omega t) \qquad i_R(t) = \frac{V_0}{R}\cos(\omega t)$$

$$p(t) = V_R(t) i_R(t) \;= \frac{V_0^2}{R}\cos^2(\omega_t)$$





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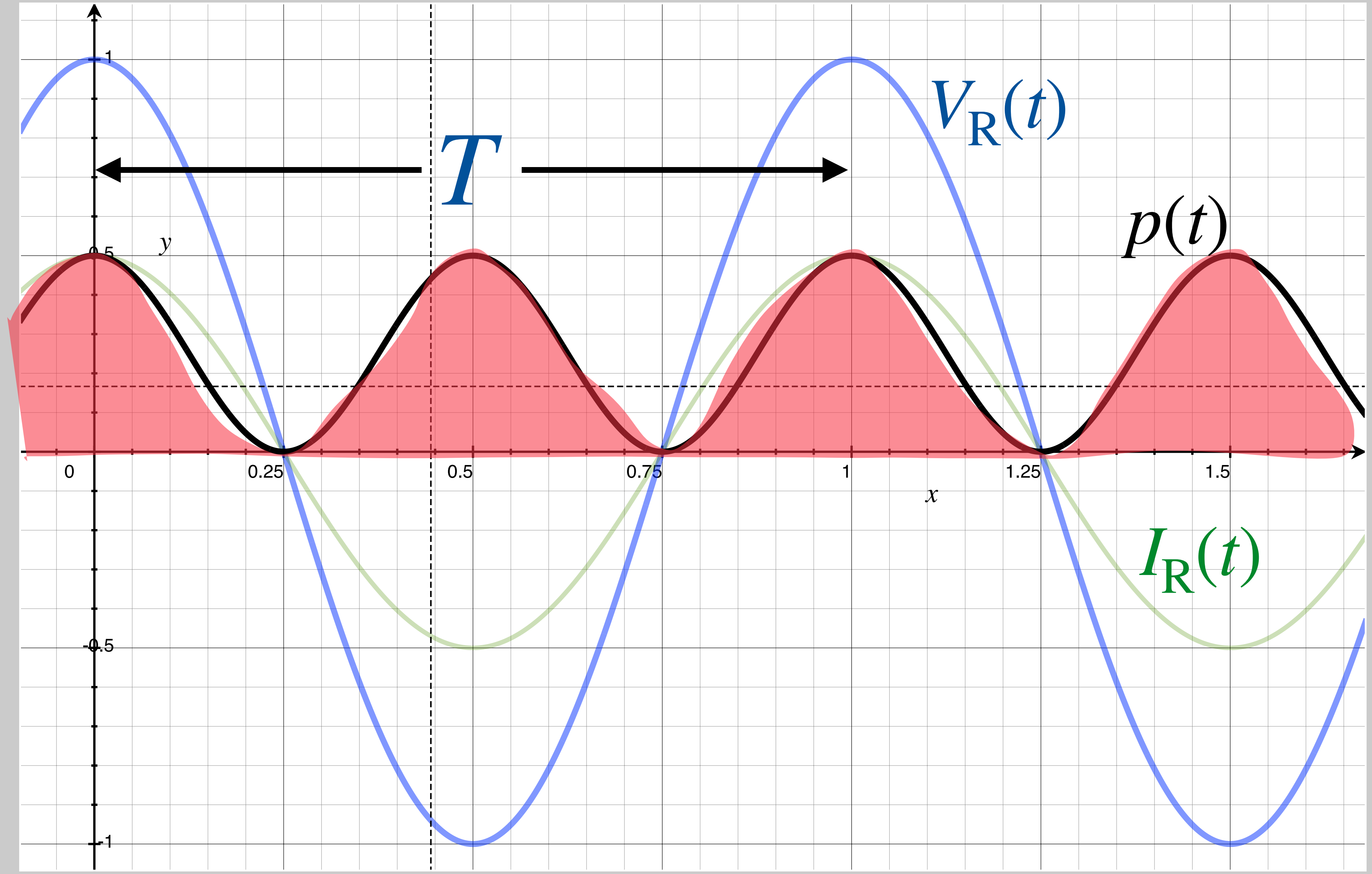
Power consumed on positive and negative cycles!

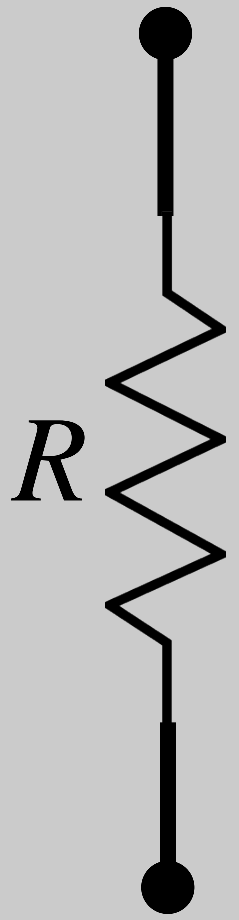
$$p_{avg} = \frac{1}{T}\int_0^T p(\tau)d\tau \; = \frac{V_0^2}{R}\frac{1}{T}\int_0^T \cos^2(\omega\tau)d\tau \; = \frac{1}{2}\frac{V_0^2}{R}$$

Phasor form: $\tilde{V}_R \qquad \tilde{I}_R = \frac{\tilde{V}_R}{R}$

$$\cancel{p(t) = \tilde{V}_R \tilde{I}_R ???}$$

$$p_{avg} = \frac{1}{2}\tilde{V}_R \tilde{I}_R^* = \frac{1}{2}\frac{|\tilde{V}_R|^2}{R} = \frac{1}{2}\frac{V_0^2}{R}$$



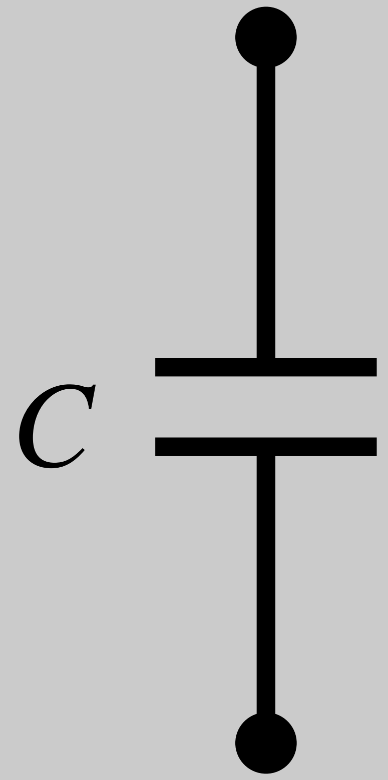


AC Voltage/Current of a Capacitive Load

$$V_c(t) = V_0 \cos(\omega t)$$

Phasor form:

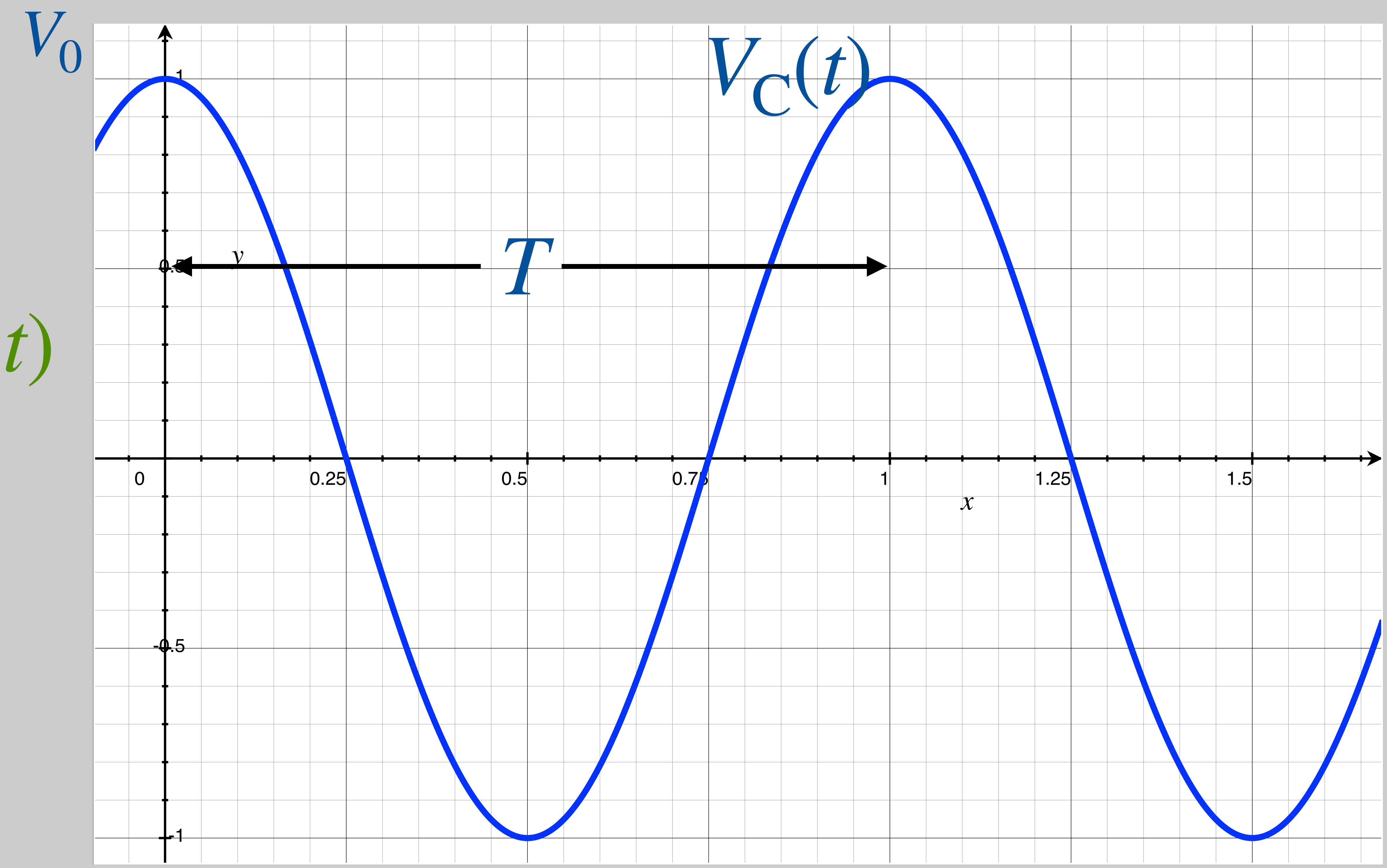
$$V_c(t) = \tilde{V}_c e^{j\omega t}$$



Capacitive Load:

$$\tilde{I}_c = \frac{\tilde{V}_c}{Z_c} = \frac{\tilde{V}_c}{\frac{1}{j\omega C}} = \omega C \tilde{V}_c e^{j\frac{\pi}{2}}$$

$$\Rightarrow i_C(t) = \omega C V_0 \cos(\omega t + \frac{\pi}{2}) = -\omega C V_0 \sin(\omega t)$$



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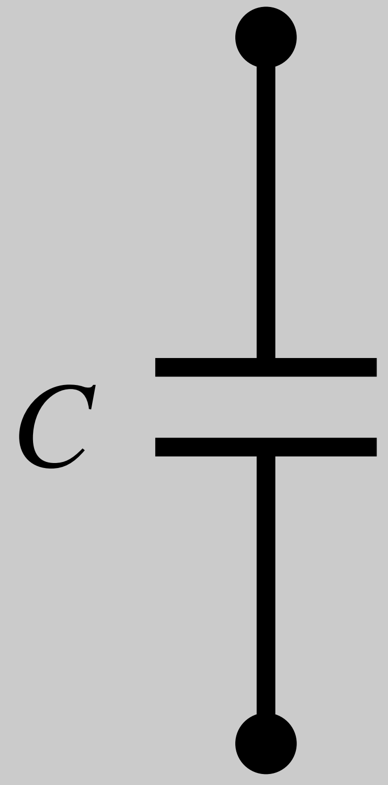

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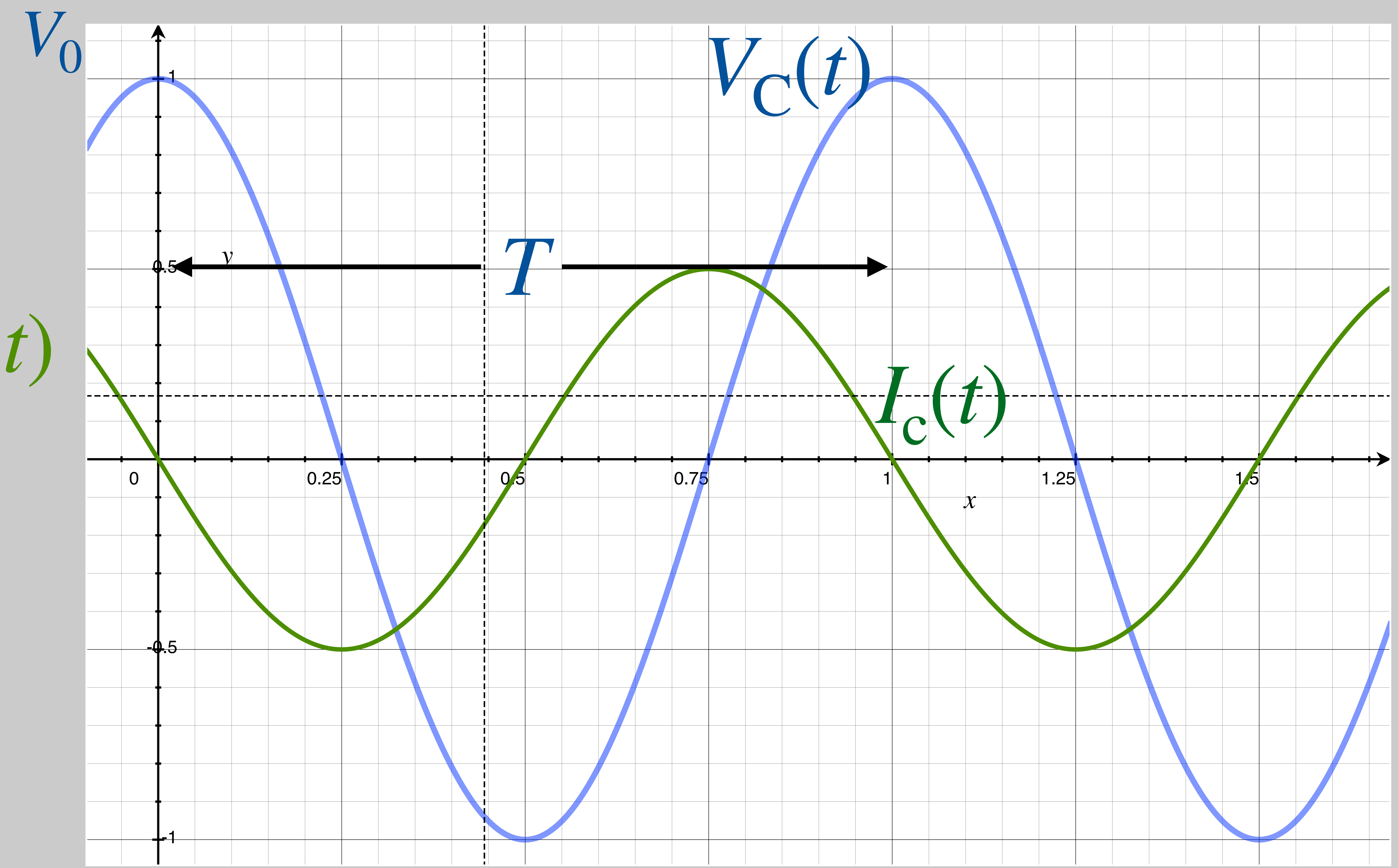
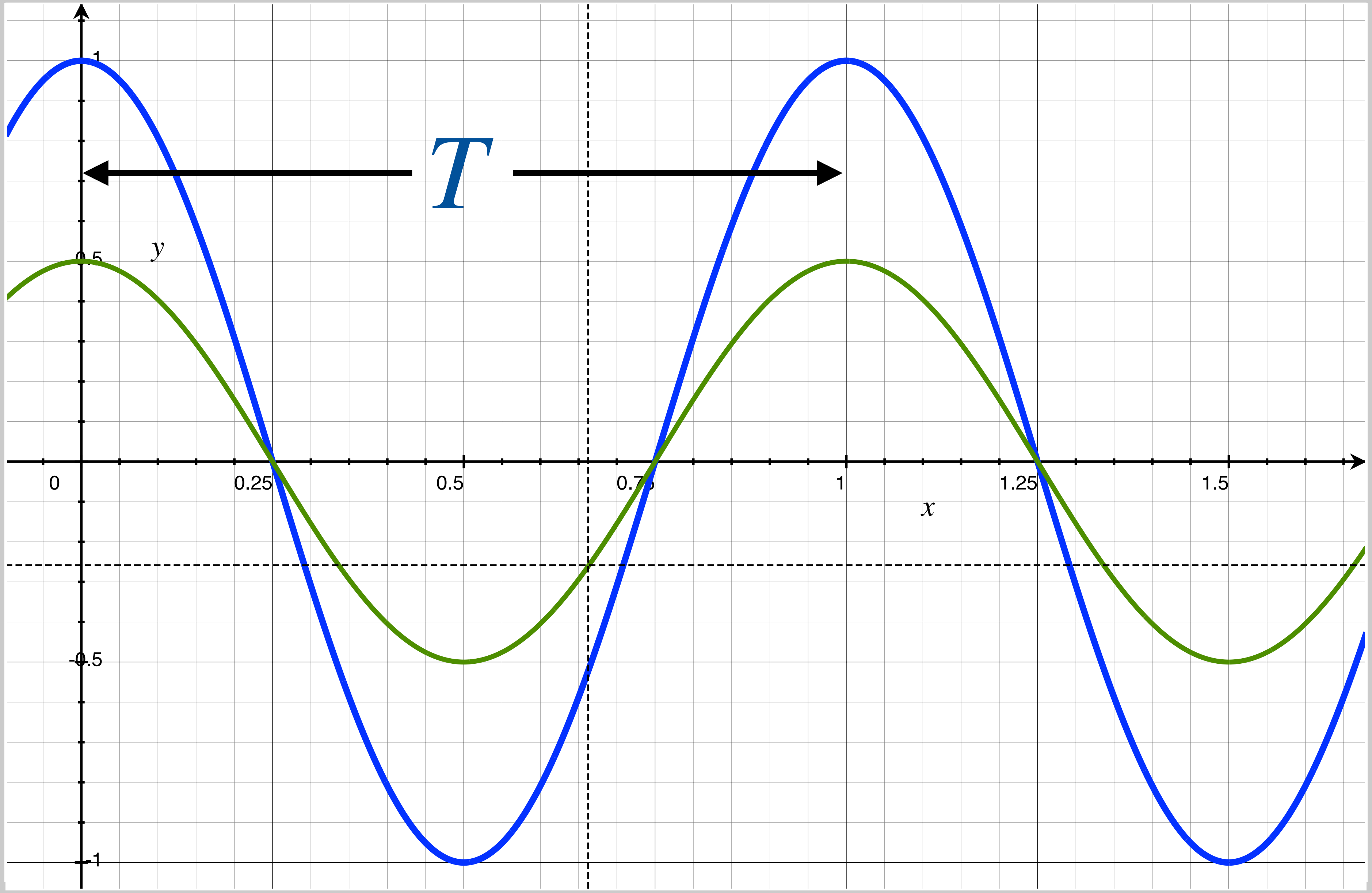
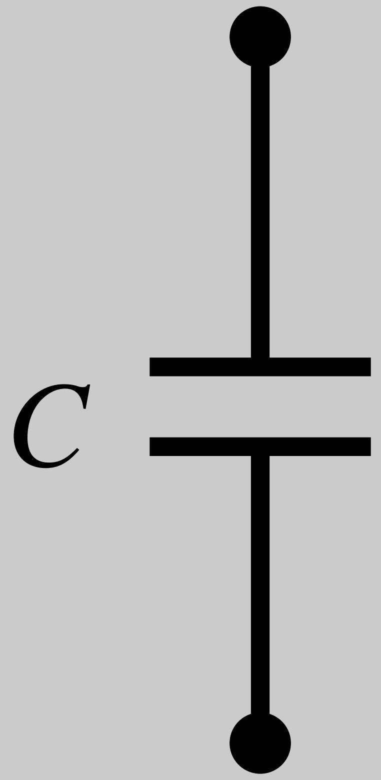


AC Power of a Capacitive Load

$V_C(t) = V_0 \cos(\omega t)$ $\qquad$ $i_C(t) = -\omega C V_0 \sin(\omega t)$





$$p(t) = V_R(t) i_R(t) = -\omega C V_0^2 \cos(\omega t)\sin(\omega t)$$

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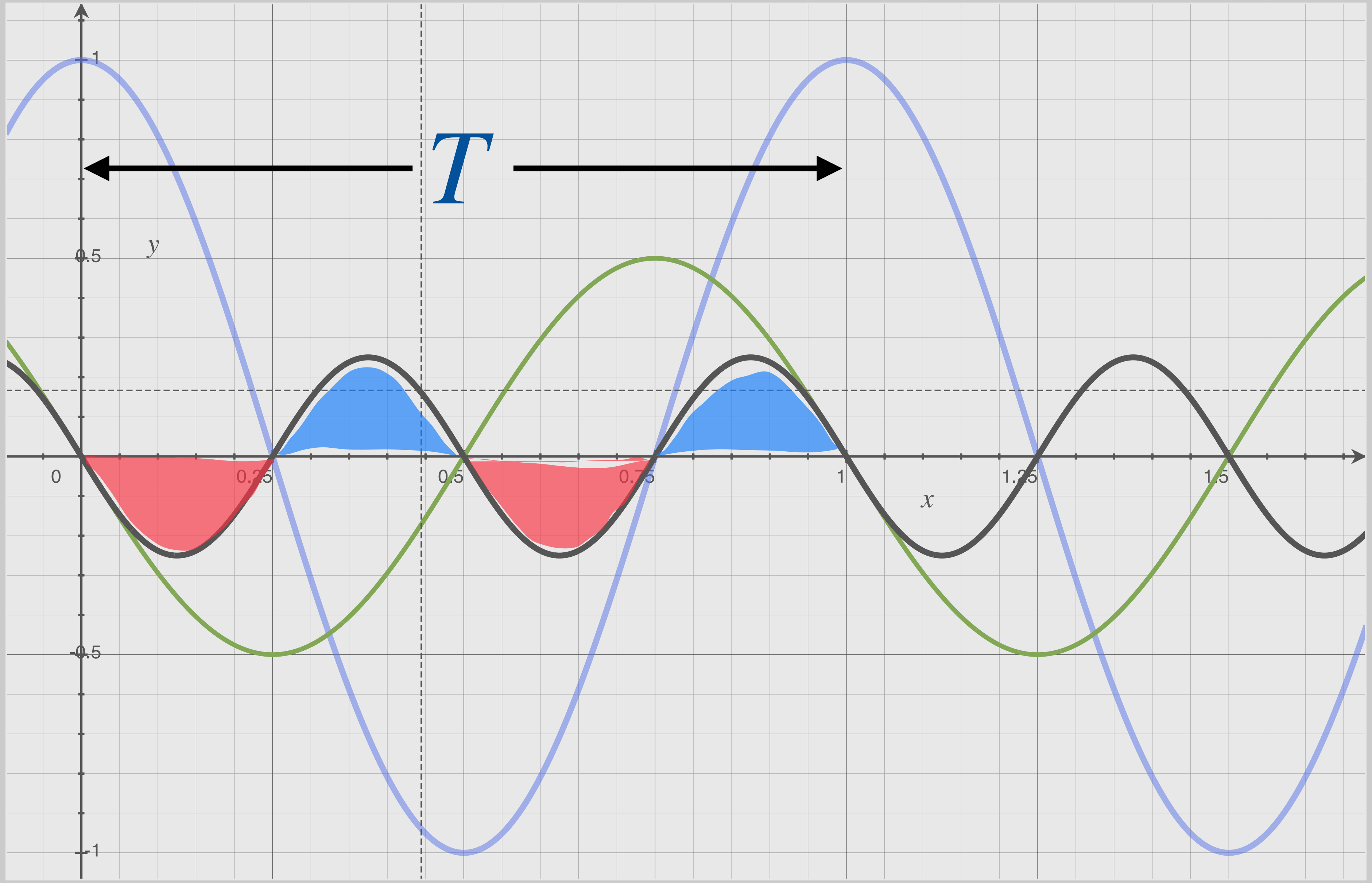
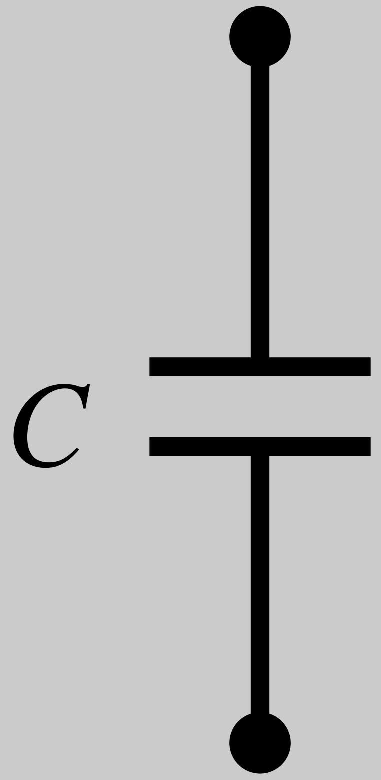




$$p(t) = V_R(t) i_R(t) = -\omega C V_0^2 \cos(\omega t) \sin(\omega t)$$

Power oscillates between positive and negative twice a cycles! Power goes back and forth!

$$p_{\mathrm{avg}} = \frac{1}{T} \int_0^T p(\tau) d\tau \ = 0$$

Phasor form: $\tilde{V}_c$ $\quad$ $\tilde{I}_c = j\omega C \tilde{V}_c$

$$p_{\mathrm{avg}} = \frac{1}{2} \tilde{V}_c \tilde{I}_c^* \ = \frac{1}{2} j\omega C V_0^2$$

Pure imaginary average power!

Also called reactive power — average power stored in a cycle

AC Power

Resistor: $\tilde{I}_R = \frac{\tilde{V}_R}{R}$ $\quad p_{avg} = \frac{1}{2}\frac{V_0^2}{R}$ — Pure real resistive/ dissipating power

Capacitor $\tilde{I}_c = j\omega C \tilde{V}_c$ $\quad p_{avg} = jC\frac{1}{2}V_0^2$ — Pure imaginary Reactive power

Inductor $\tilde{I}_L = \frac{\tilde{V}_L}{j\omega L}$ $\quad p_{avg} = \frac{1}{2}\frac{V_0^2}{j\omega L}$ — Pure imaginary Reactive power

General complex valued power: $\tilde{I} = I_0 e^{j\theta_1}$ $\quad \tilde{V} = V_0 e^{j\theta_2}$

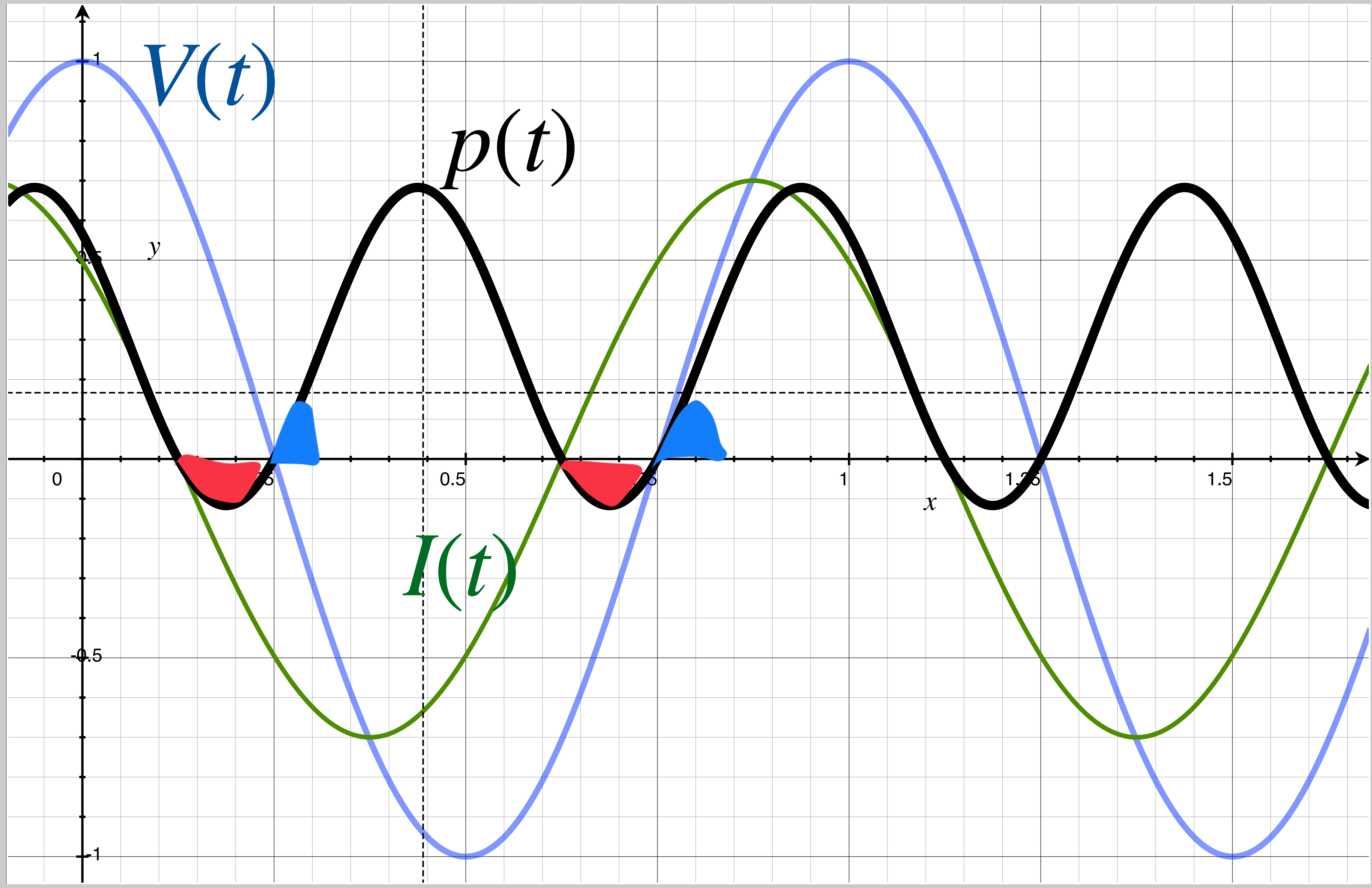
$$p_{avg} = \frac{1}{2}\tilde{V}\tilde{I}^* = V_0 I_0 e^{j(\theta_2-\theta_1)}$$

AC Power

Resistor: $\tilde{I}_\mathrm{R} = \dfrac{\tilde{V}_\mathrm{R}}{R}$ $\qquad p_{\mathrm{avg}} = \dfrac{1}{2}\dfrac{V_0^2}{R}$ Pure real resistive/ dissipating power

Capacitor $\tilde{I}_\mathrm{c} = j\omega C\tilde{V}_\mathrm{c}$ $\qquad p_{\mathrm{avg}} = jC\dfrac{1}{2}V_0^2$ Pure imaginary Reactive power

Inductor $\tilde{I}_\mathrm{L} = \dfrac{\tilde{V}_\mathrm{L}}{j\omega L}$ $\qquad p_{\mathrm{avg}} = \dfrac{1}{2}\dfrac{V_0^2}{j\omega L}$ Pure imaginary Reactive power



General complex valued power: $\tilde{I} = I_0 e^{j\theta_1} \qquad \tilde{V} = V_0 e^{j\theta_2}$

$$p_{\mathrm{avg}} = \frac{1}{2}\tilde{V}\tilde{I}^* = V_0 I_0 e^{j(\theta_2-\theta_1)}$$

Resistive dissipating average power: $\mathrm{Real}\{\frac{1}{2}\tilde{V}\tilde{I}^*\} = \frac{1}{2}V_0 I_0 \cos(\theta_2-\theta_1)$

Reactive (storage) average power: $\mathrm{Imag}\{\frac{1}{2}\tilde{V}\tilde{I}^*\} = \frac{1}{2}V_0 I_0 \sin(\theta_2-\theta_1)$

Grid AC Voltage and Power

$$V_{\mathrm{RMS}} = \frac{1}{\sqrt{2}} V_{\mathrm{magn}}$$

$$V_{\mathrm{pp}} = 2V_{\mathrm{magn}}$$

$$p_{\mathrm{avg}} = \frac{1}{2}\tilde{V}\tilde{I}^* = \tilde{V}_{\mathrm{RMS}}\tilde{I}^*_{\mathrm{RMS}}$$