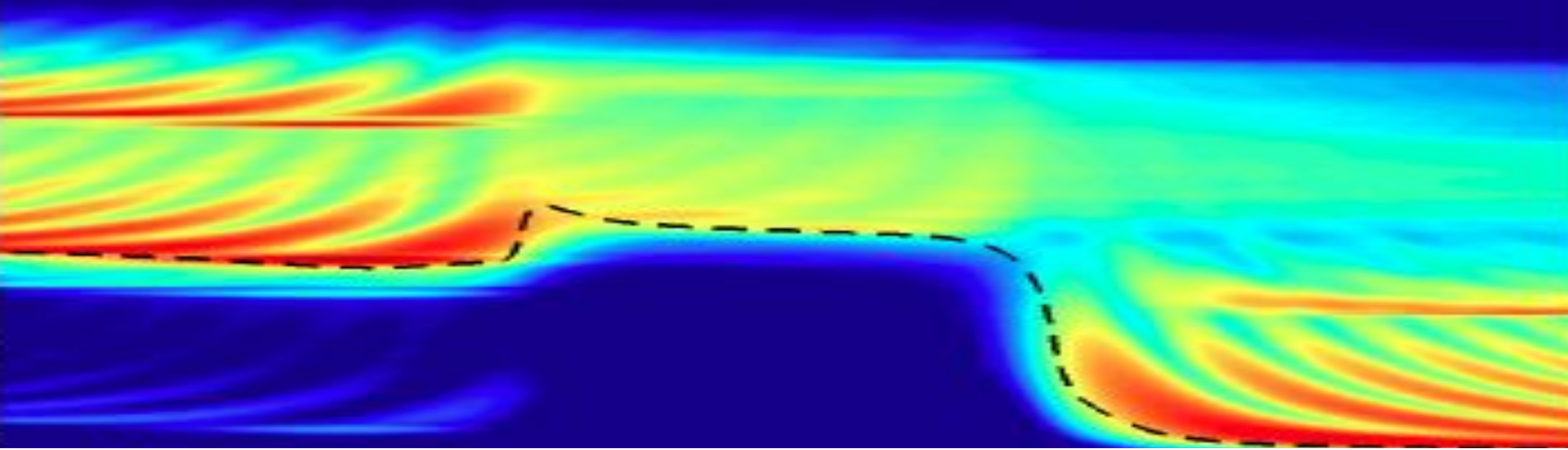

EECS 16B
Designing Information Devices and Systems II
Lecture 4

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Transient Response

- Outline
  - R-L-C Circuits
  - Phasors

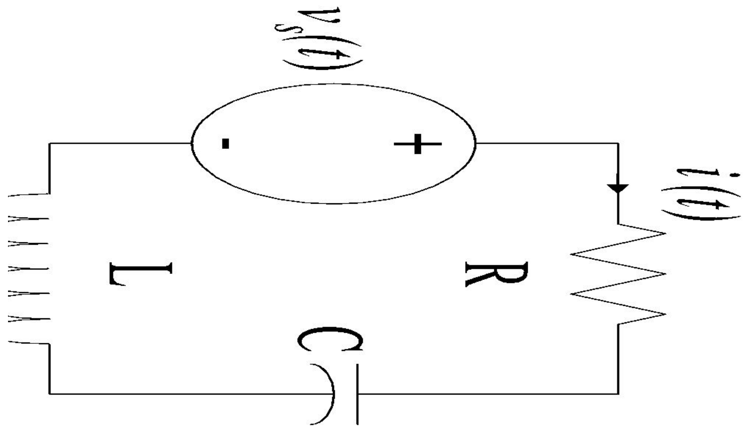
- Reading- Hambley text sections 4.5, 5.1,5.2slides

R-L-C circuits: Response in time

$v_s = iR + v_c + v_L$

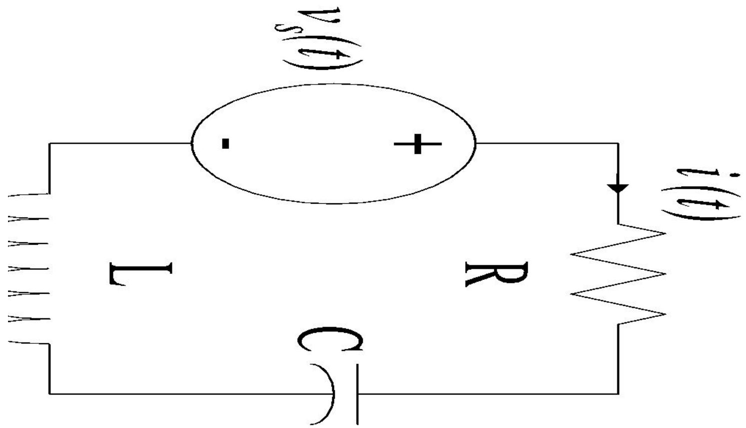
$$i = C\frac{dv_c}{dt}$$

$$v_s = RC\frac{dv_c}{dt} + v_c + L\frac{di}{dt}$$



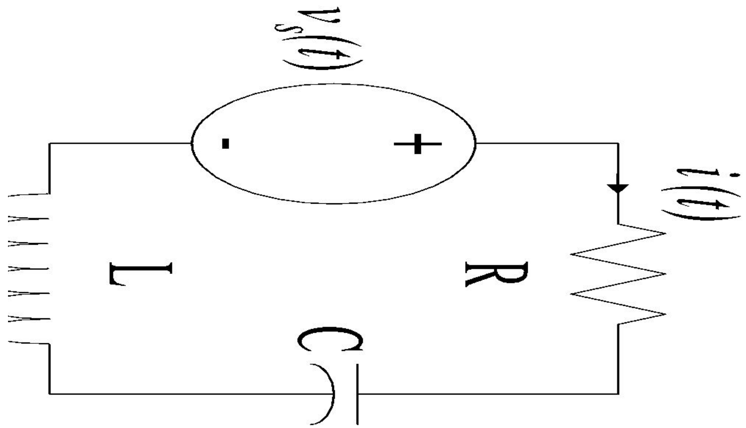
Filters, antennas, resonances

R-L-C circuits: Response in time



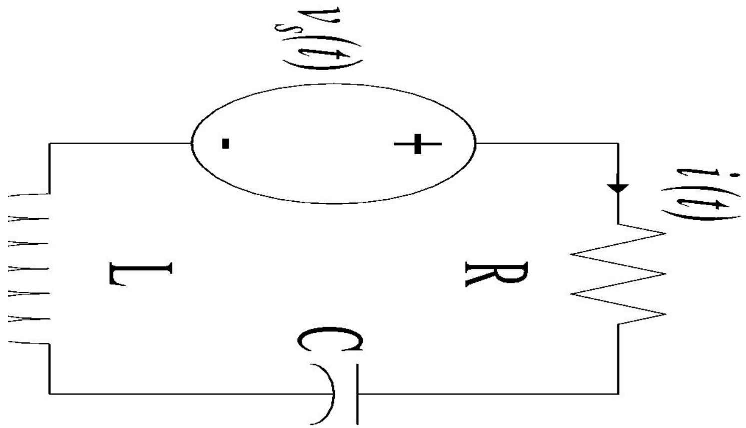
Filters, antennas, resonances

R-L-C circuits: Response in time



Filters, antennas, resonances

R-L-C circuits: Response in time



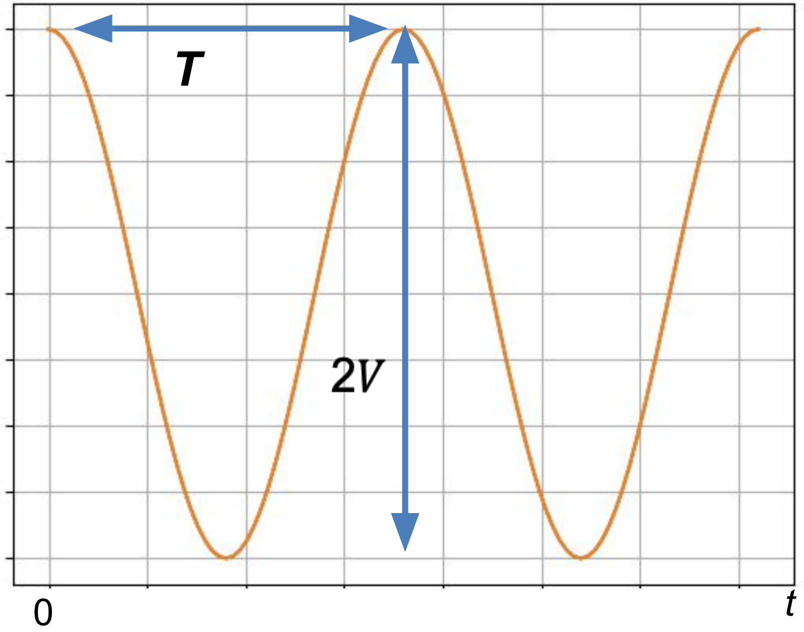
Filters, antennas, resonances

Summary

- Overdamped – real unequal roots
- Critically damped – Real Equal roots
- Underdamped – Complex roots

Sinusoidal voltages

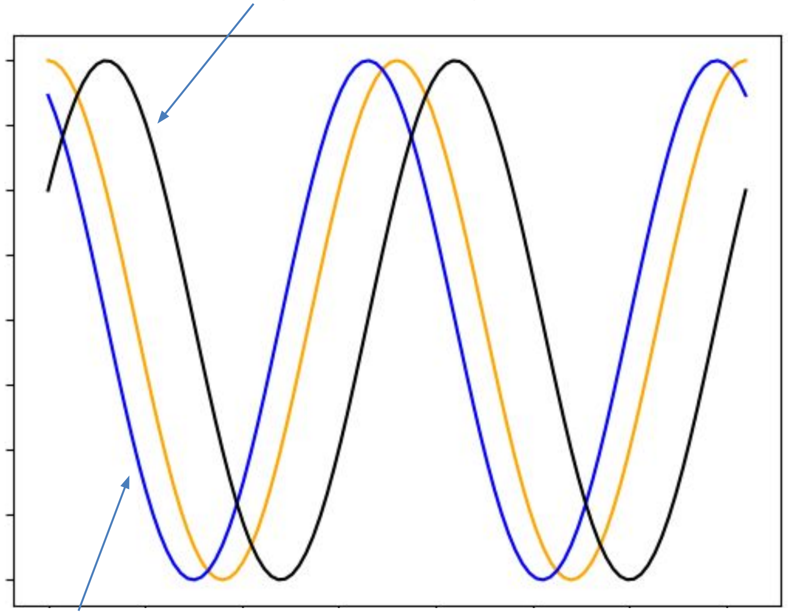
$v(t) = Vcos(\omega t)$



T : Period

$$\omega = \frac{2\pi}{T}$$

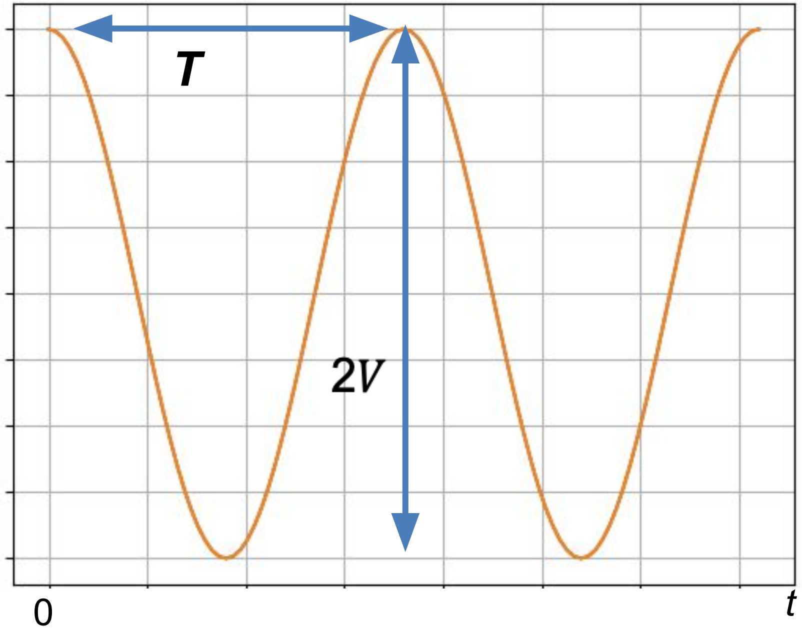
$v(t) = Vcos(\omega t - 60^0)$



$v(t) = Vcos(\omega t + 30^0)$

Root Mean Square Values

$v(t) = V cos(\omega t)$



T : Period

$$\omega = \frac{2\pi}{T}$$

Average Power over one period:

$$P = \frac{1}{T}\int_0^T \frac{v^2}{R} dt$$

$$P = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v^2 dt}\right]^2}{R}$$

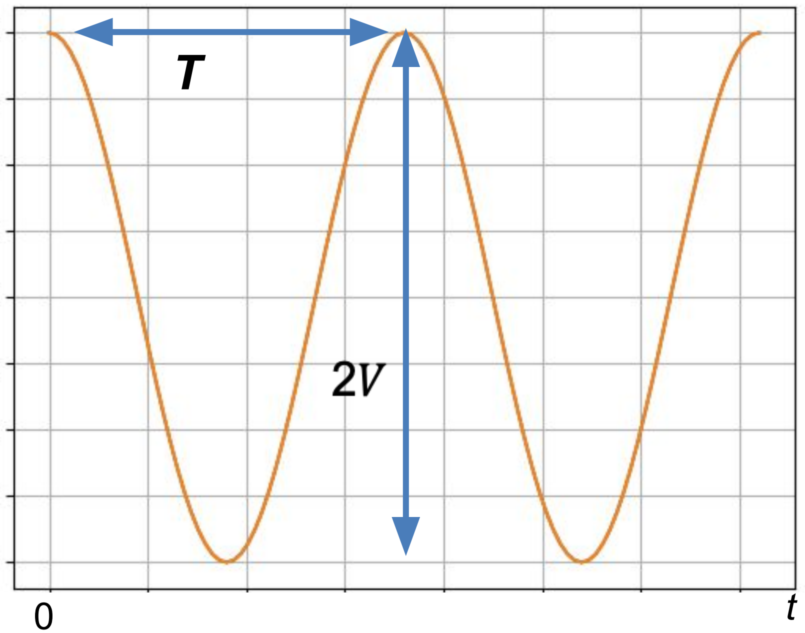
Comparing with conventional equation: P=voltage^2/R

A new quantity is defined for time-varying voltages known as the root-mean-square voltage

$$v_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

Root Mean Square Value for Sinusoidal Voltage

$$v(t) = V cos(\omega t)$$



T : Period

$$\omega = \frac{2\pi}{T}$$

Average Power over one period:

$$v_{rms}^2 = \frac{1}{T}\int_0^T v^2 dt = \frac{1}{T}\int_0^T V^2 \cos^2(\omega t) dt = \frac{1}{2}\frac{1}{T} V^2 \int_0^T (1 + cos2\omega t) dt$$

$$v_{rms}^2 = \frac{V^2}{2T}(t + 2sin2\omega t)_0^T = \frac{V^2}{2T}[T - 0 + sin2\omega T - 0] = \frac{V^2}{2}$$

$$v_{rms} = \frac{V}{\sqrt{2}}$$

How do we add arbitrary sinusoids?

$v(t) = 10cos\omega t + 5sin\omega t - 5\cos(\omega t - 30^0)$

$v(t) = 10cos\omega t + 5\cos(\omega t - 90) - 5\cos(\omega t - 30^0)$

Remember? cos (a + b) = cos a cos b - sin a sin b

Lets do it it differently $e^{j\theta} = cos\theta + jsin\theta$

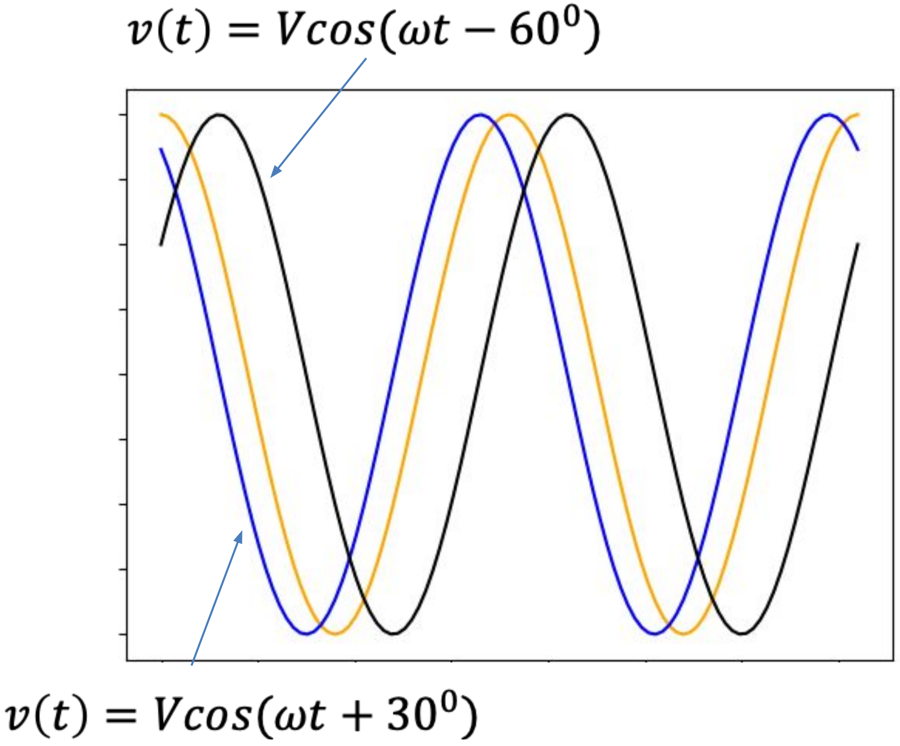
$e^{-j\theta} = cos\theta - jsin\theta$

Then

$$cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$sin\theta = \frac{1}{2}\left(e^{j\theta} - e^{-j\theta}\right)$$

$v(t) = 10cos\omega t + 5\cos(\omega t - 90) - 5\cos(\omega t - 30^0) = 1/2e^{j\theta}[10 + 5e]$



How do we add arbitrary sinusoids?

$$v(t) = 10cos\omega t + 5\cos(\omega t - 90^0) - 5\cos(\omega t - 30^0)$$

$$= \frac{1}{2}e^{j\omega t}[10] + \frac{1}{2}e^{j(\omega t - 90^0)}[5] - \frac{1}{2}e^{j(\omega t - 30^0)}[5]$$

$$+ \frac{1}{2}e^{-j\omega t}[10] + \frac{1}{2}e^{-j(\omega t - 90^0)}[5] - \frac{1}{2}e^{-j(\omega t - 30^0)}$$

$$cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

$$sin\theta = \frac{1}{2}(e^{j\theta} - e^{-j\theta})$$

$$= \frac{1}{2}e^{j\omega t}[10 + 5e^{-j90} - 5e^{-j30}] + \frac{1}{2}e^{-j\omega t}[10 + 5e^{+j90} - 5e^{+j30}]$$

$$= \frac{1}{2}e^{j\omega t}[10 + 5cos90 - j5sin90 - 5cos30 + j5sin30] + cc$$

$$= \frac{1}{2}e^{j\omega t}\left[10 + 0 - j5 - 5\frac{\sqrt{3}}{2} + \frac{j5}{2}\right] + cc$$

$$= \frac{1}{2}e^{j\omega t}[5.66 - j2.5] + cc$$

$$= \frac{1}{2}6.18e^{j(\omega t - 23^0)} + cc$$

$$= 6.18\cos(\omega t - 23^0)$$

$$Ae^{-j\theta} = 5.66 - j2.5$$

$$Ae^{j\theta} = 5.66 + j2.5$$

$$A^2 = 5.66^2 - j^2 2.5^2$$

$$A^2 = 5.66^2 + 2.5^2$$

$$A = \sqrt{5.66^2 + 2.5^2} = 6.18$$

$$cos\theta = 5.66/6.18; sin\theta = 2.5/6.18$$

$$tan\theta = \frac{2.5}{5.66} = 0.44$$

$$\theta = 0.41 = 23^0$$

Some Observations

$$v(t) = \frac{1}{2} e^{j\omega t}[5.66 - j2.5] + cc$$

$$= \frac{1}{2} 6.18 e^{j(\omega t - 23^0)} + \frac{1}{2} 6.18 e^{-j(\omega t - 23^0)}$$

$$= \frac{1}{2} 6.18[\cos(\omega \mathrm{t} - 23^0) + jsin(\omega t - 23^0) \cos(\omega \mathrm{t} - 23^0) - jsin(\omega t - 23^0)]$$

$$= Real\ [6.18 e^{j(\omega t - 23^0)}]$$

Phasors

In short hand, it is represented as $6.18\ \angle - 23^0$

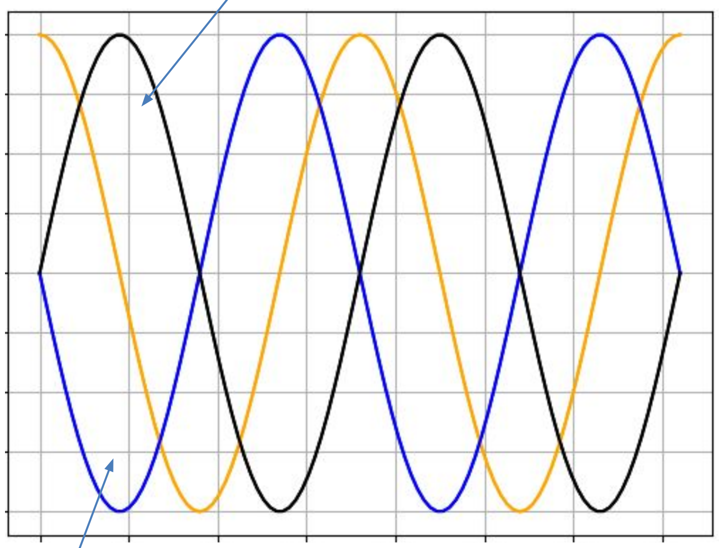
Some Observations

$$A(t) = 5\cos(\omega t) = Real[5e^{j(\omega t)}]$$

$$B(t) = 5\cos(\omega t - 90^0) = Real[5e^{j(\omega t - 90^0)}]$$

$$C(t) = 5\cos(\omega t + 90^0) = Real[5e^{j(\omega t + 90^0)}]$$

$v(t) = V cos(\omega t - 90^0)$



$v(t) = V cos(\omega t + 90^0)$

At any given time *t, B(t)* is trailing or lagging behind *A(t)* by 90^0 while *C(t)* is leading *A(t)* by the same amount

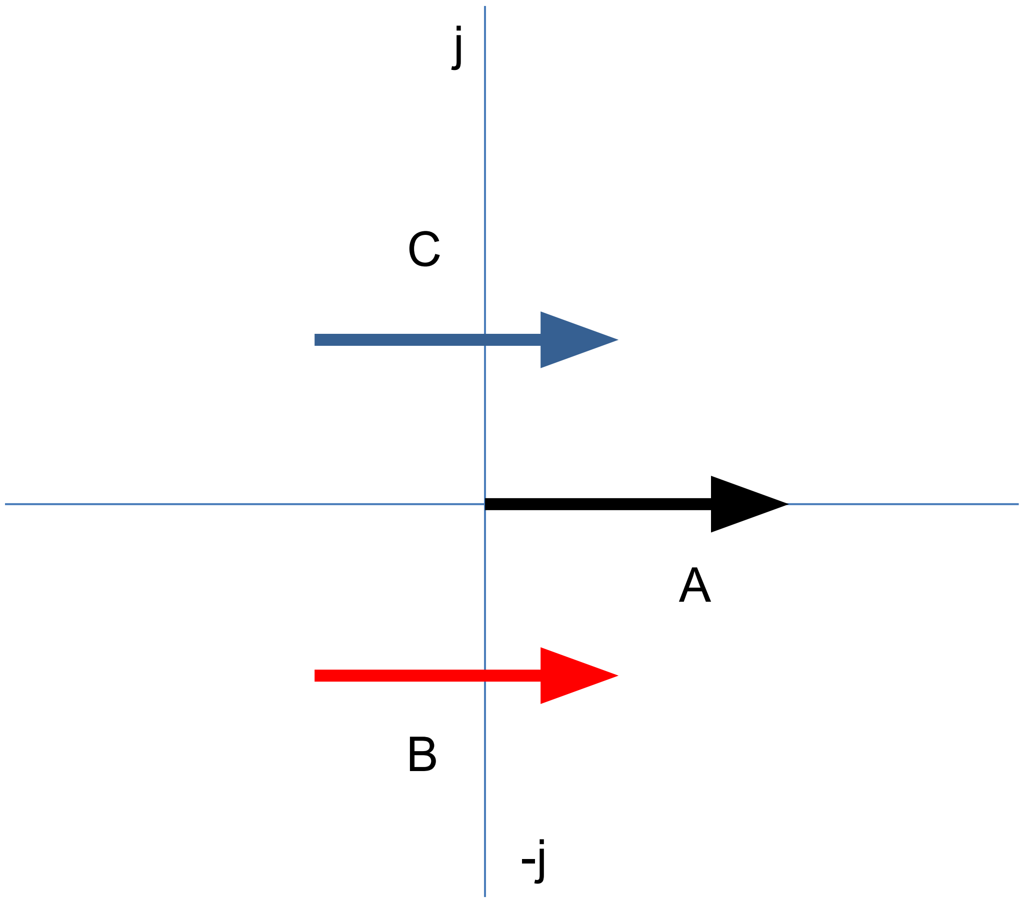
Let us now look at the phasors at *t=0*

$$A(0) = 5$$
$$B(0) = 5(cos 90^0 - j sin 90^0) = -j5$$
$$C(0) = 5(cos 90^0 + j sin 90^0) = +j5$$

Therefore *+j or –j* signifies signals having 90^0 phase lead or lag respectively.

Some Observations

+*j or* –*j* signifies signals having 90^0 phase lead or lag respectively.



$$A(t) = 5\cos(\omega t) = Real[5e^{j(\omega t)}]$$

$$B(t) = 5\cos(\omega t - 90^0) = Real[5e^{j(\omega t - 90^0)}]$$

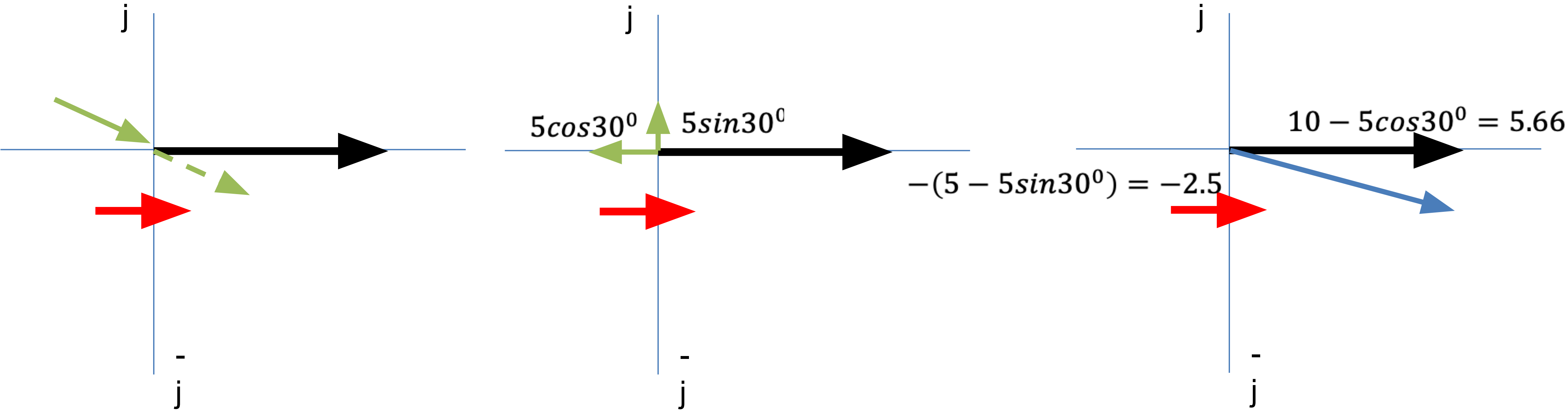
$$C(t) = 5\cos(\omega t + 90^0) = Real[5e^{j(\omega t + 90^0)}]$$

Some Observations

$$v(t) = 10cos\omega t + 5\cos(\omega t - 90^0) - 5\cos(\omega t - 30^0)$$

$$= 10 + 5\angle - 90^0 - 5\angle - 30^0$$ In phasor notation

$$= 6.18\angle - 23^0$$

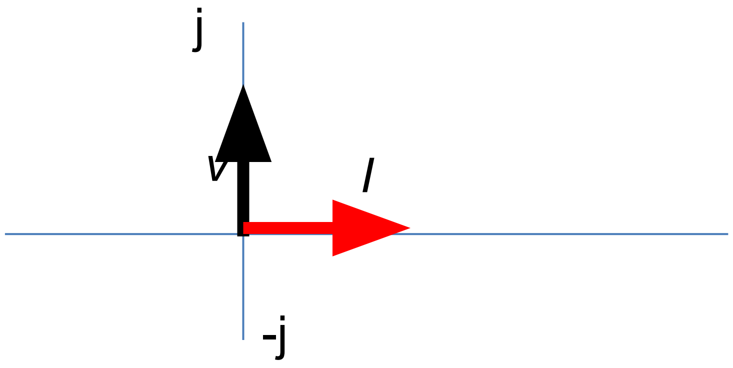


Phasors are like vectors where the phase angle denotes the angle between coordinate axes with *j* representing 90^0

Complex Impedances

Inductance:

Say a sinusoidal current is flowing in a circuit with inductance



$$i(t) = I_0 \sin(\omega t) = I_0 \cos(\omega t - 90^0)$$

$$v_L(t) = L\frac{di}{dt} = \omega L I_0 cos\omega t$$

Therefore, the current in an inductor lags the voltage by 90^0

In the phasor notation

$$\boldsymbol{V} = \omega L I_0$$
$$\boldsymbol{I} = I_0 \angle - 90^0$$

Then, inductive impedance

$$\boldsymbol{Z_L} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{\omega L}{\angle - 90^0} = \frac{\omega L}{-j} = j\omega L$$

We could have obtained the same result working directly with exponentials

$$v_L(t) = L\frac{d}{dt} Real[I_0 e^{j(\omega t - 90^0)}]$$

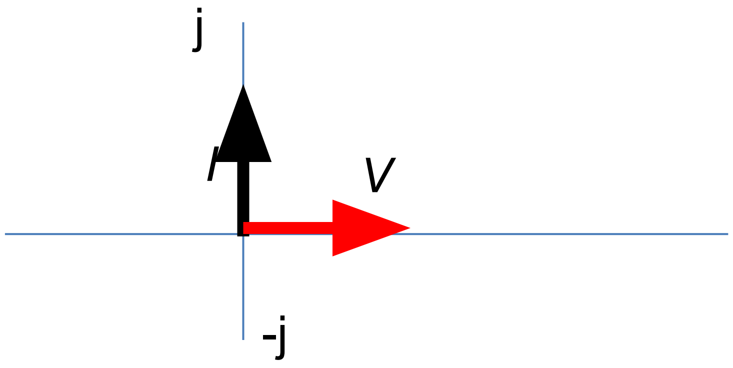
$$v_L(t) = Real[I_0 L j\omega e^{j(\omega t - 90^0)}]$$

$$v_L(t) = j\omega L I_0 \angle - 90^0$$

$$\boldsymbol{V} = j\omega L \boldsymbol{I}$$

Complex Impedances

Capacitance:



Complex Impedances

- AC steady-state analysis using phasors allows us to express the relationship between current and voltage using a formula that looks likes Ohm's law:

$$\boldsymbol{V} = \boldsymbol{I}\boldsymbol{Z}$$

- Impedance depends on the frequency ω.
- Impedance is a complex number.
- Impedance allows us to use the same solution techniques for AC steady state as