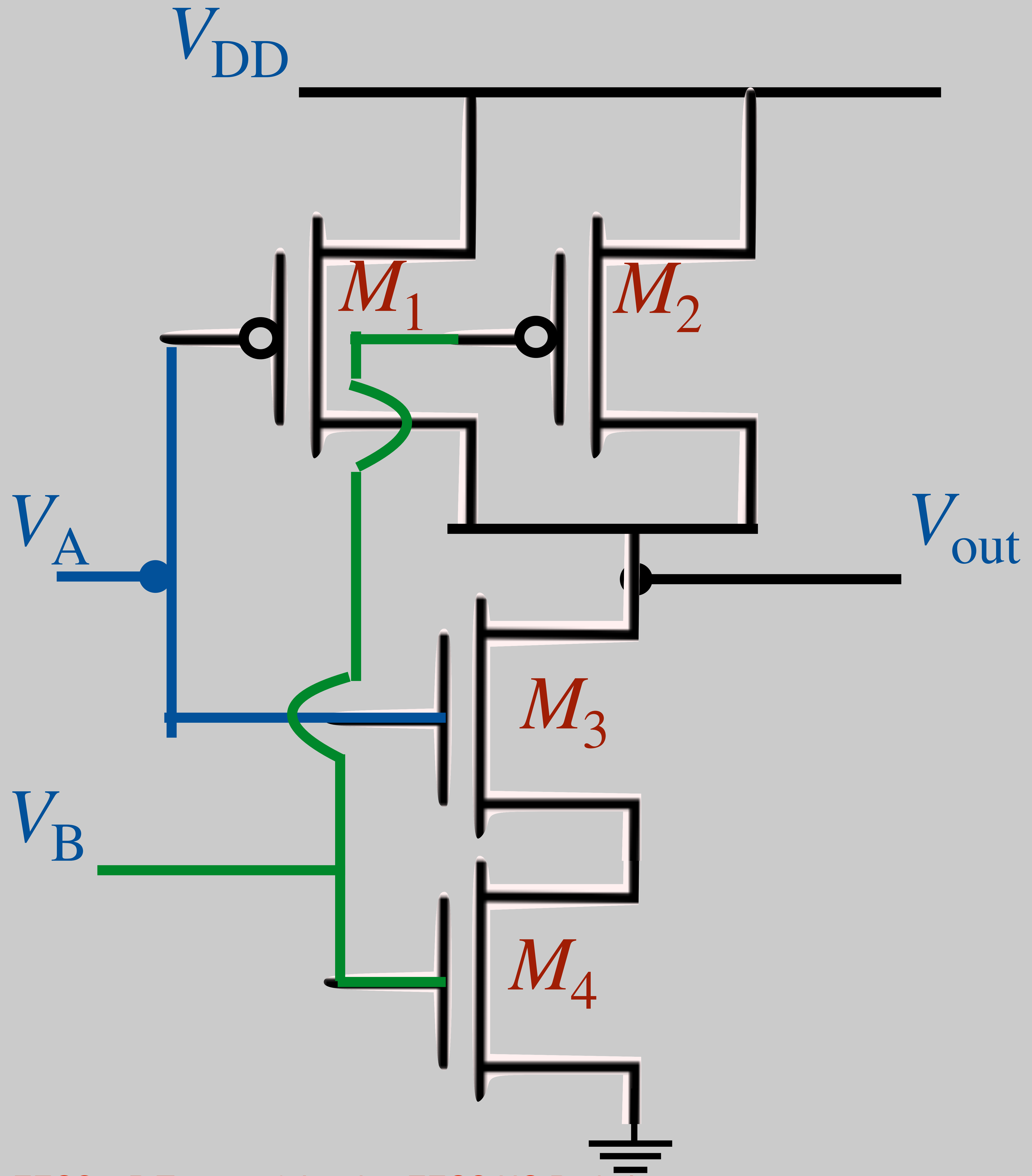
EE16B
Designing Information Devices and Systems II

Lecture 3A
Transistors II

Announcements

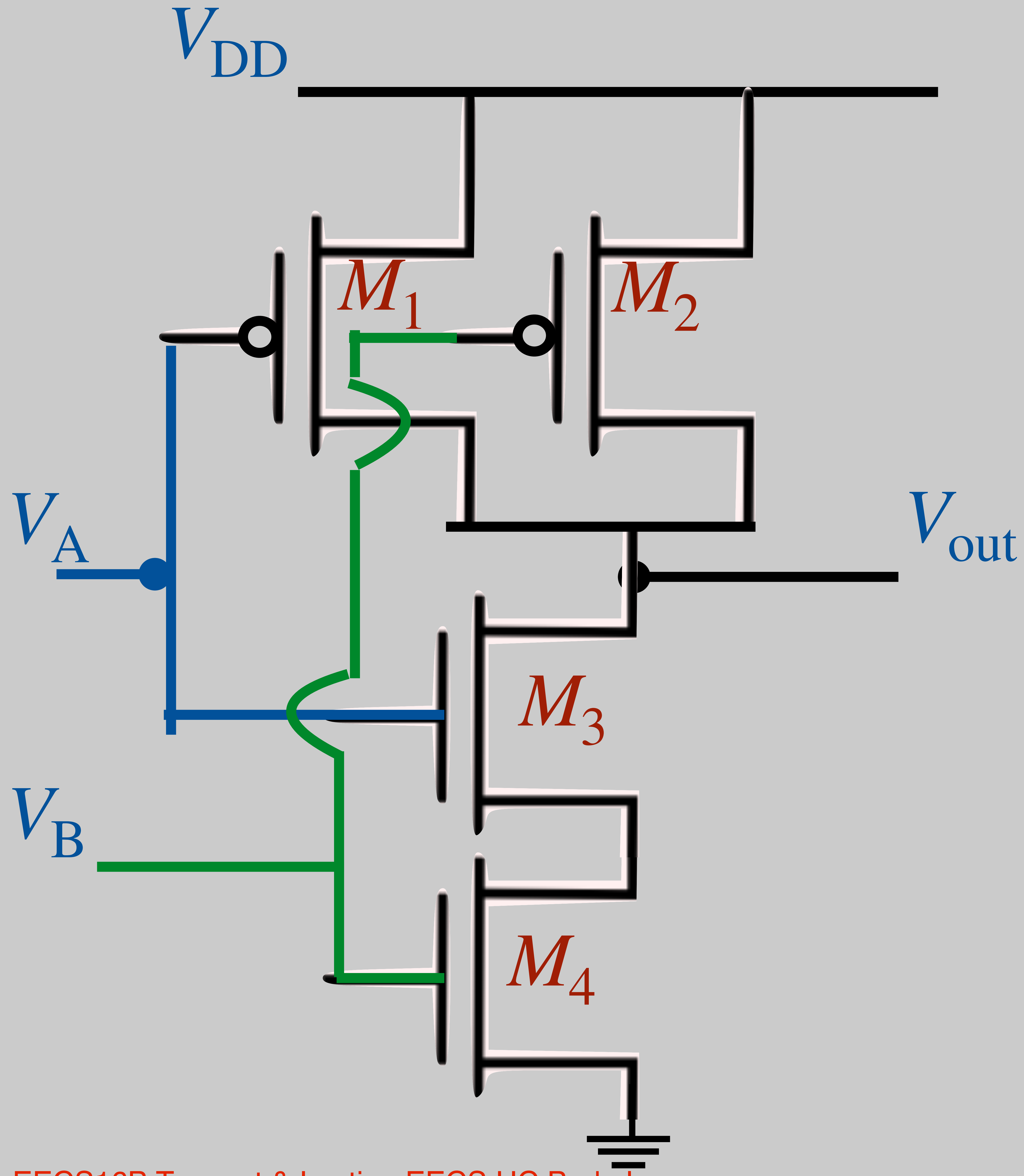
- Last time:
  - Lab 2
  - Transistors as a switch
- Today:
  - Transistors Switch RC model
  - Finish review of complex numbers
    - Euler Formula
  - Solution for sinusoidal input

CMOS (Complementary MOS)



V_A	V_B	M_1	M_2	M_3	M_4	V_{out}
0	0					
0	1					
1	0					
1	1					

CMOS (Complementary MOS)



V_A	V_B	M_1	M_2	M_3	M_4	V_{out}
0	0	On	On	Off	Off	V_{DD}
0	1					
1	0					
1	1					

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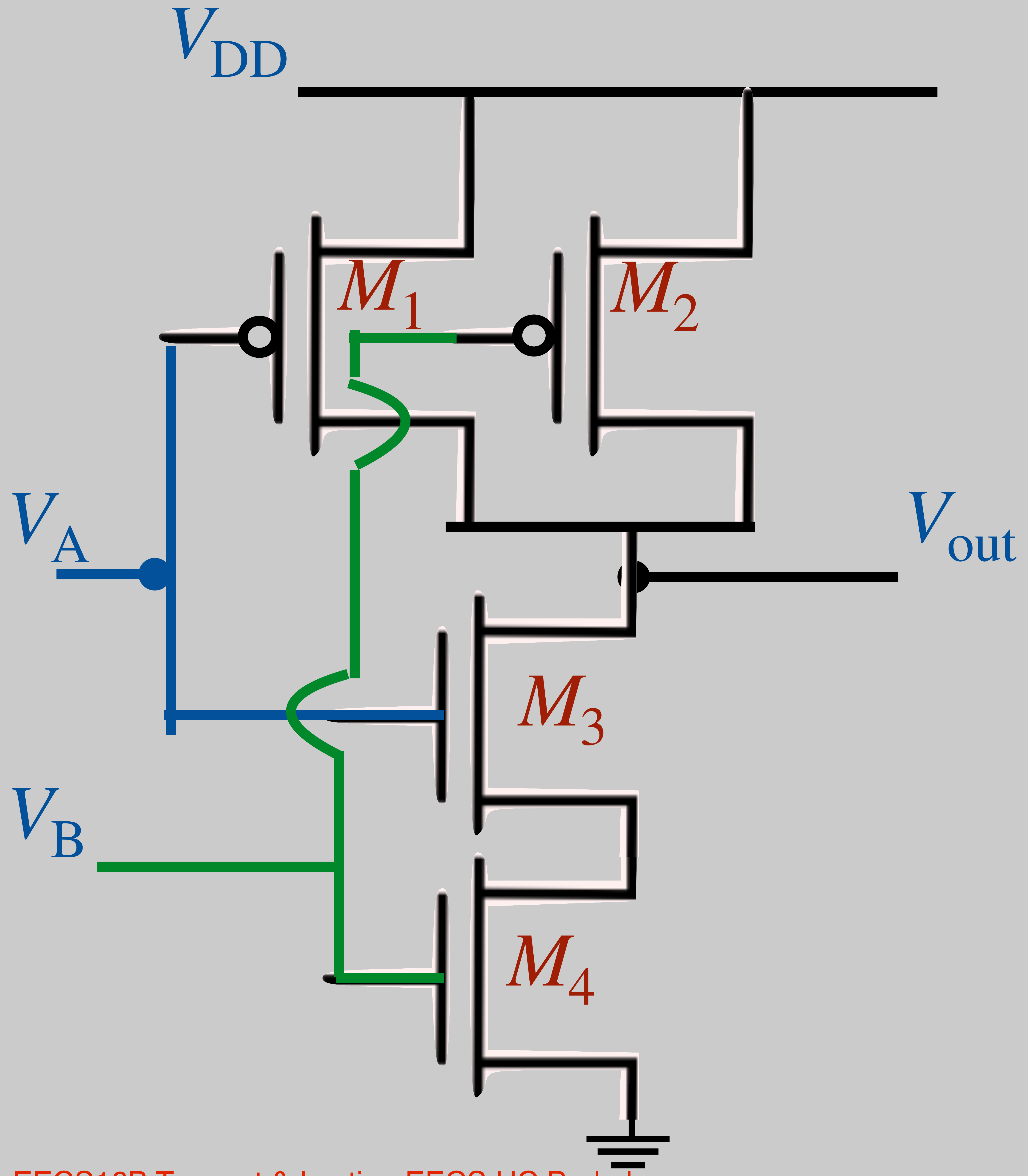
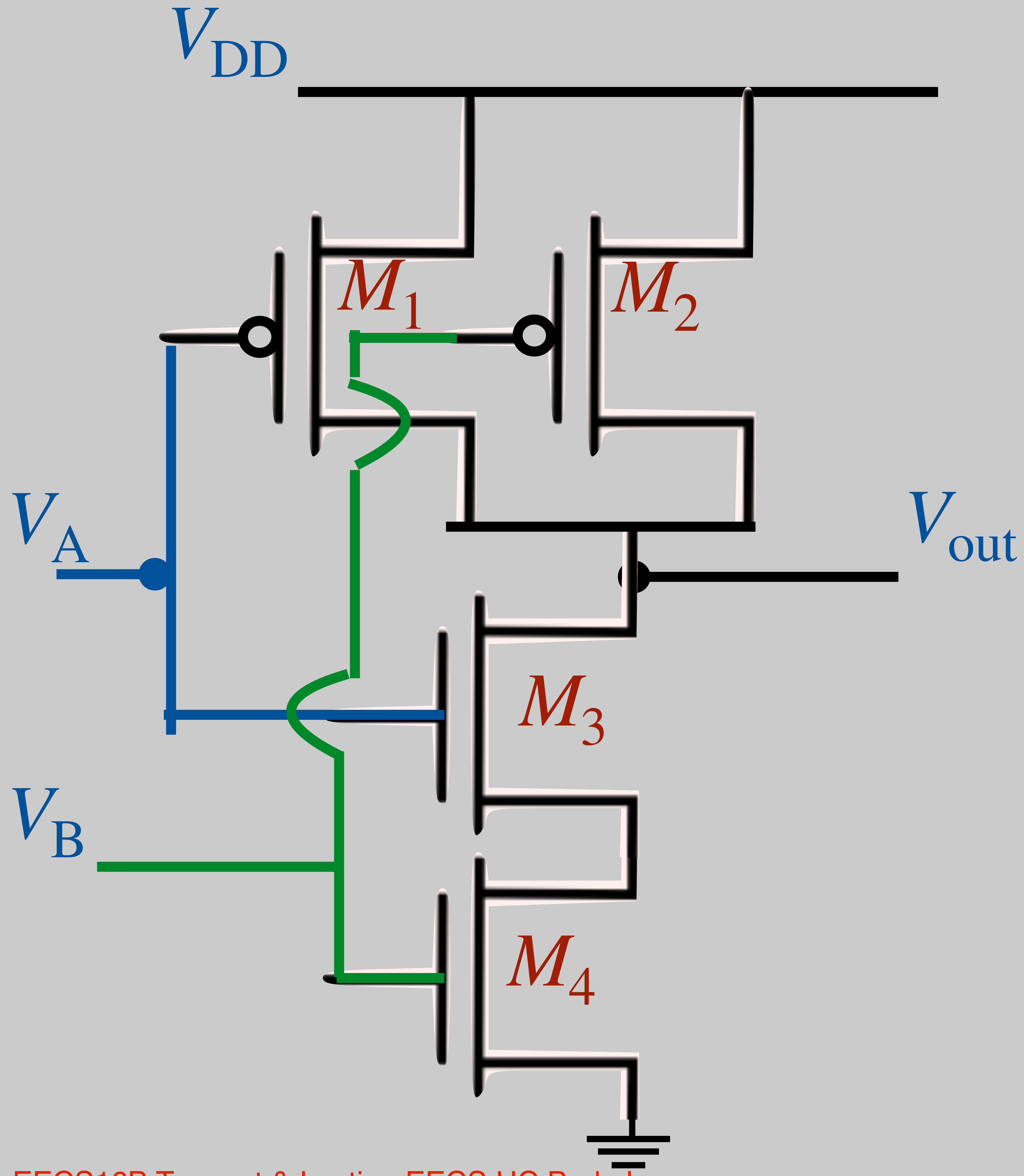


V_A	V_B	M_1	M_2	M_3	M_4	V_{out}
0	0	On	On	Off	Off	V_{DD}
0	1	On	Off	Off	On	V_{DD}
1	0					
1	1					

CMOS (Complementary MOS)



V_A	V_B	M_1	M_2	M_3	M_4	V_{out}
0	0	On	On	Off	Off	V_{DD}
0	1	On	Off	Off	On	V_{DD}
1	0	Off	On	On	Off	V_{DD}
1	1					

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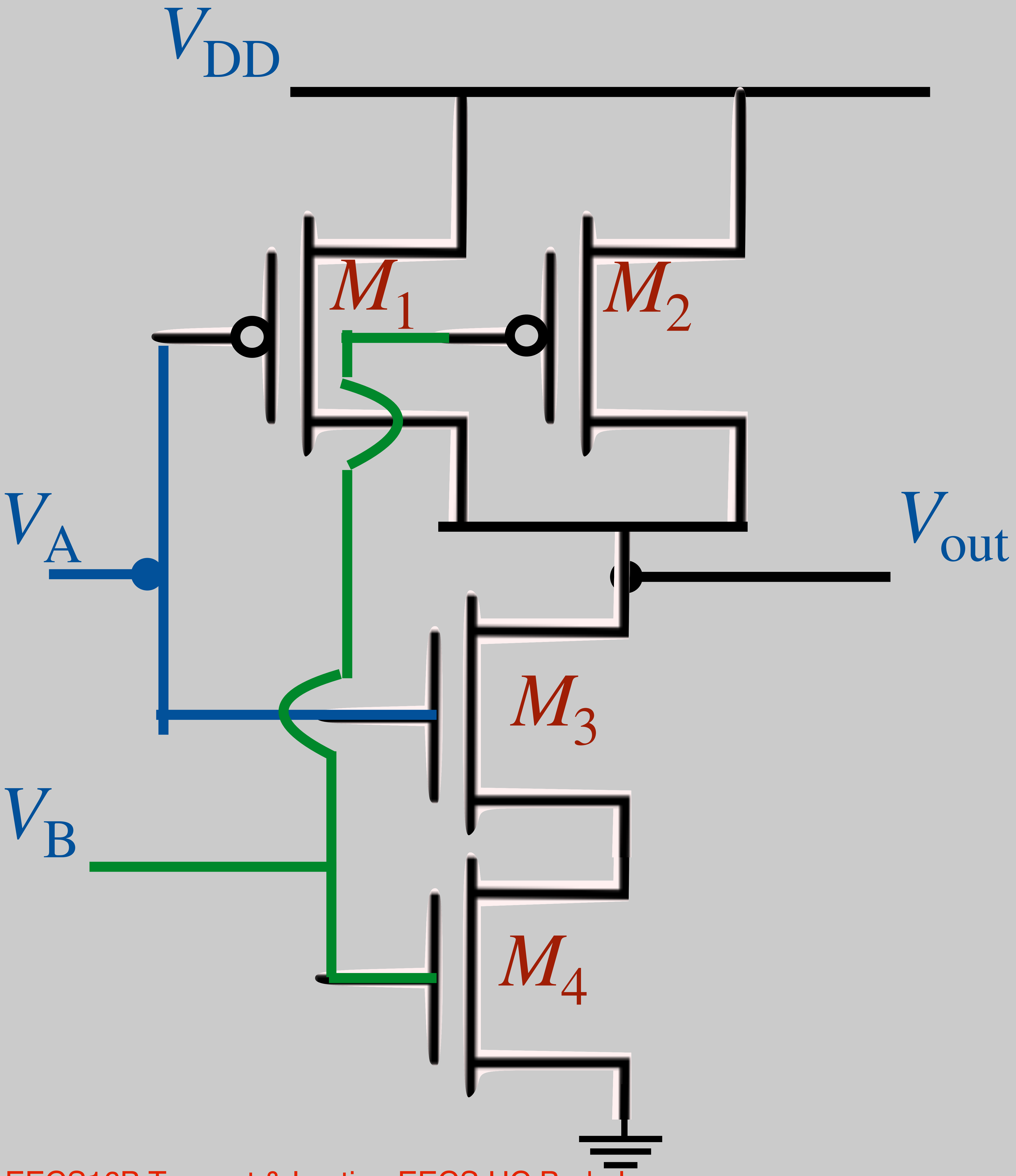
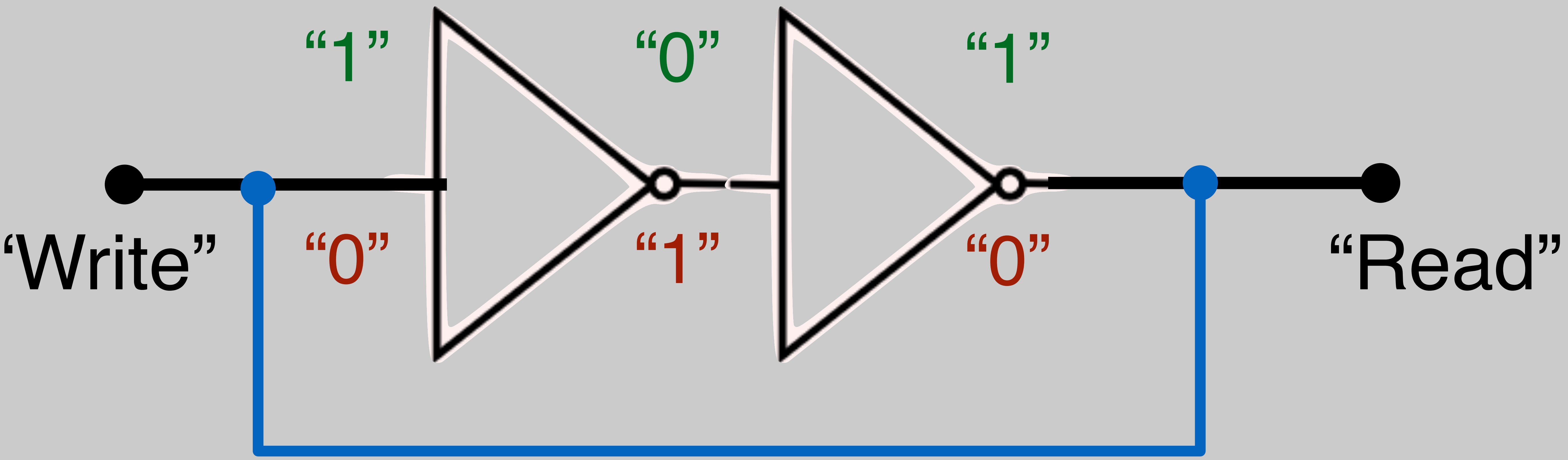


V_A	V_B	M_1	M_2	M_3	M_4	V_{out}
0	0	On	On	Off	Off	V_{DD}
0	1	On	Off	Off	On	V_{DD}
1	0	Off	On	On	Off	V_{DD}
1	1	Off	Off	On	On	0

NAND gate!

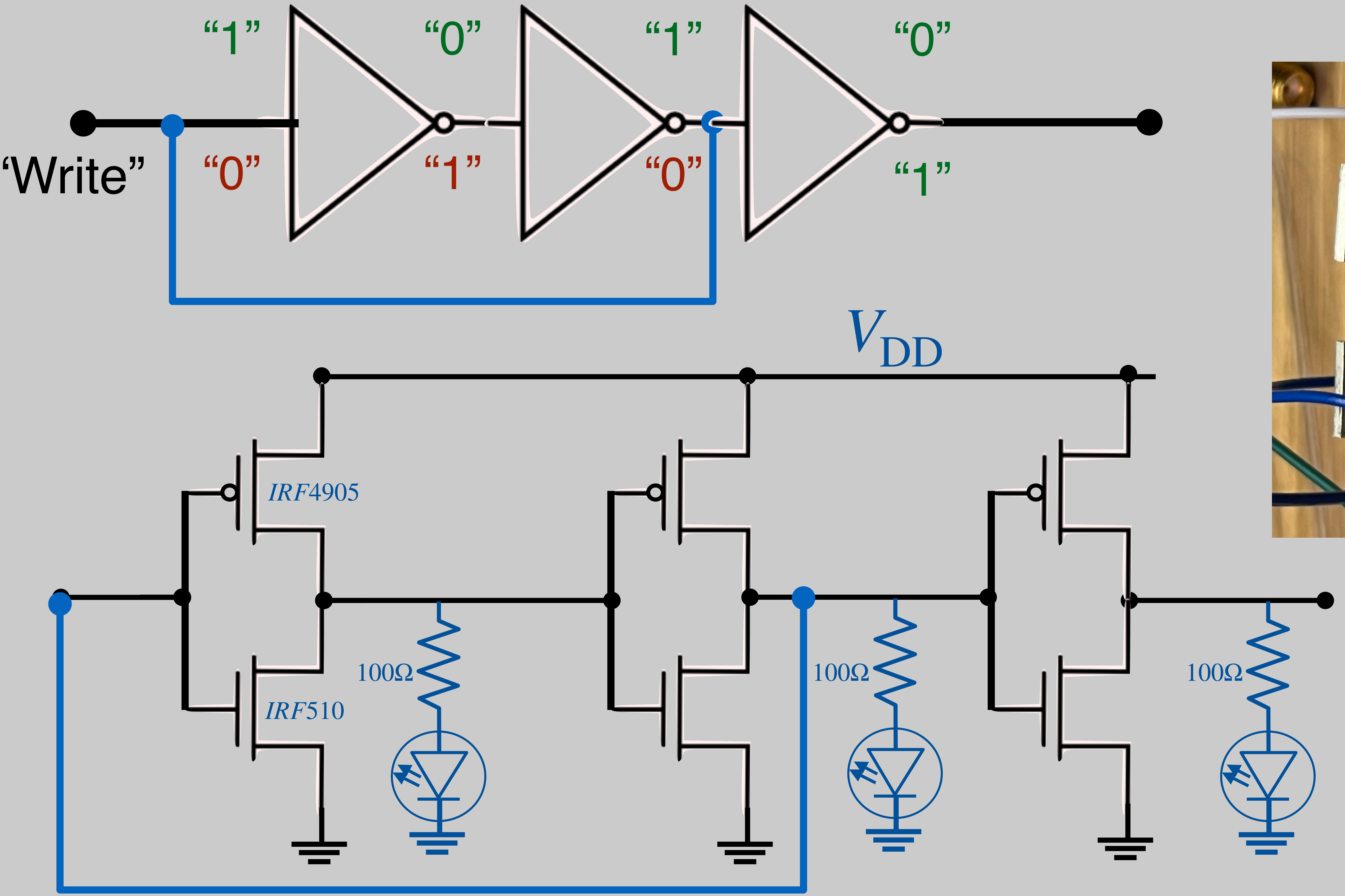
Static RAM (SRAM)

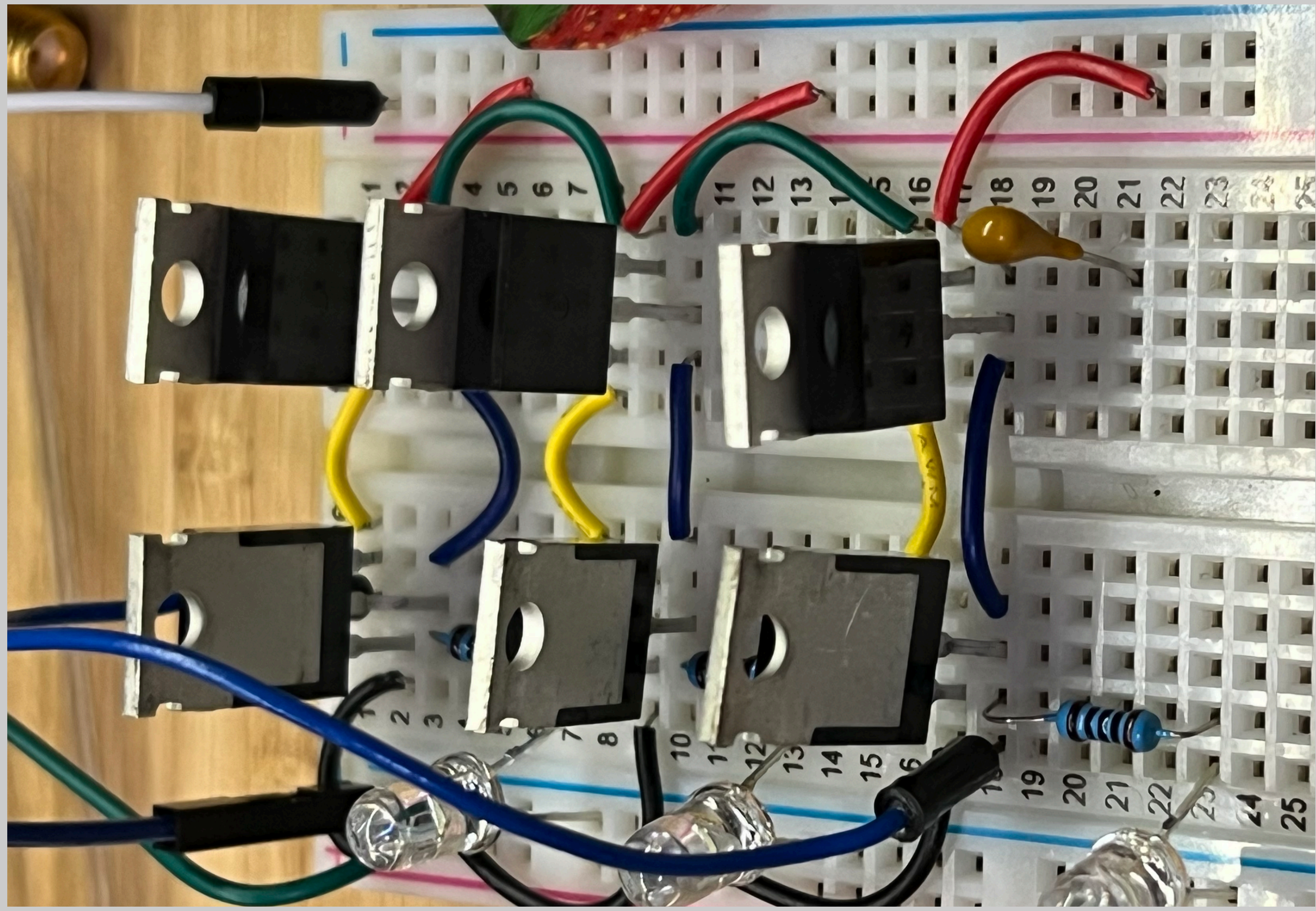


2 stable states (bistable)

“Write” signal forces circuit to change states…. Memory!

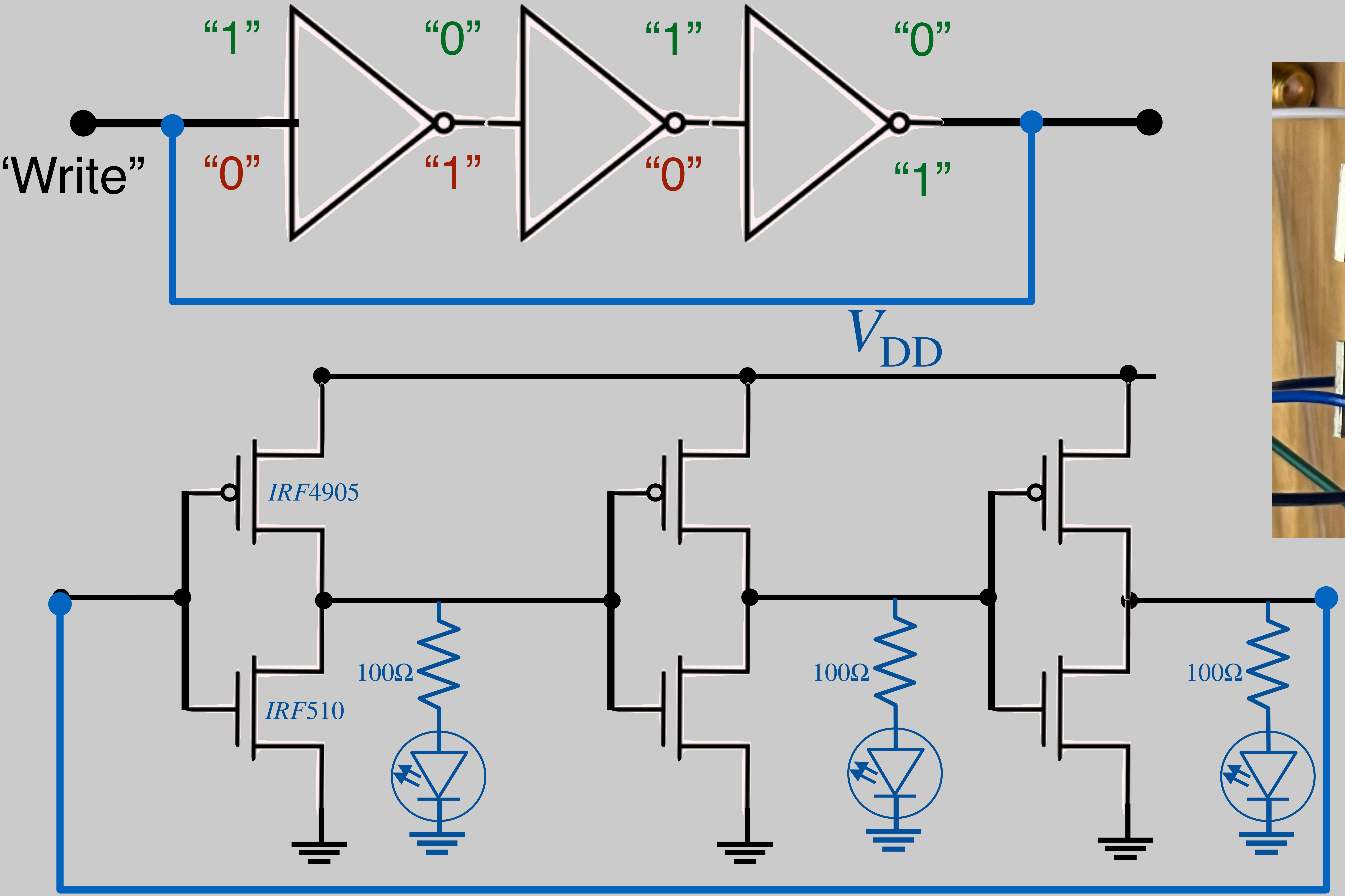
Demo

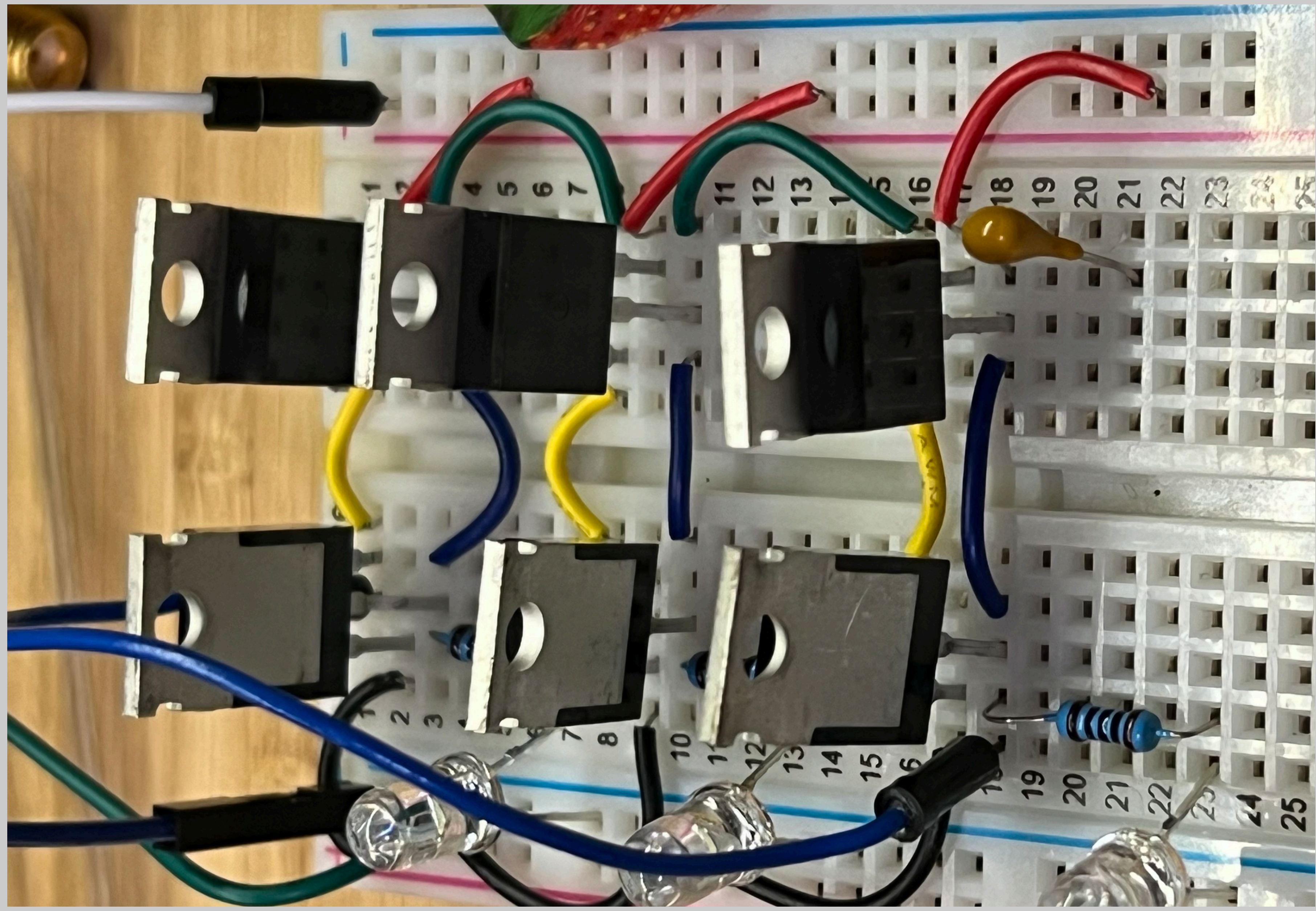




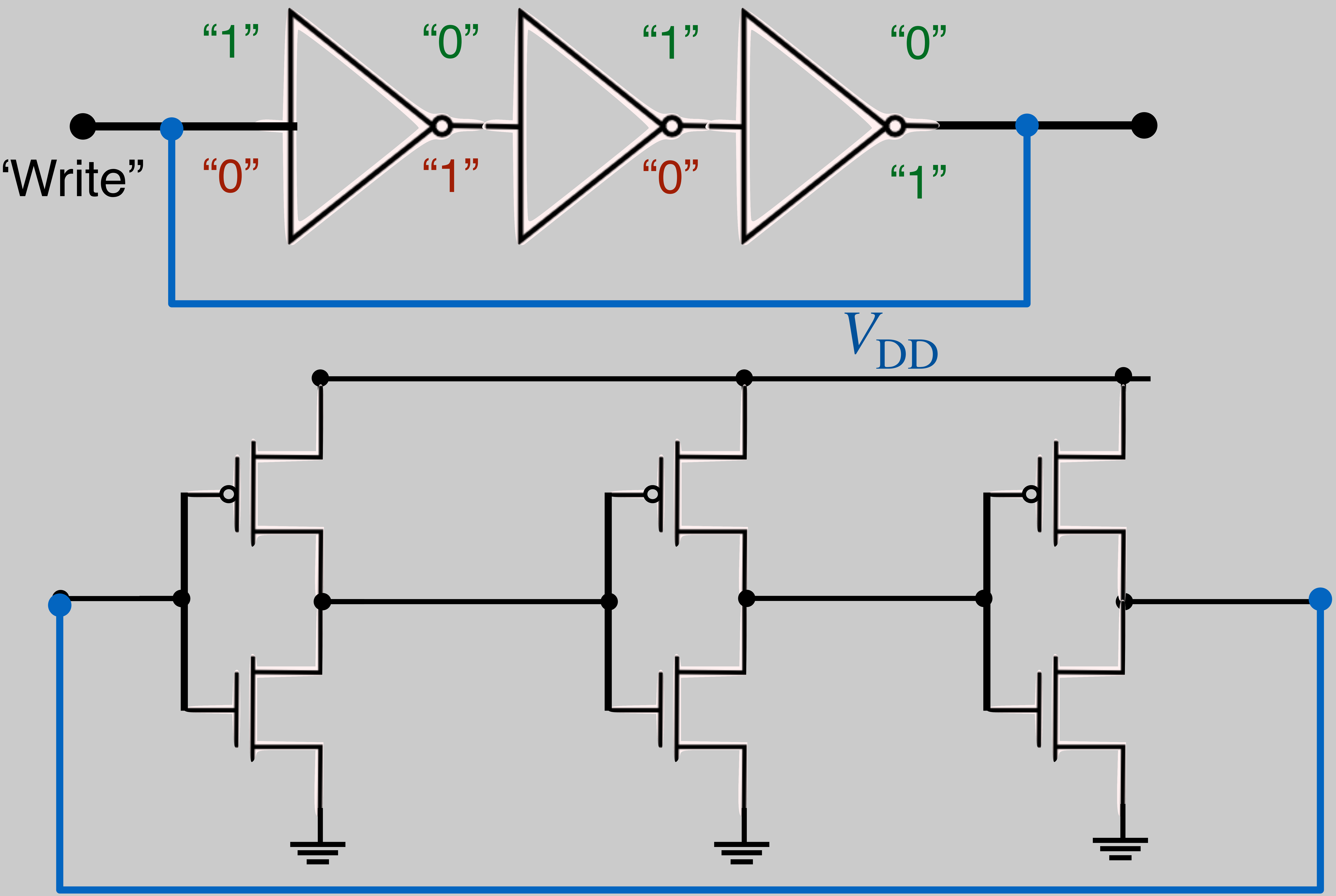
Demo

Inconsistant!





Ring Oscilator



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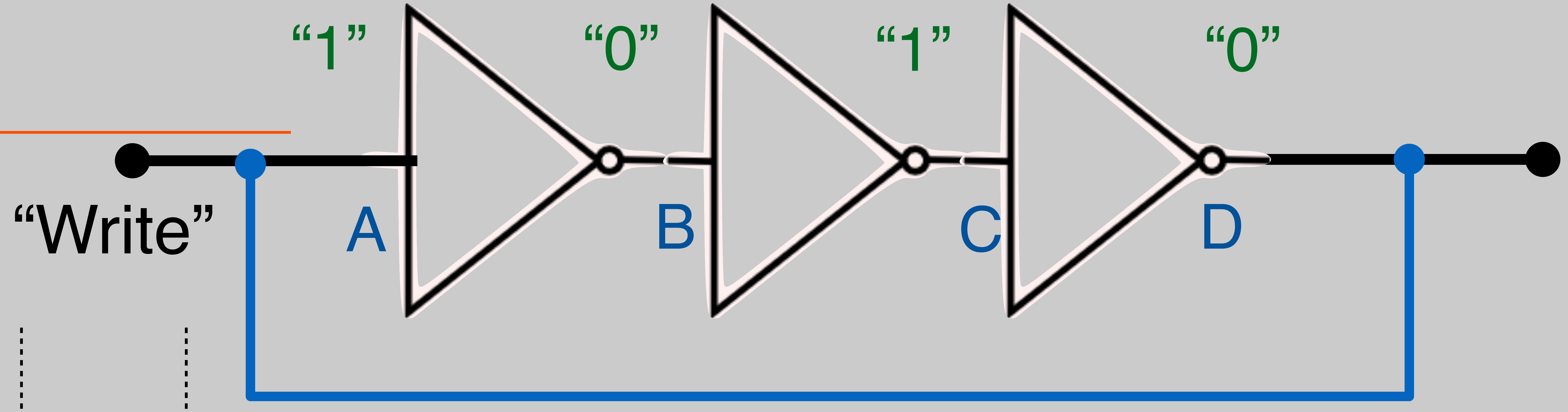

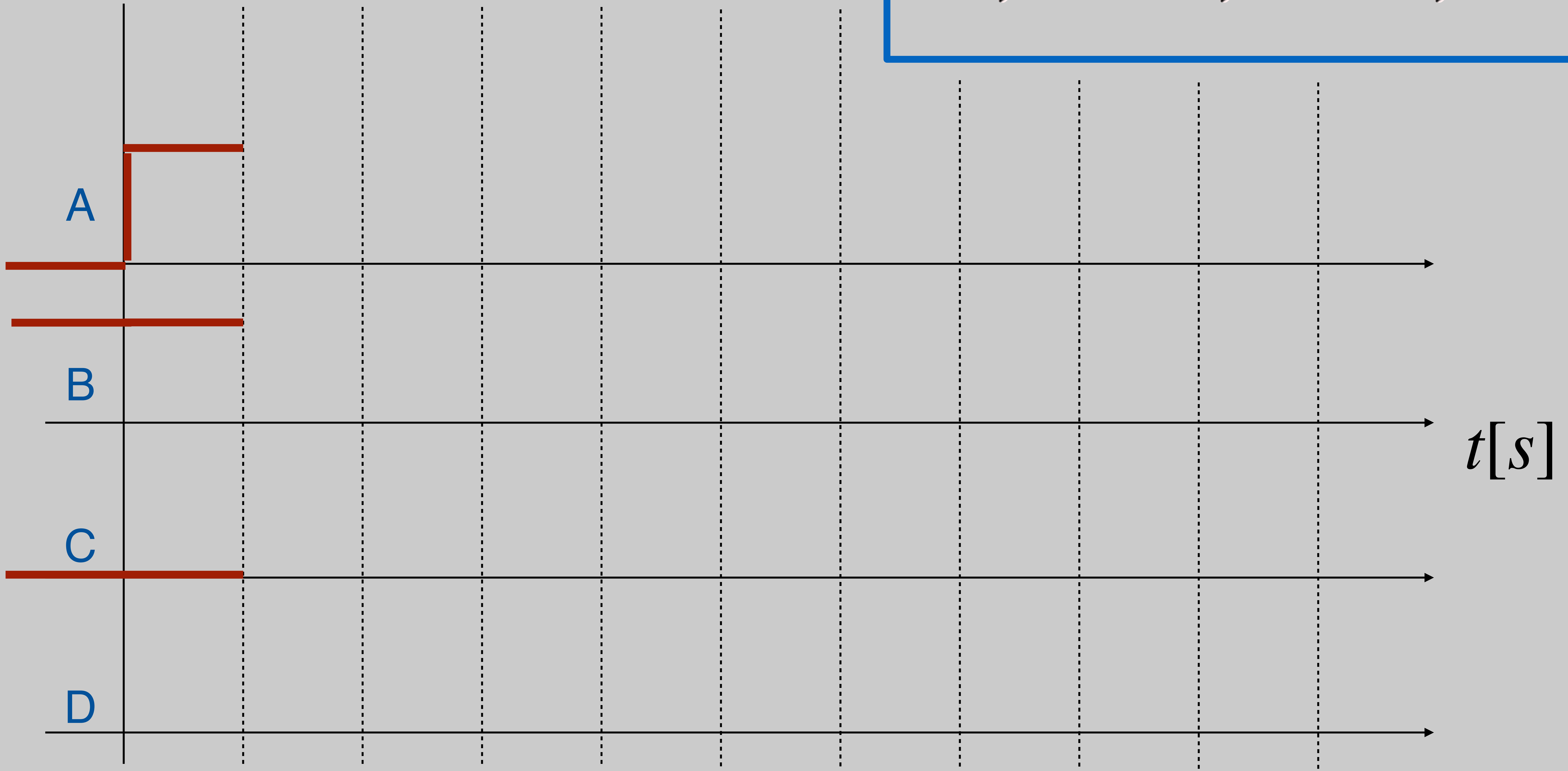

Ring Oscilator

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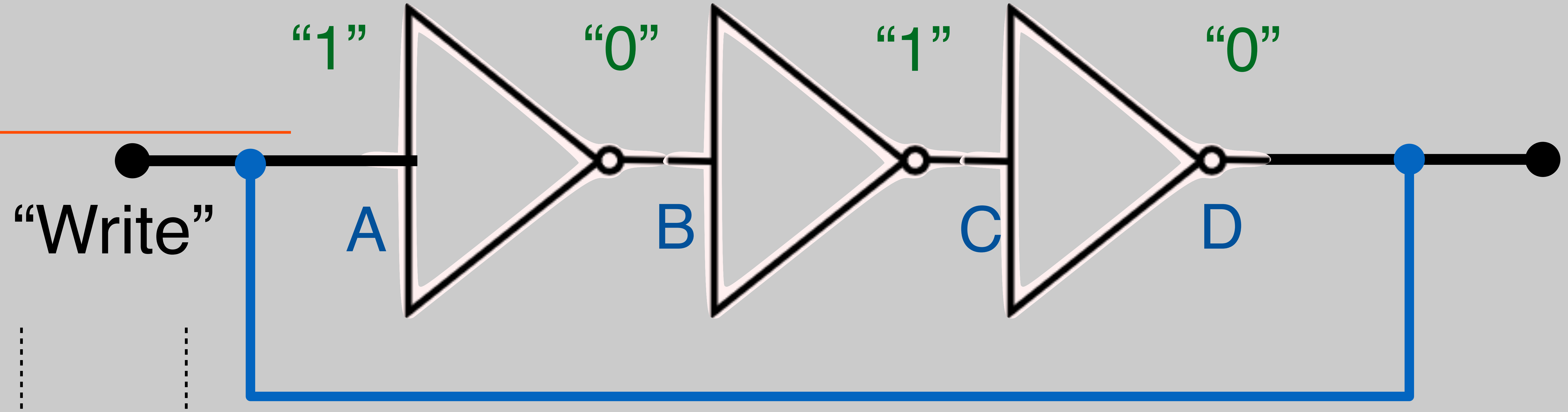

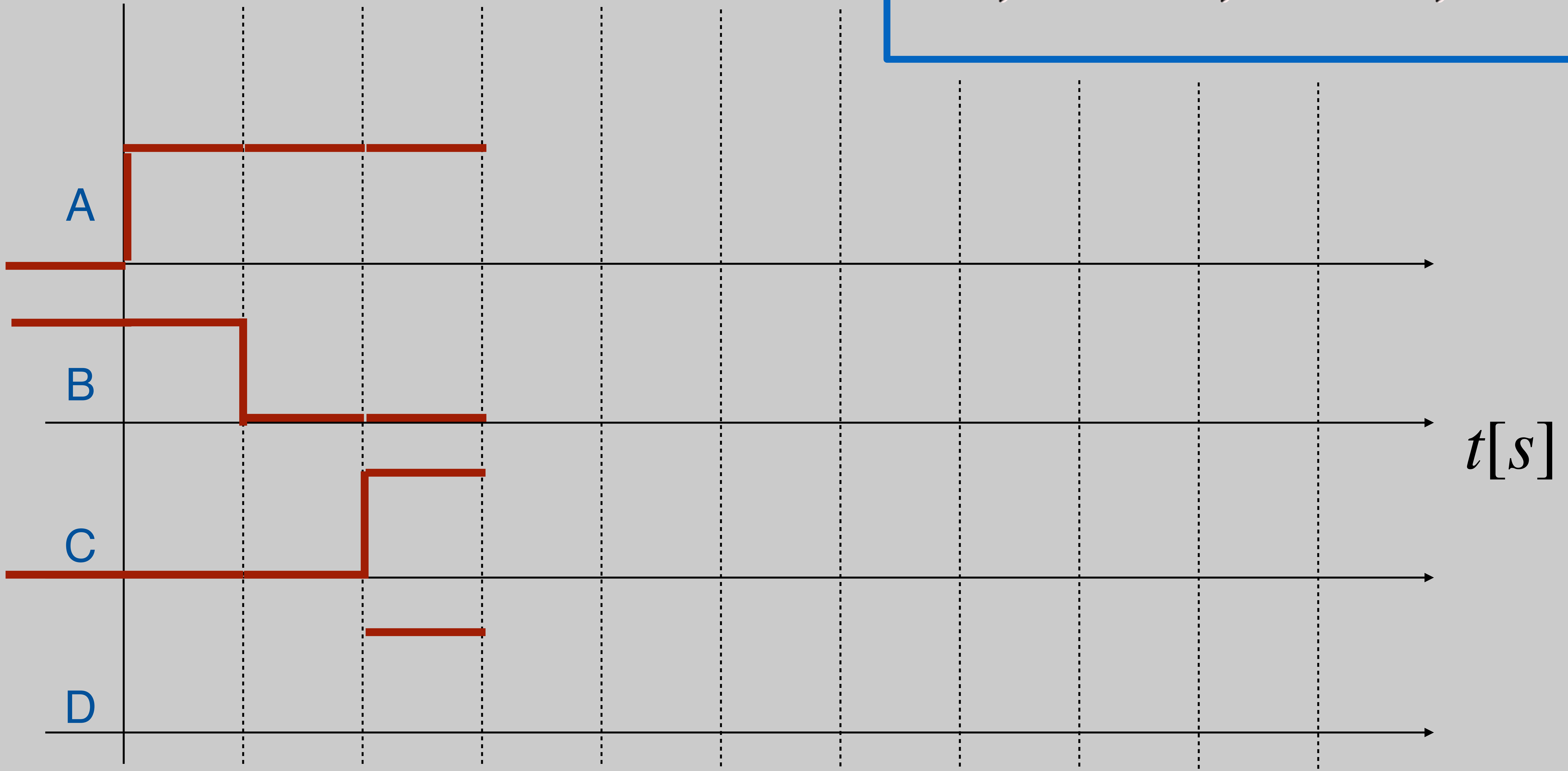

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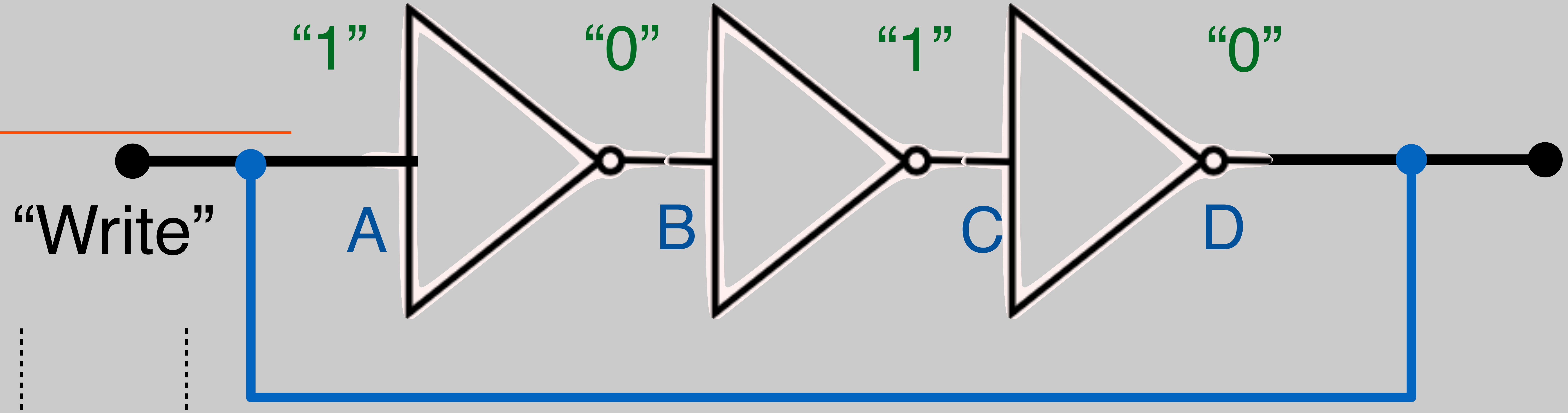
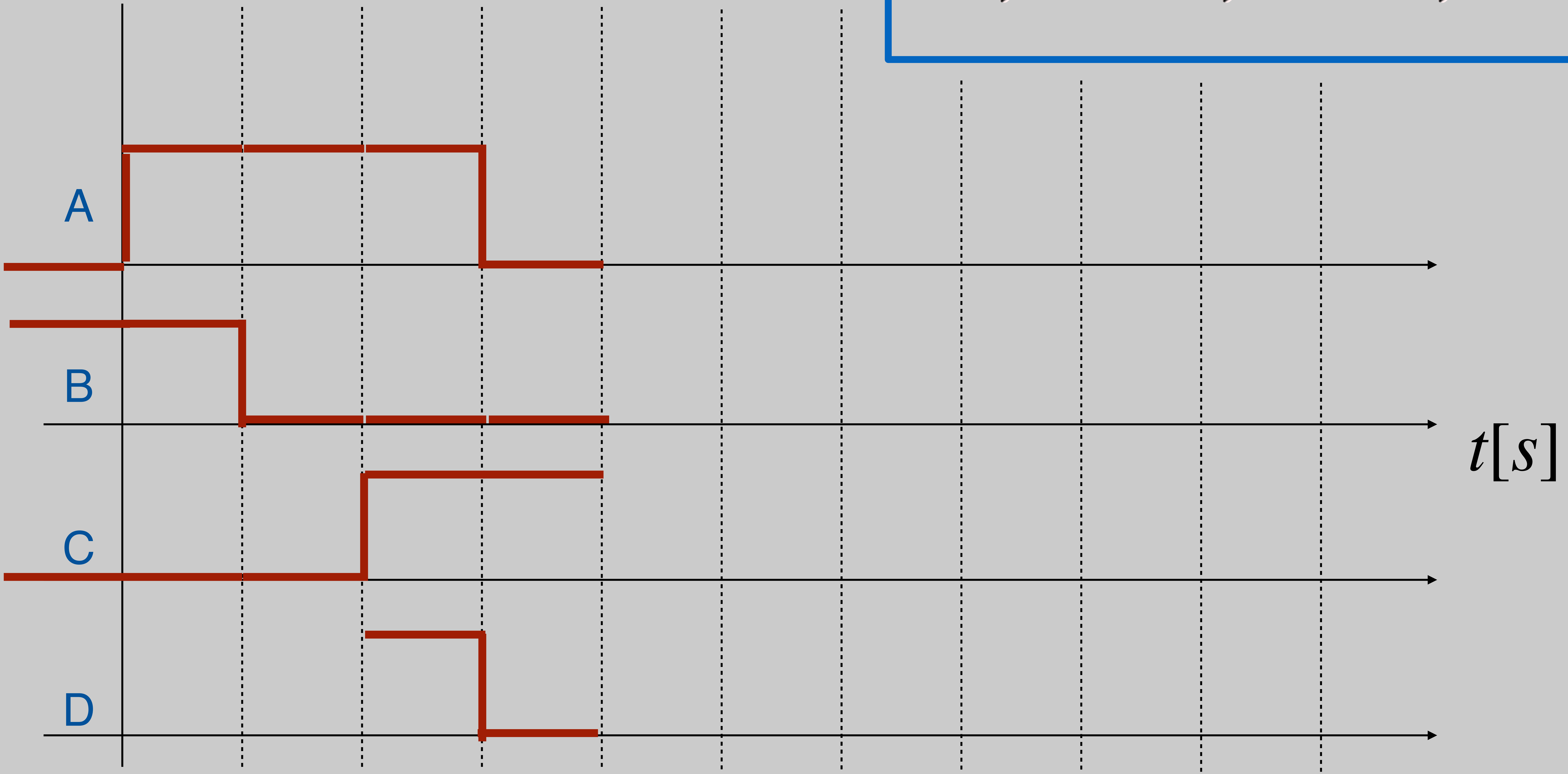
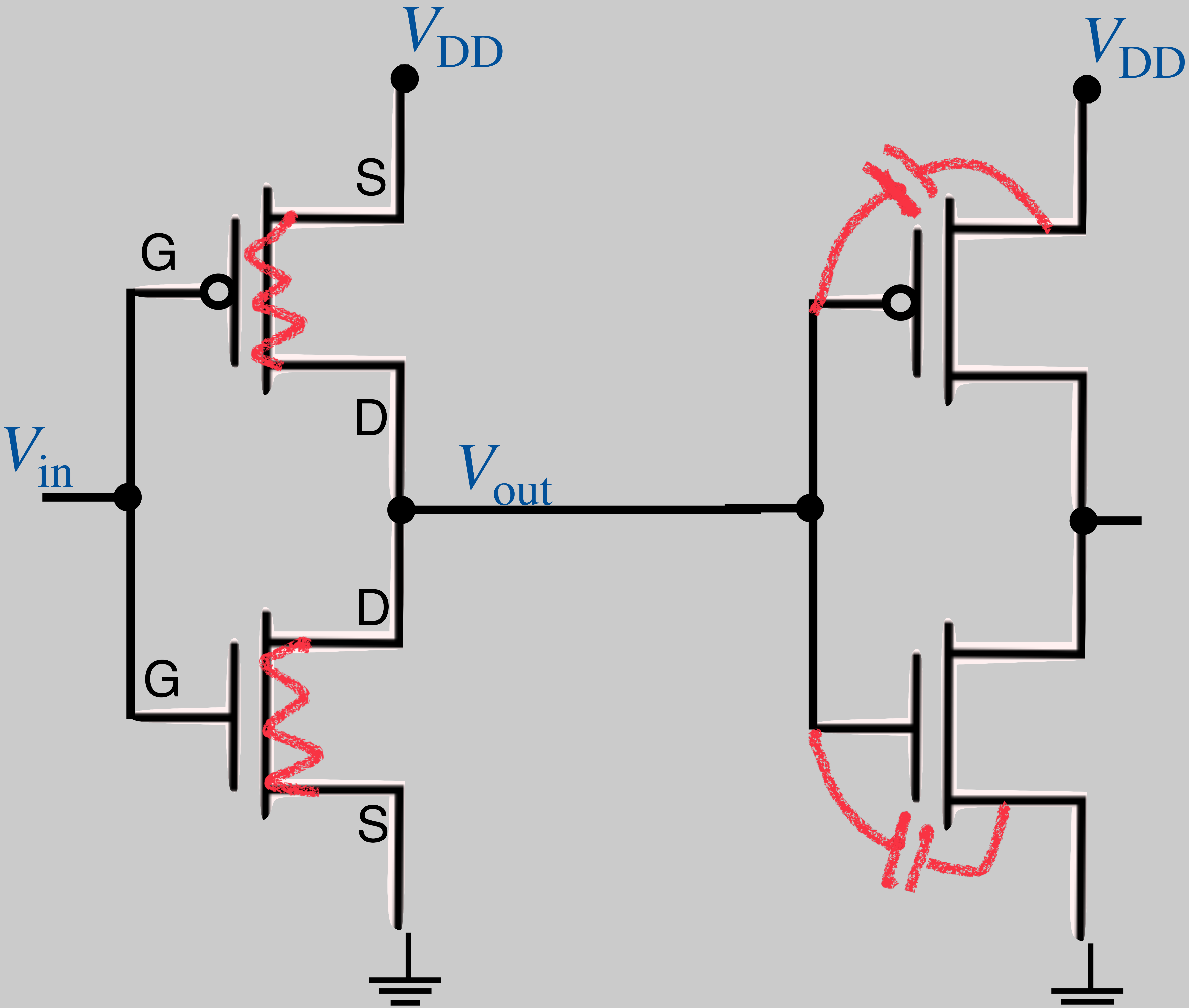


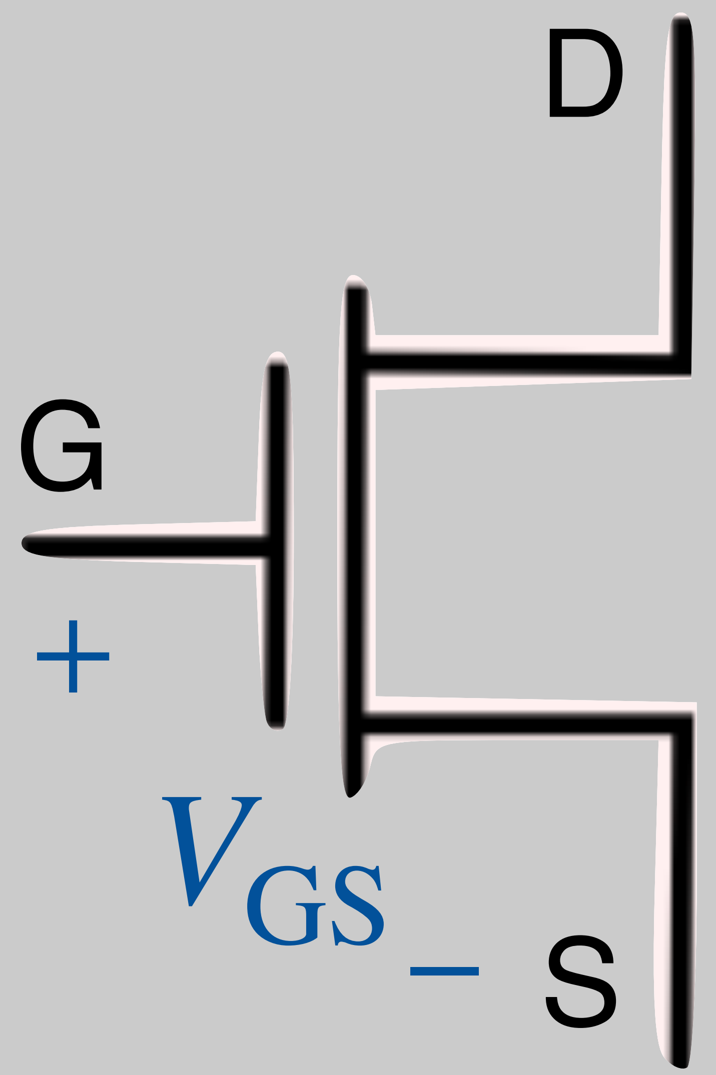


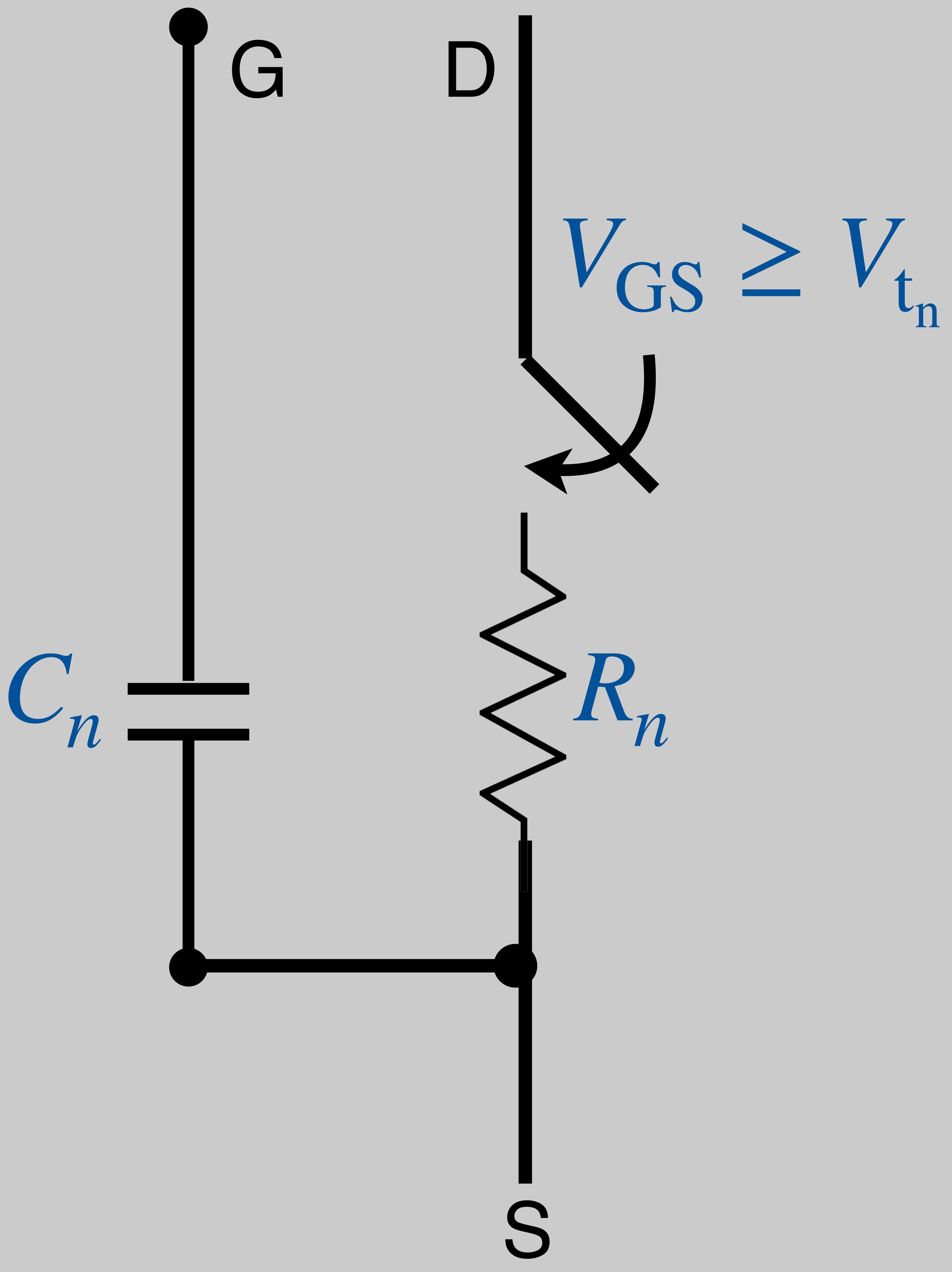
Ring Oscilator

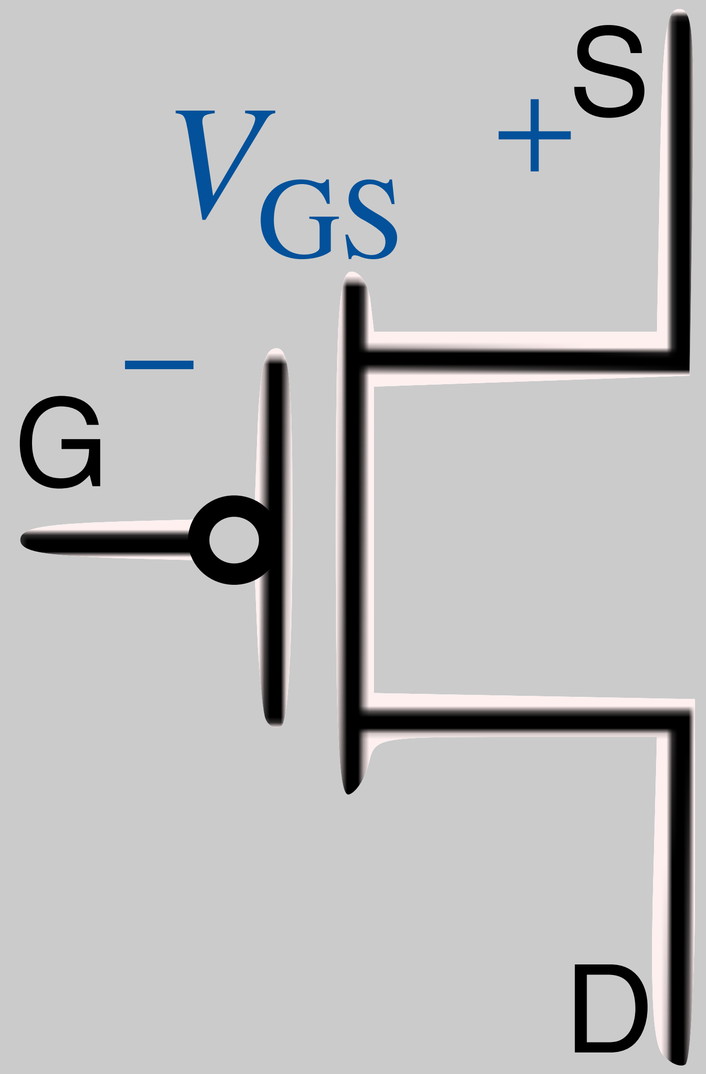
RC Delay in MOS Circuits



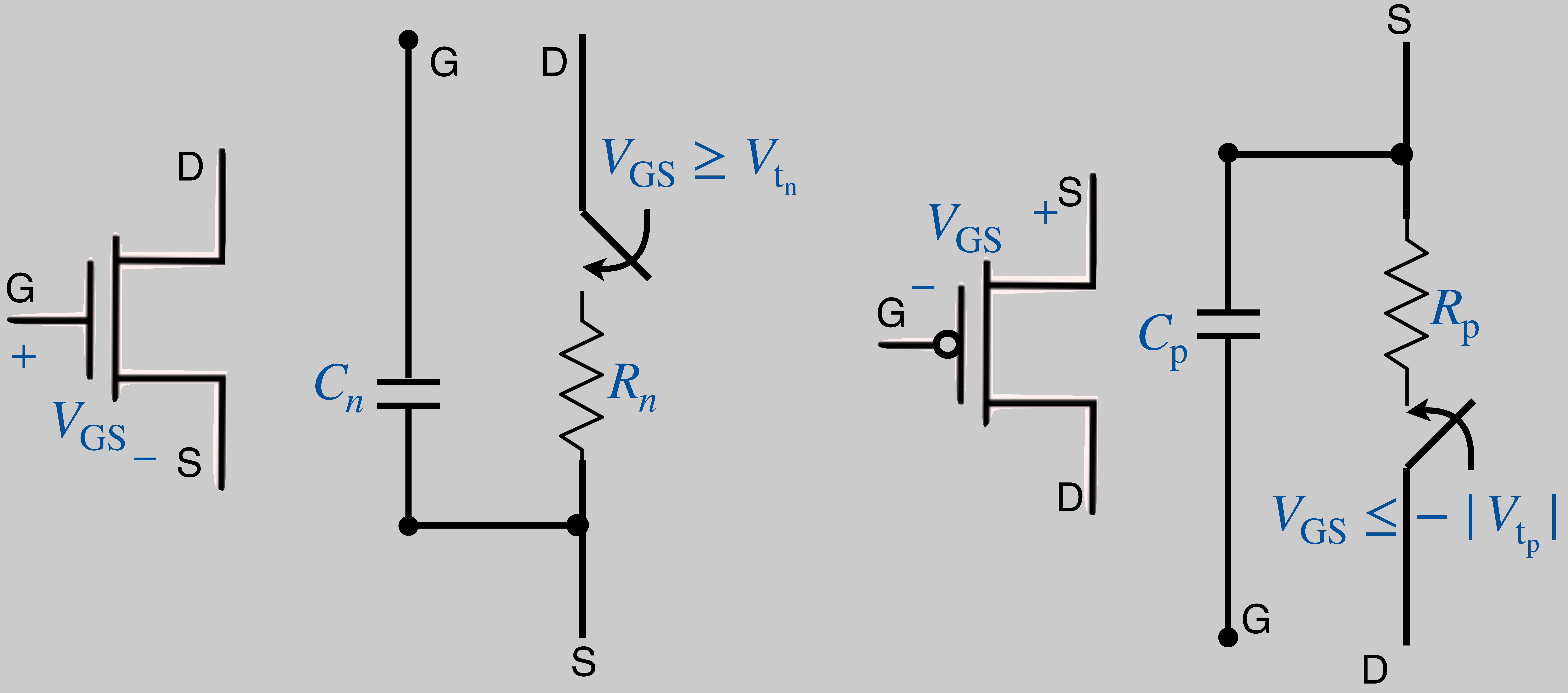
RC Model of NMOS and PMOS



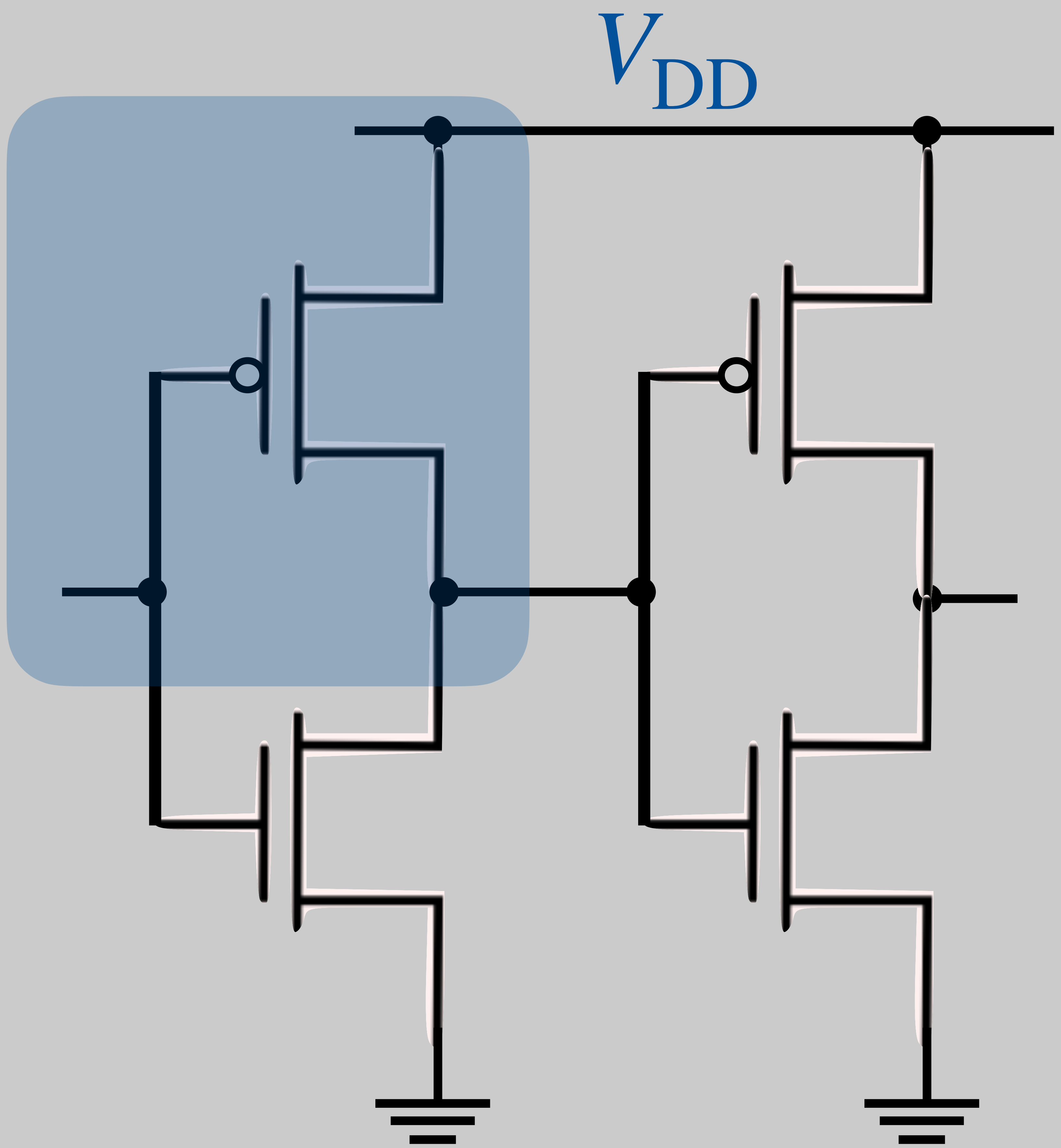


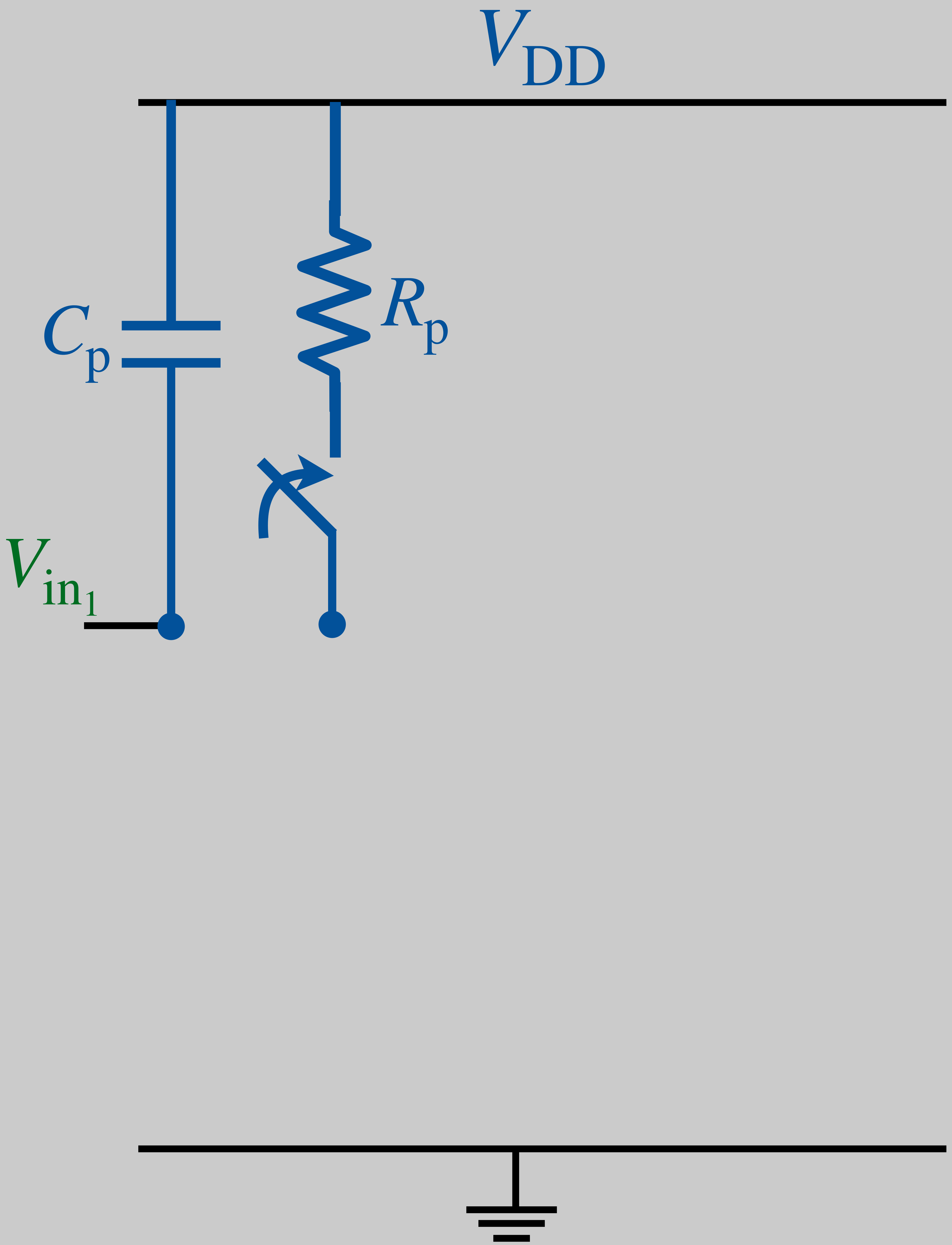


RC Model of NMOS and PMOS



RC Model of CMOS





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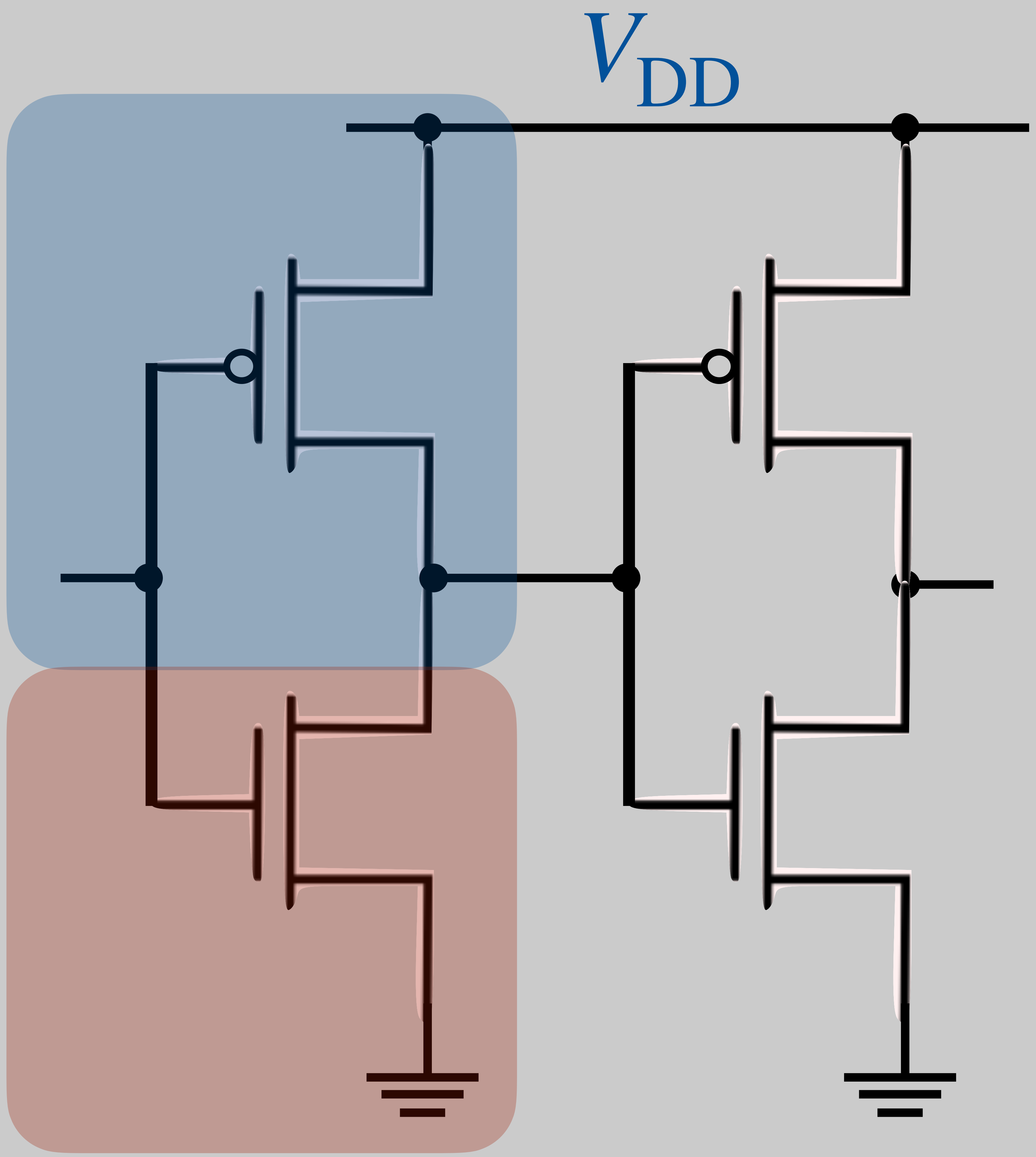
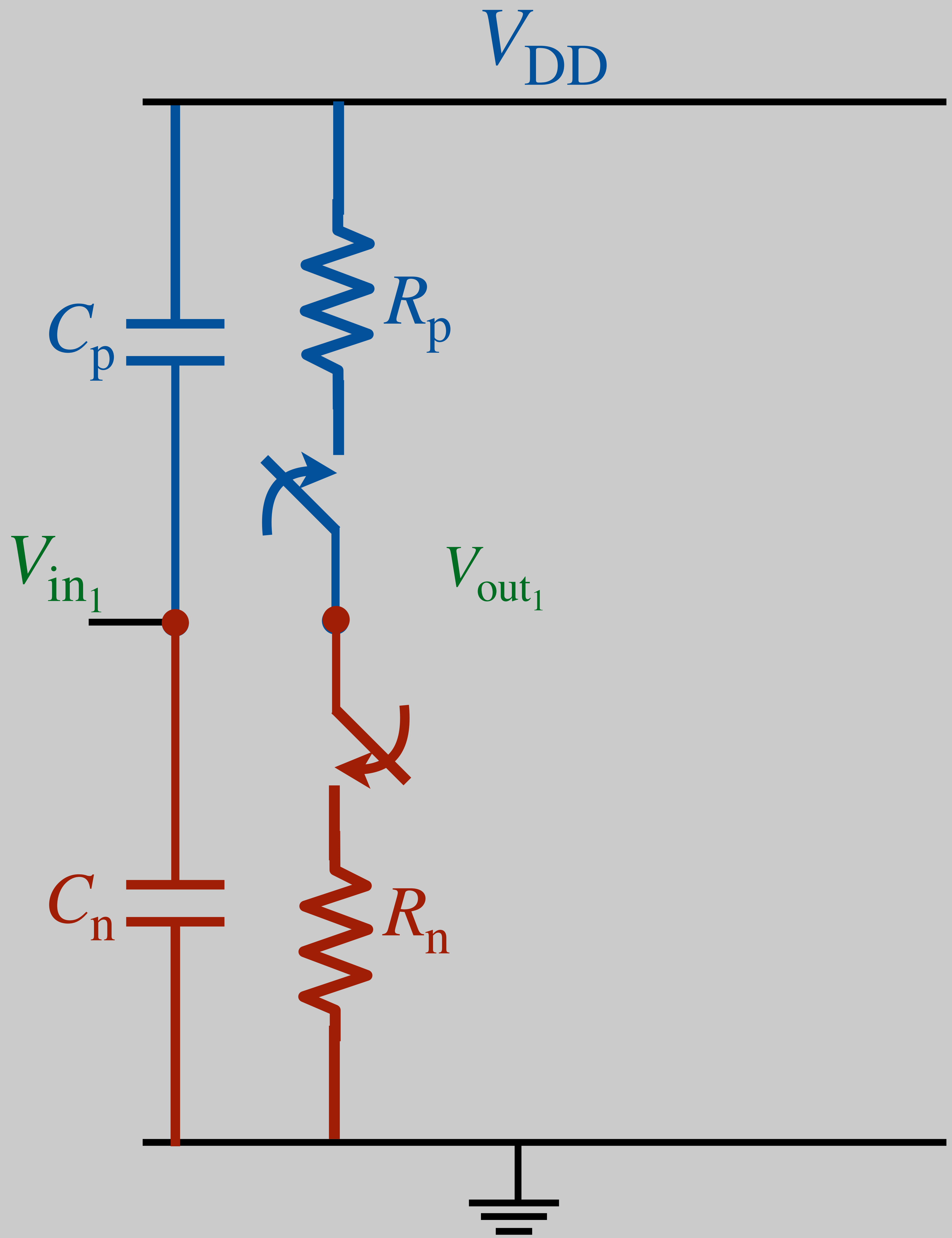
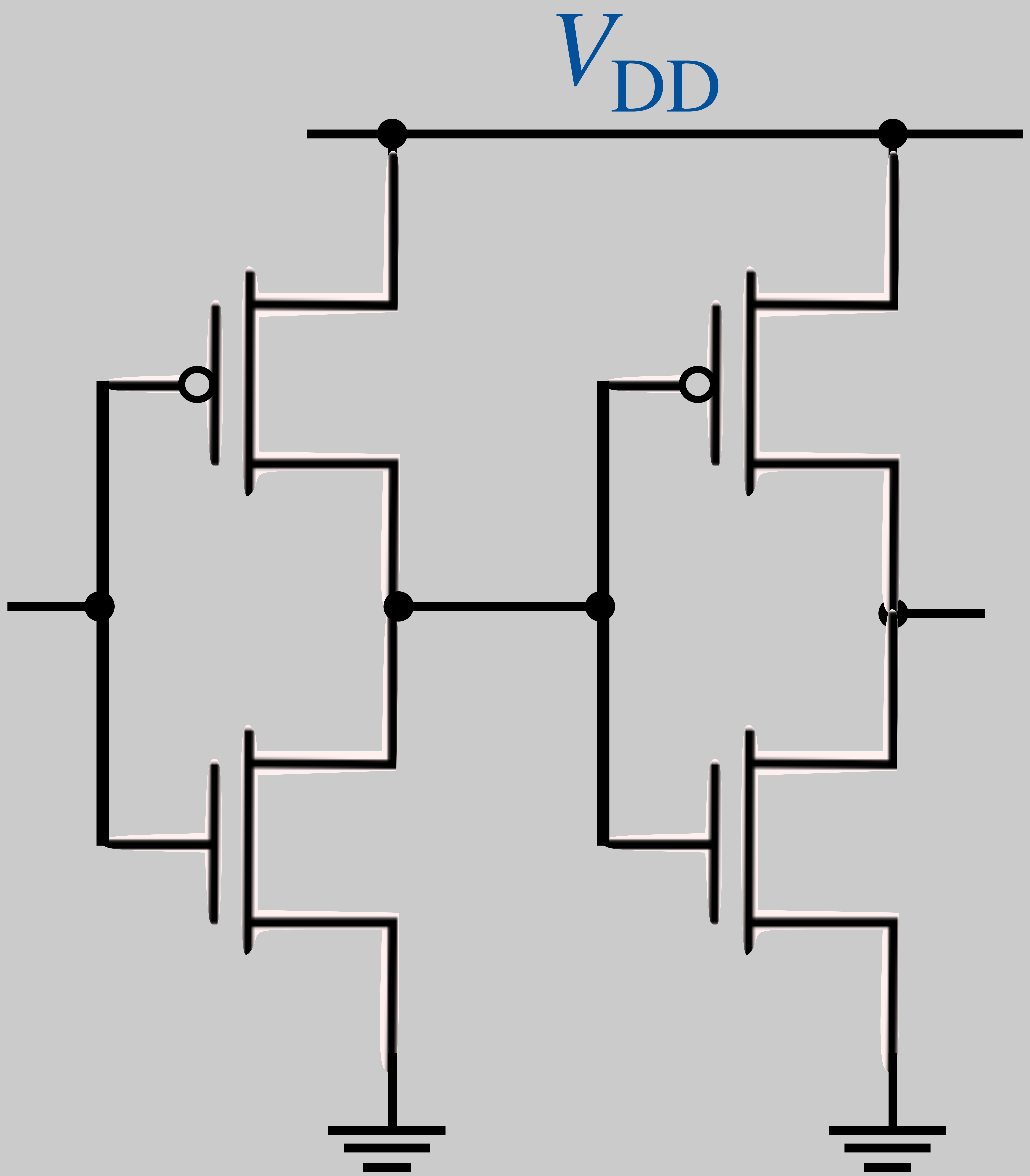
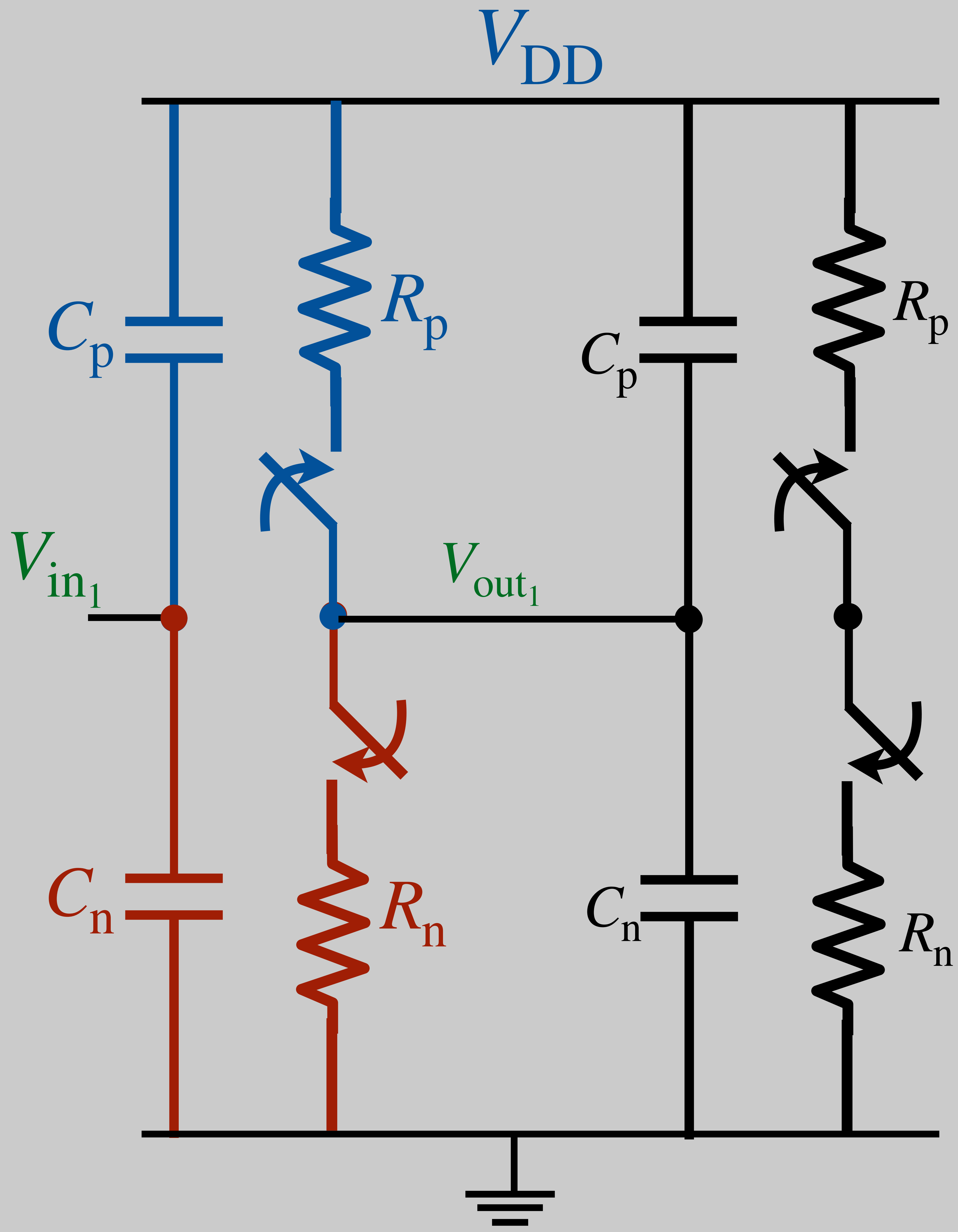




RC Model of CMOS

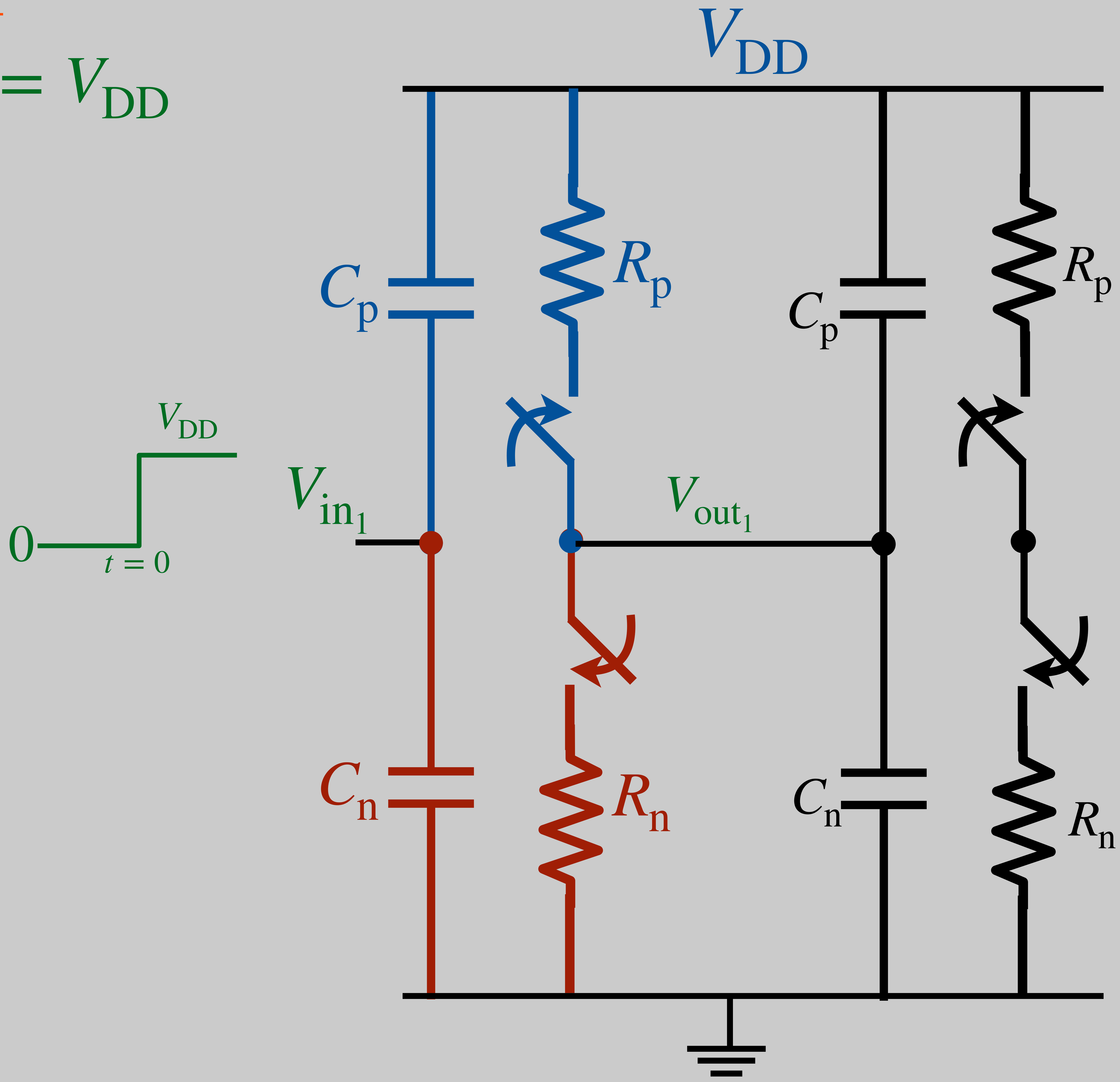




RC Model of CMOS

Example 1: switches from $V_{\mathrm{in}_1} = 0 \quad \Rightarrow V_{\mathrm{in}_1} = V_{DD}$

$t < 0$

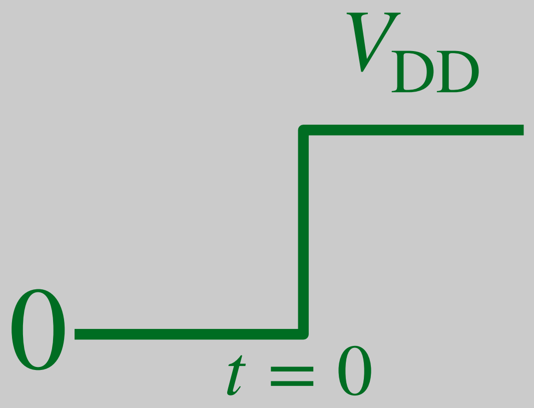


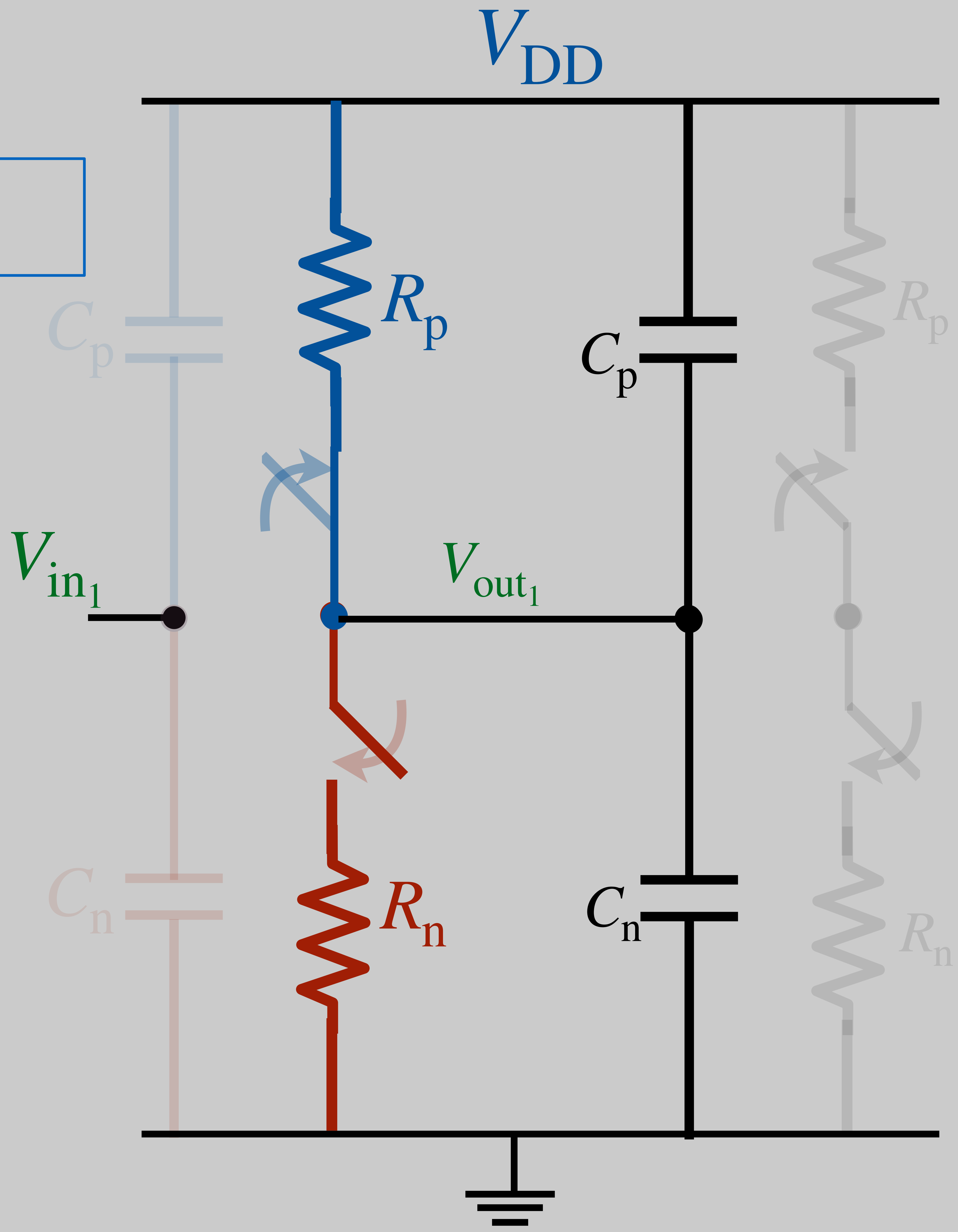
RC Model of CMOS

Example 1: switches from $V_{\text{in}_1} = 0 \quad \Rightarrow V_{\text{in}_1} = V_{\text{DD}}$

$t < 0 \quad \Rightarrow \boxed{V_{\text{in}_1} = \qquad , V_{\text{out}_1} = \qquad , V_{C_p} = \qquad , V_{C_n} = \qquad}$

$t \geq 0$



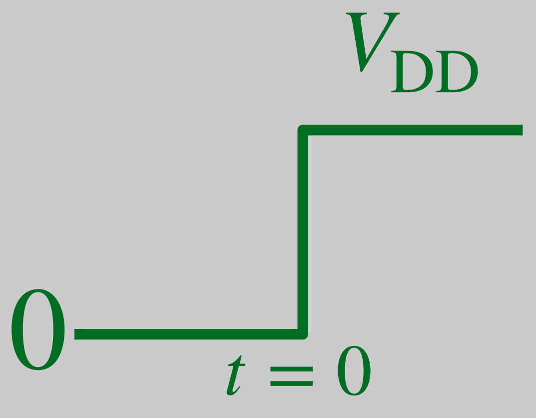


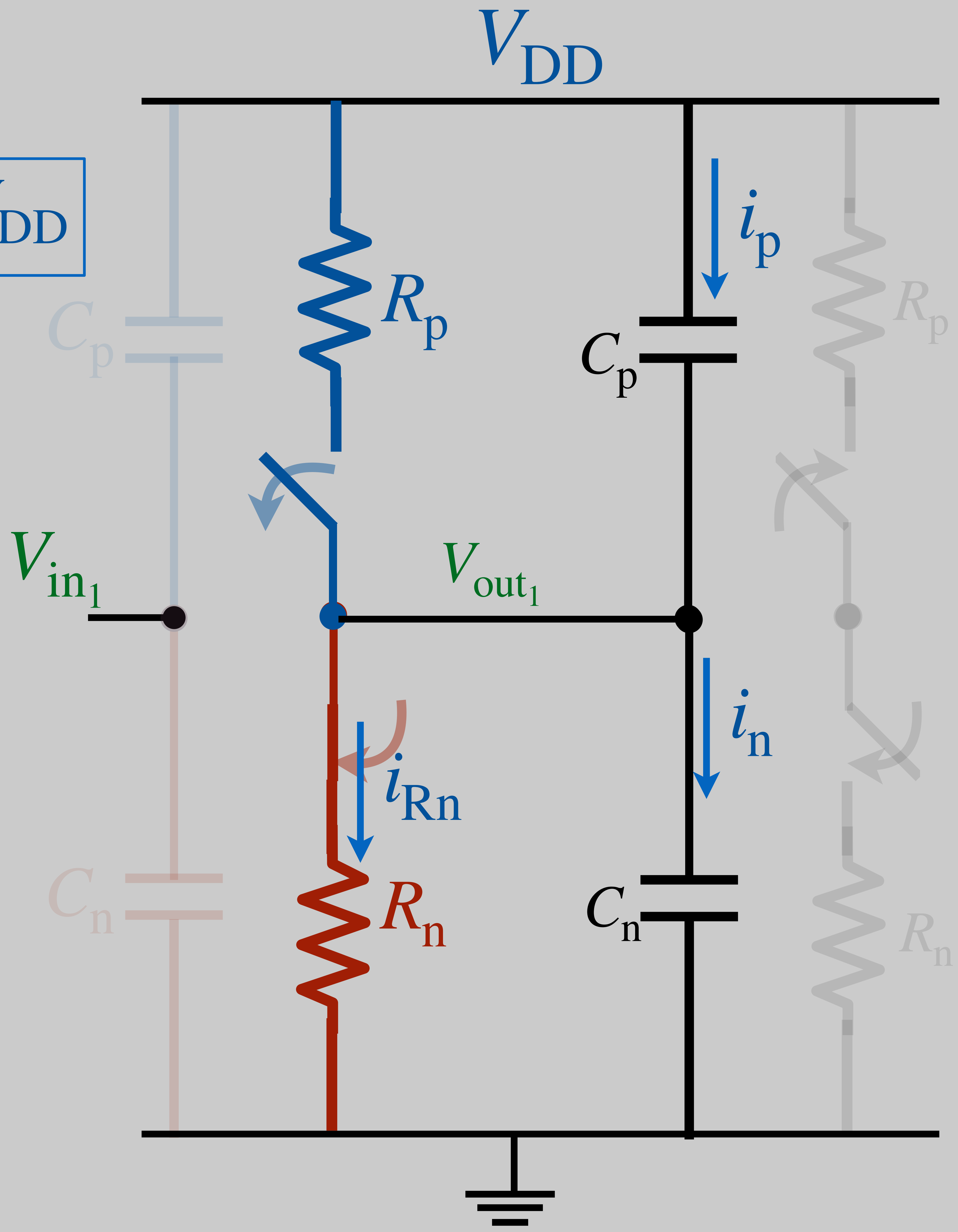
RC Model of CMOS

Example 1: switches from $V_{\text{in}_1} = 0 \quad \Rightarrow V_{\text{in}_1} = V_{\text{DD}}$

$t < 0 \quad \Rightarrow \boxed{V_{\text{in}_1} = 0 \quad , V_{\text{out}_1} = V_{\text{DD}} \ , V_{C_\text{p}} = 0 \quad , V_{C_\text{n}} = V_{\text{DD}}}$

$t \geq 0$





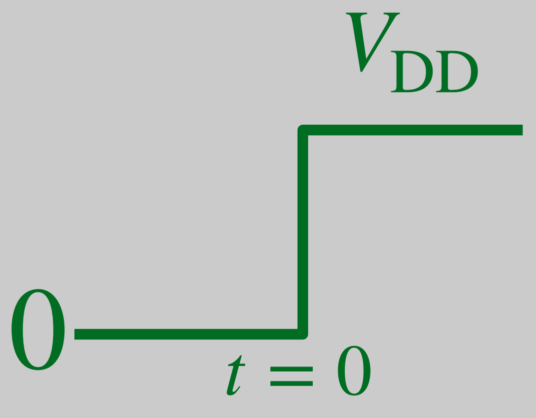
RC Model of CMOS

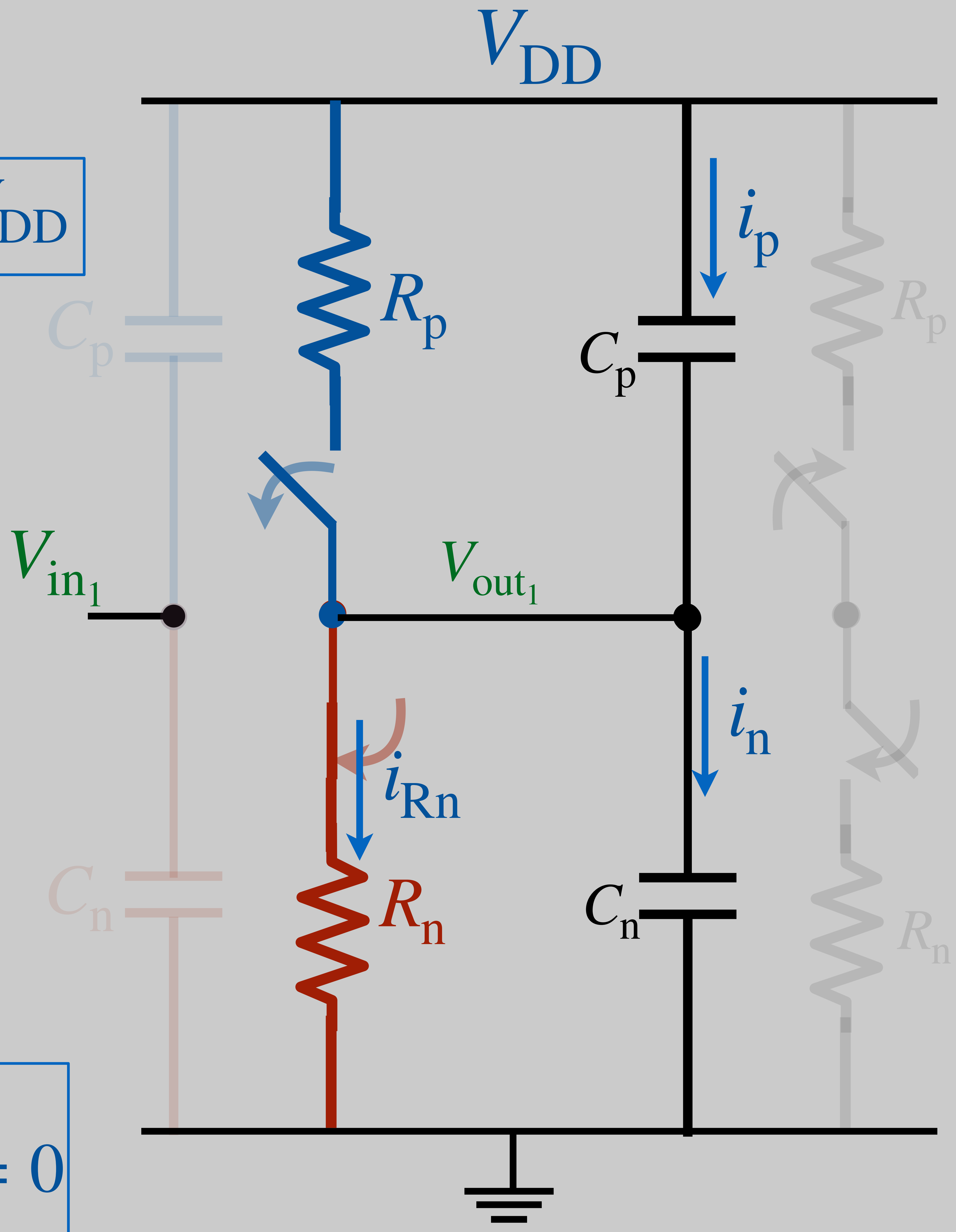
Example 1: switches from $V_{in_1} = 0 \quad \Rightarrow V_{in_1} = V_{DD}$

$$t < 0 \quad \Rightarrow \boxed{V_{in_1} = 0 \quad , V_{out_1} = V_{DD} \; , V_{C_p} = 0 \quad , V_{C_n} = V_{DD}}$$

$$t \geq 0 \qquad i_{Rn} + i_n = i_p$$

$$\frac{V_{out_1}}{R_n} + C_n \frac{dV_{out_1}}{dt} = C_p \frac{d(\overset{\text{const}}{\cancel{V_{DD}}} - V_{out_1})}{dt}$$





$$C_n \frac{dV_{out_1}}{dt} + C_p \frac{dV_{out_1}}{dt} + \frac{V_{out_1}}{R_n} = 0$$

$$(C_n + C_p) \frac{dV_{out_1}}{dt} + \frac{V_{out_1}}{R_n} = 0$$

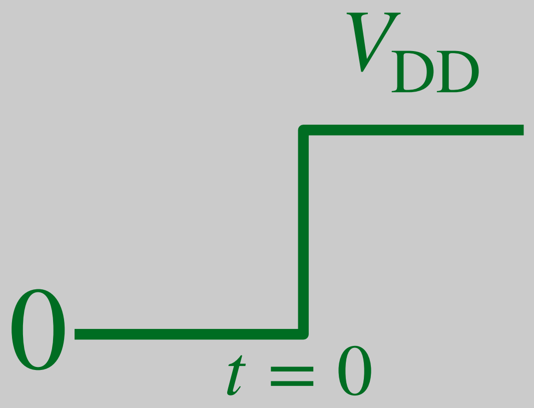
$$\boxed{\frac{dV_{out_1}}{dt} + \frac{V_{out_1}}{R_n(C_n + C_p)} = 0}$$

RC Model of CMOS

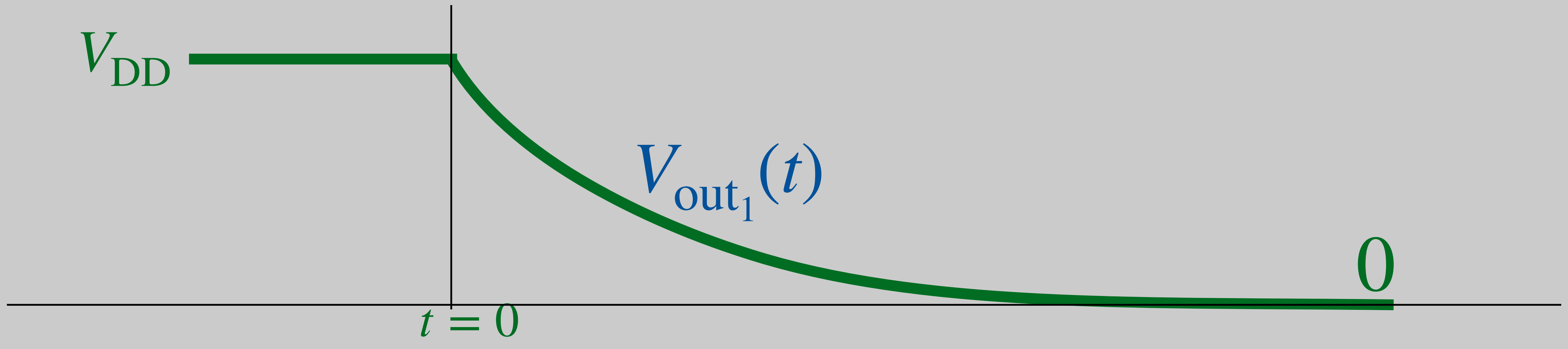
Example 1: switches from $V_{\text{in}_1} = 0 \quad \Rightarrow V_{\text{in}_1} = V_{\text{DD}}$

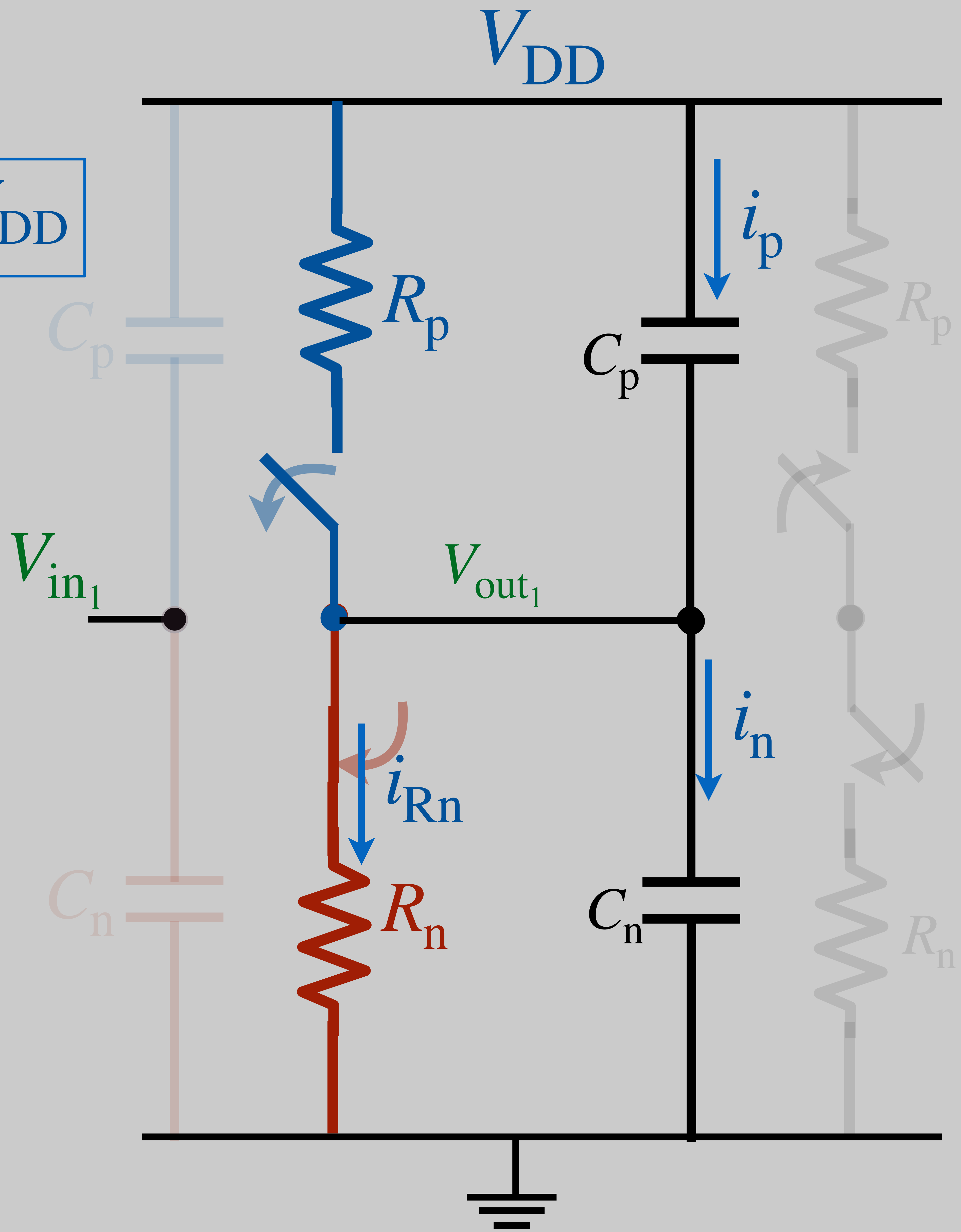
$t < 0 \quad \Rightarrow \boxed{V_{\text{in}_1} = 0 \quad , V_{\text{out}_1} = V_{\text{DD}} \ , V_{C_\text{p}} = 0 \quad , V_{C_\text{n}} = V_{\text{DD}}}$

$t \geq 0 \qquad \boxed{\dfrac{dV_{\text{out}_1}}{dt} + \dfrac{V_{\text{out}_1}}{R_\text{n}(C_\text{n} + C_\text{p})} = 0}$

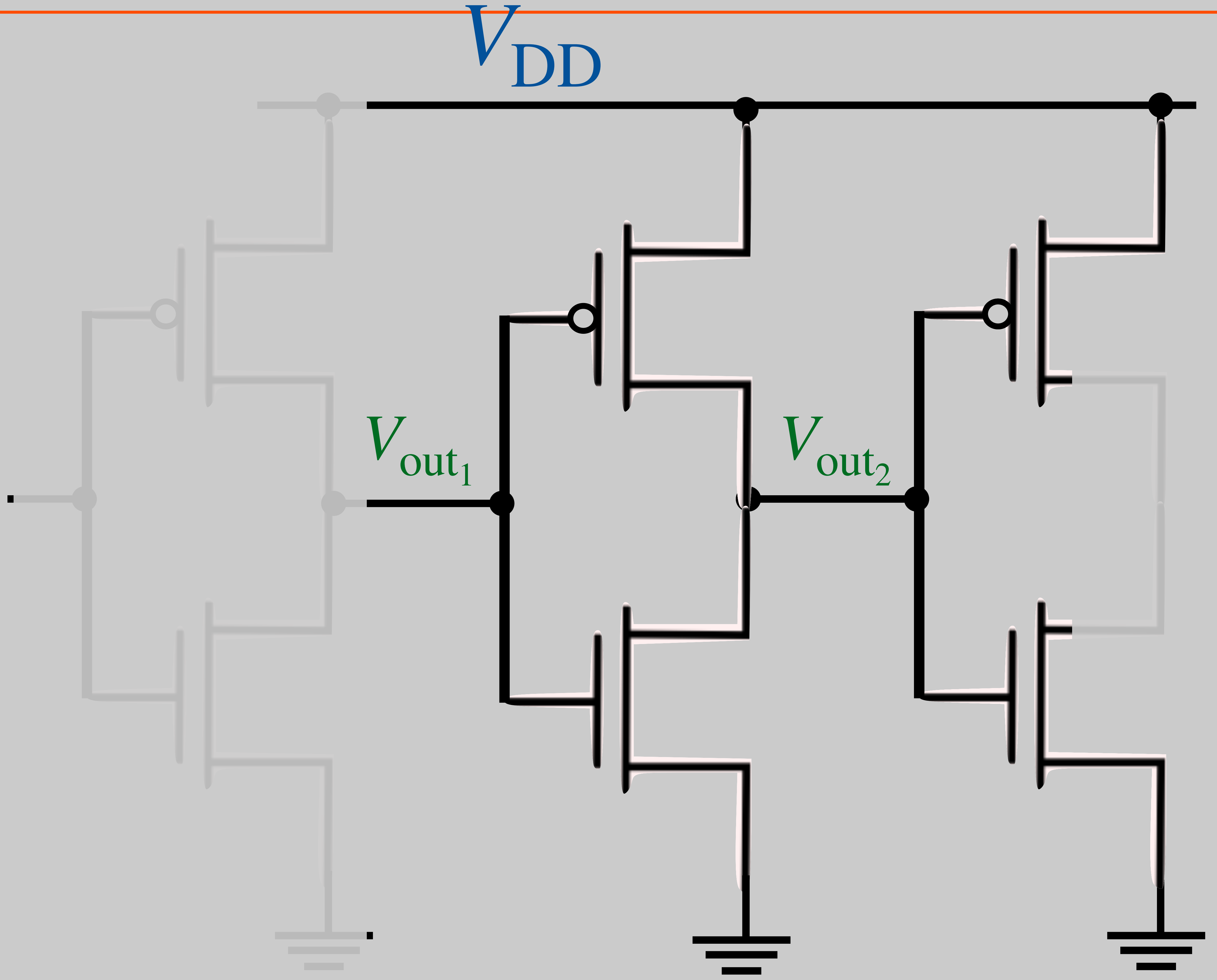


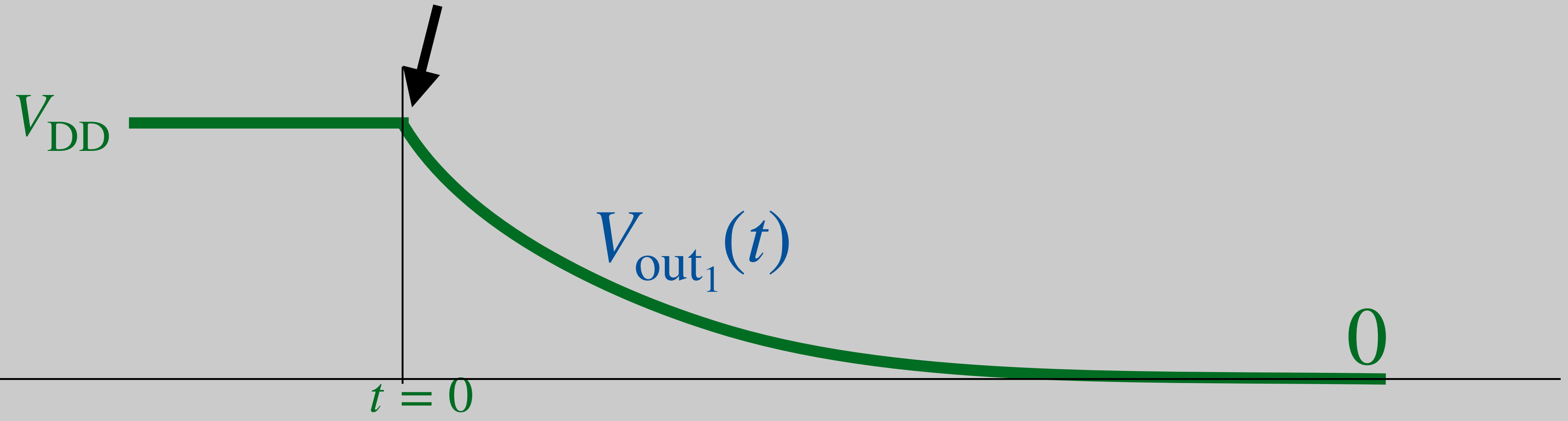
$$V_{\text{out}_1} = V_{\text{DD}} e^{-\frac{t}{R_\text{n}(C_\text{p} + C_\text{n})}}$$

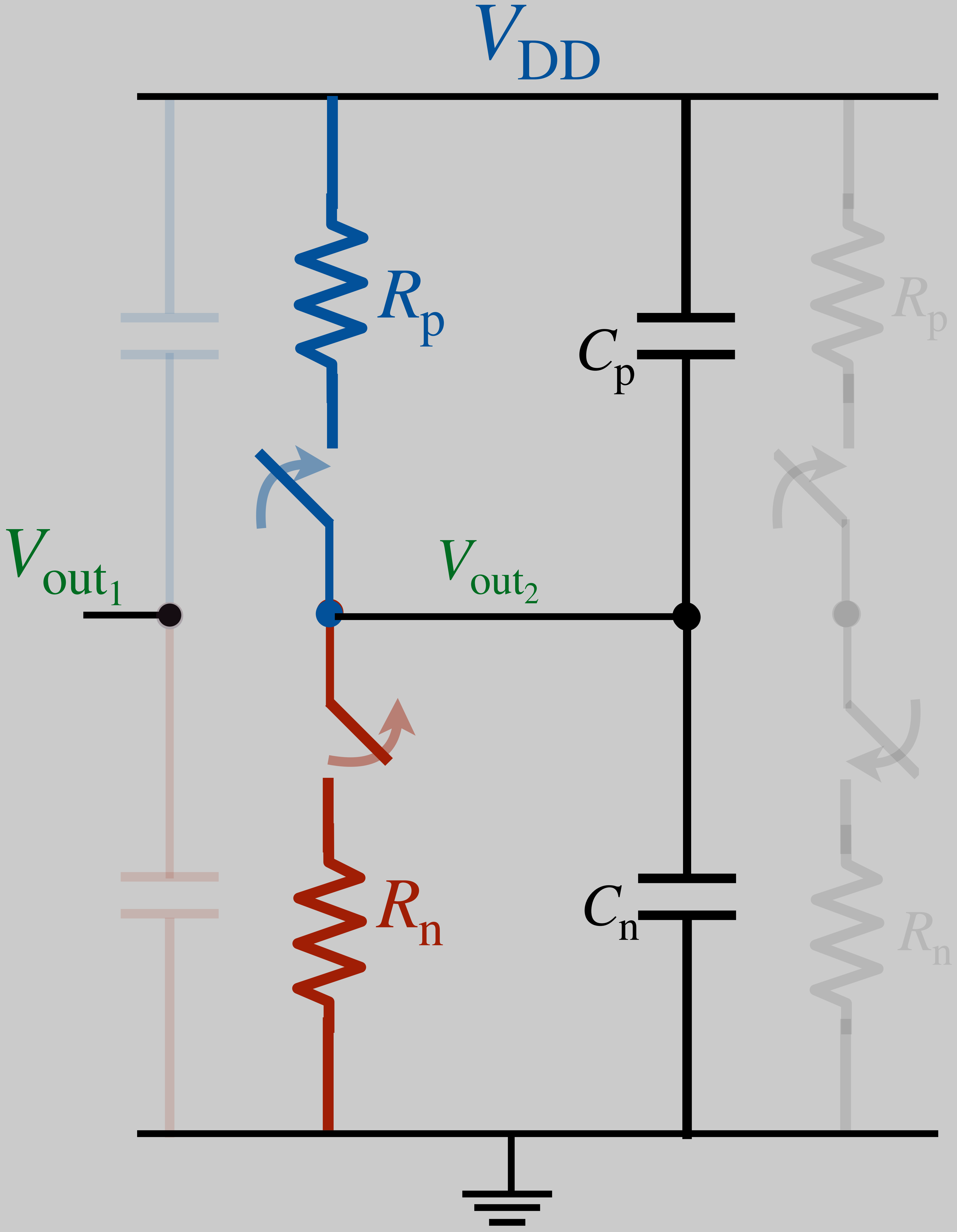




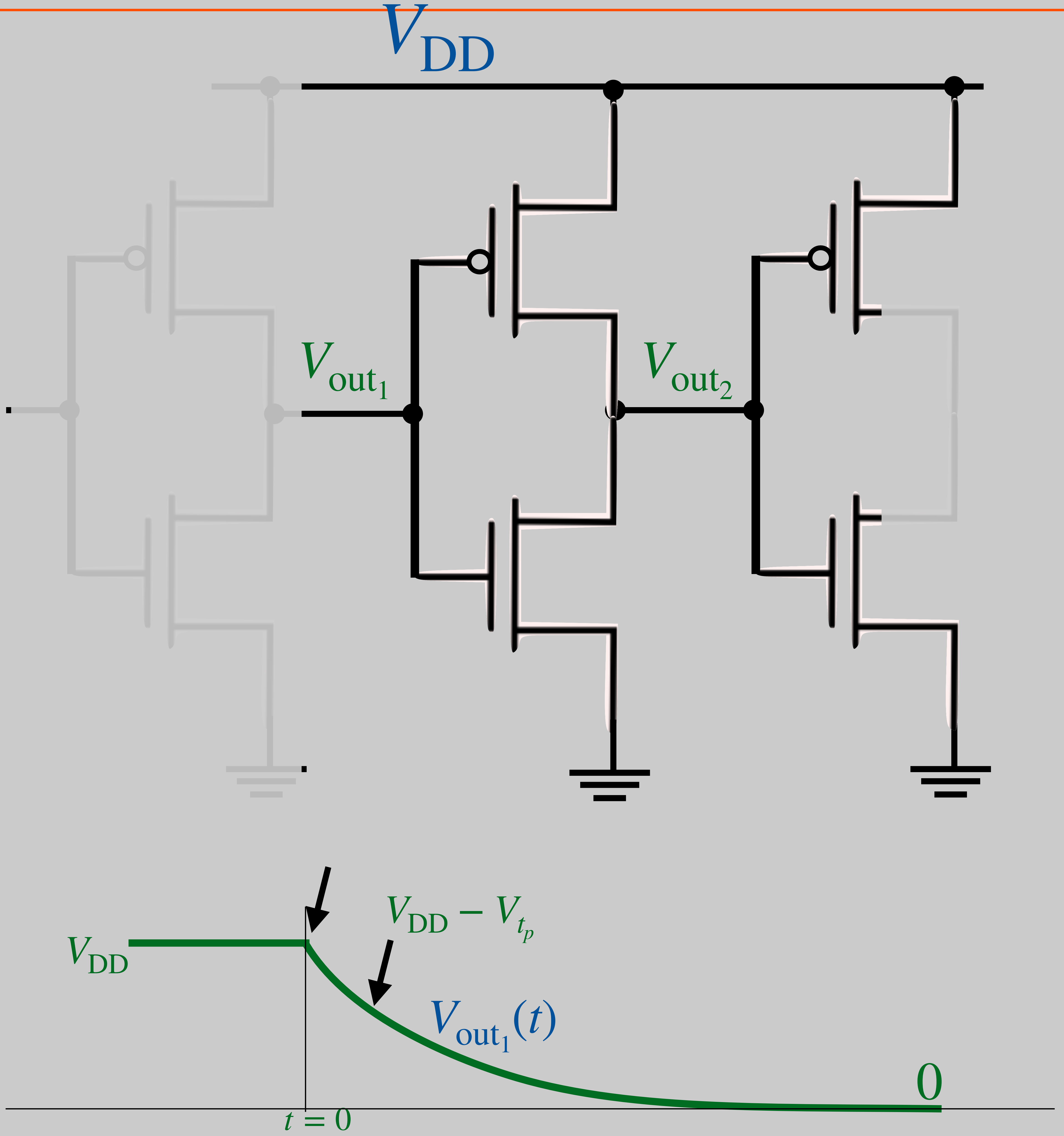
Realistic RC Delay

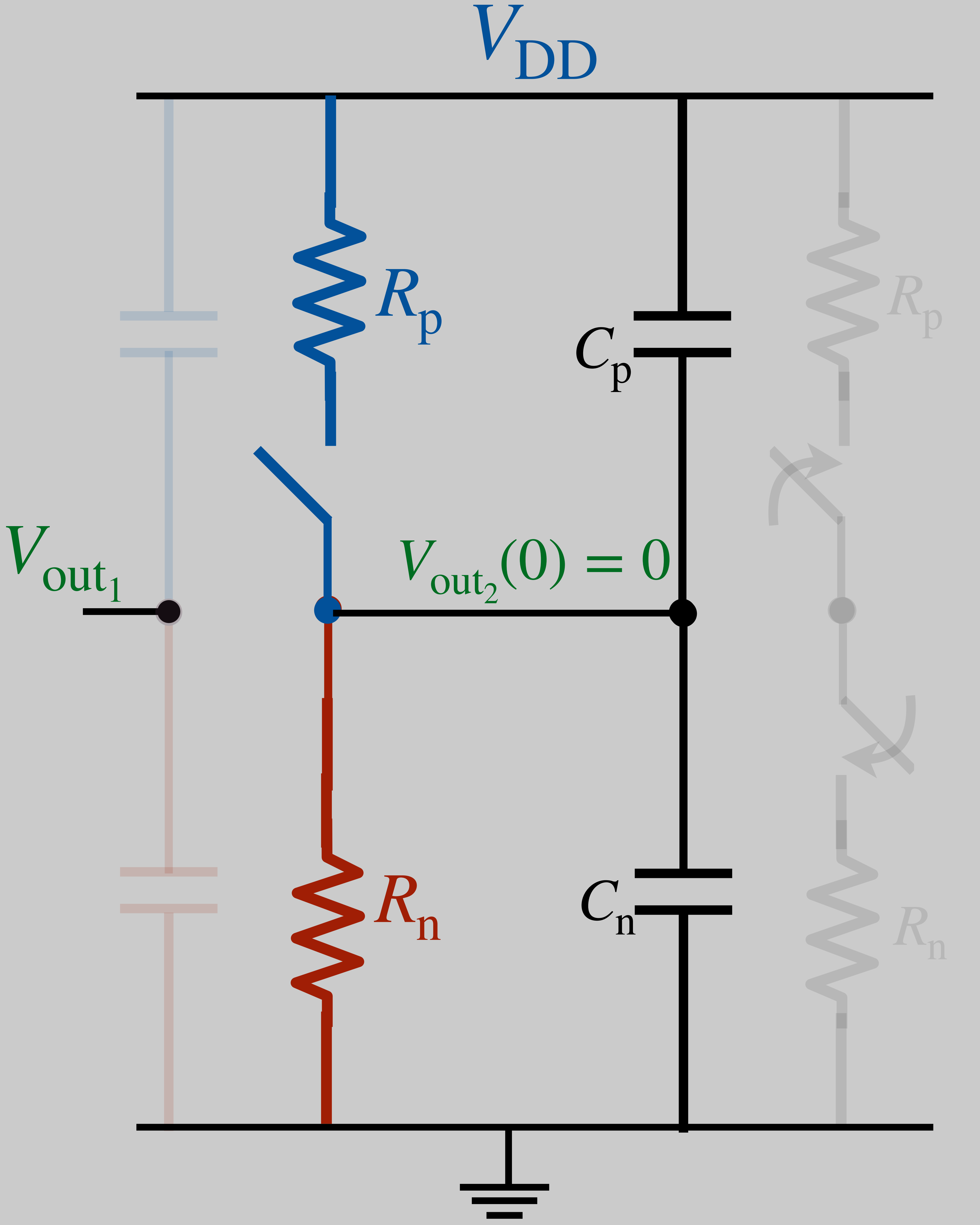




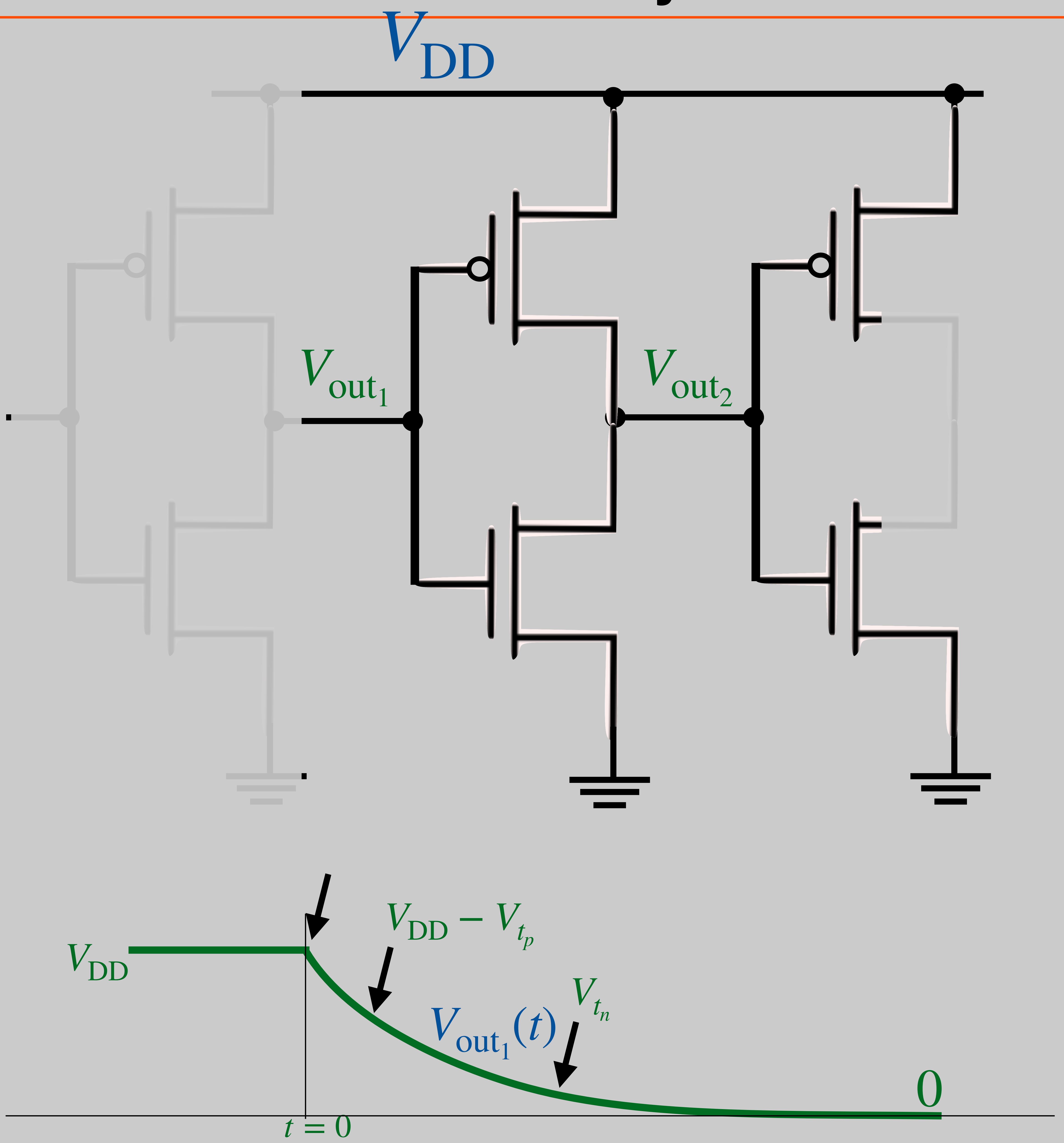


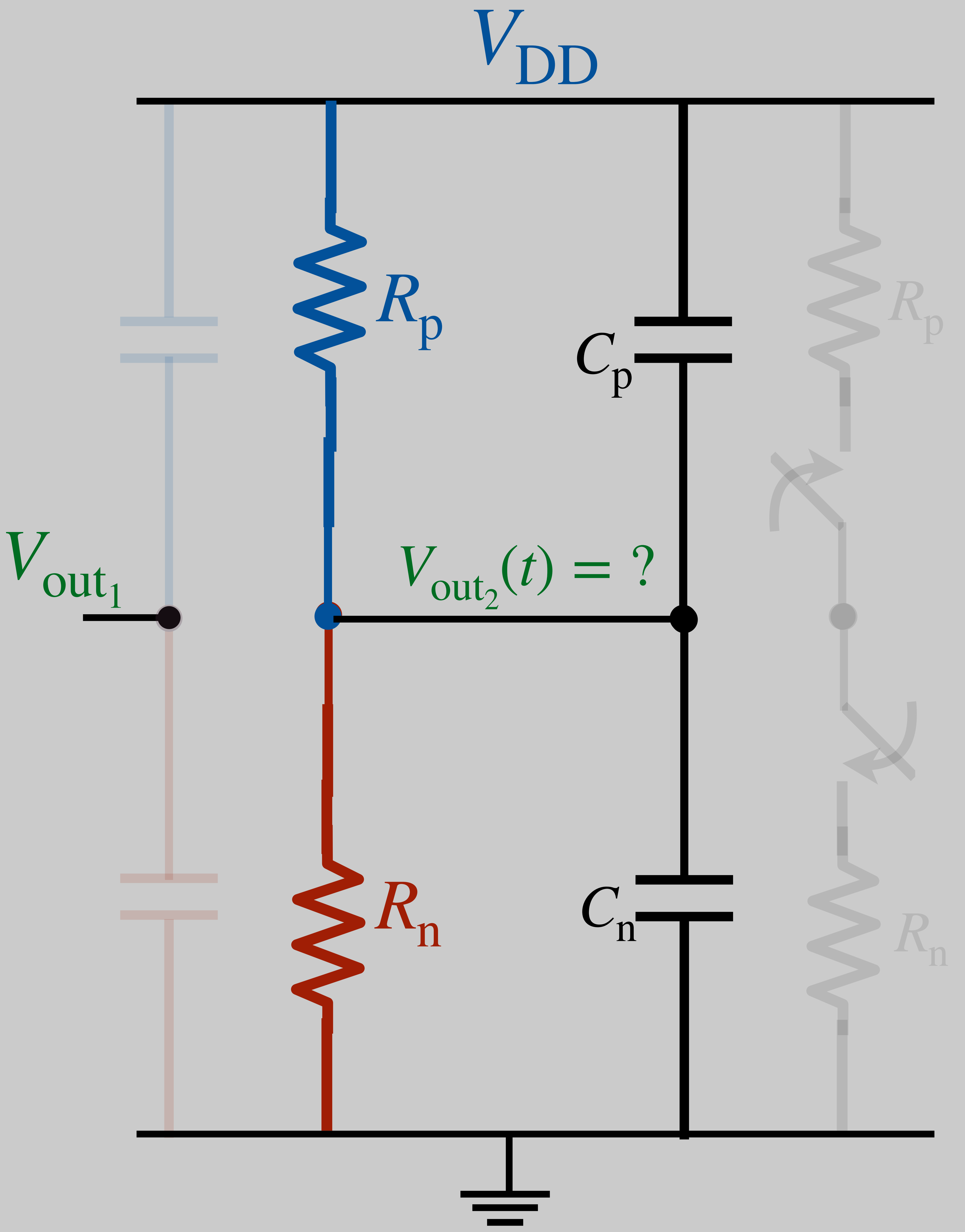
Realistic RC Delay



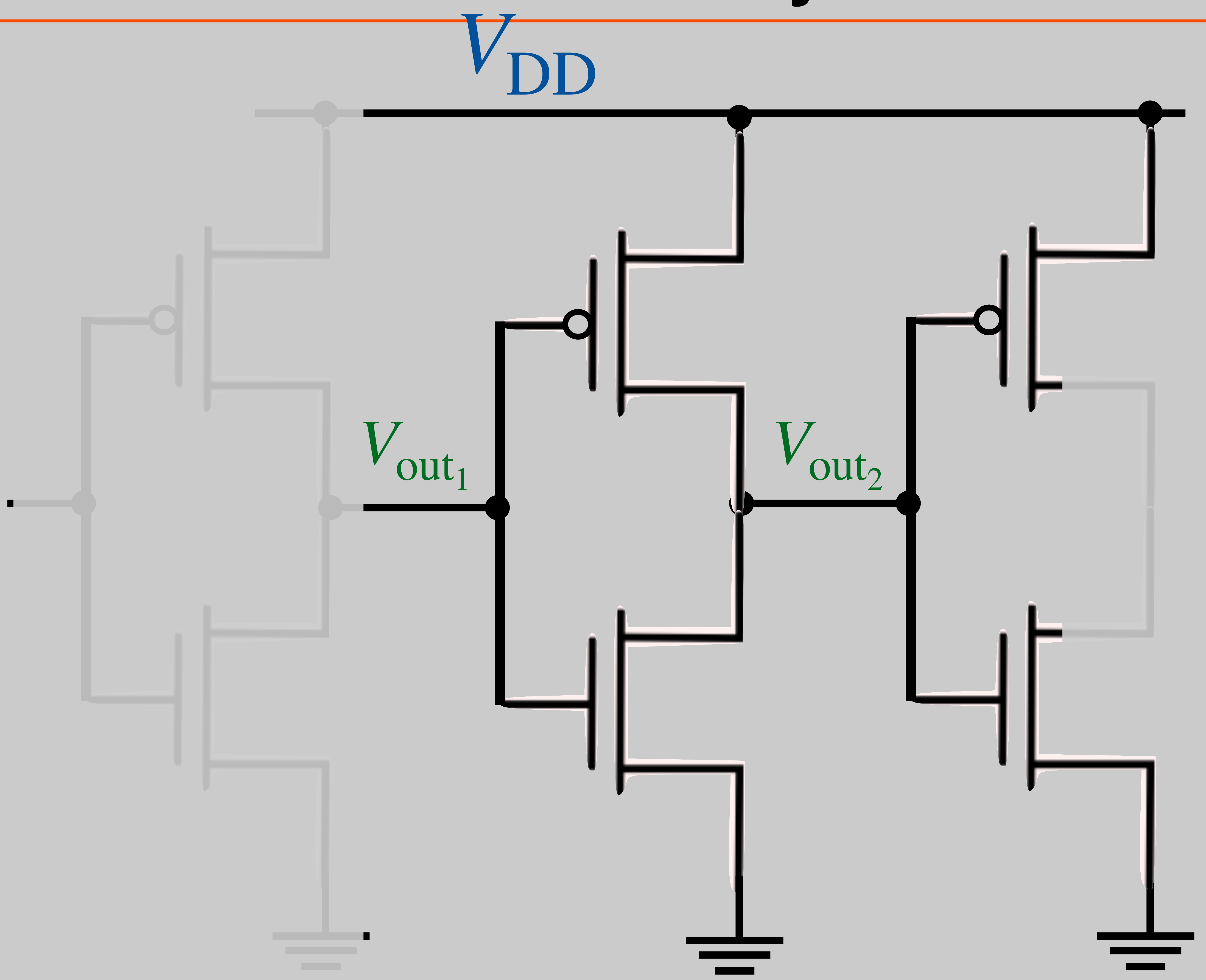


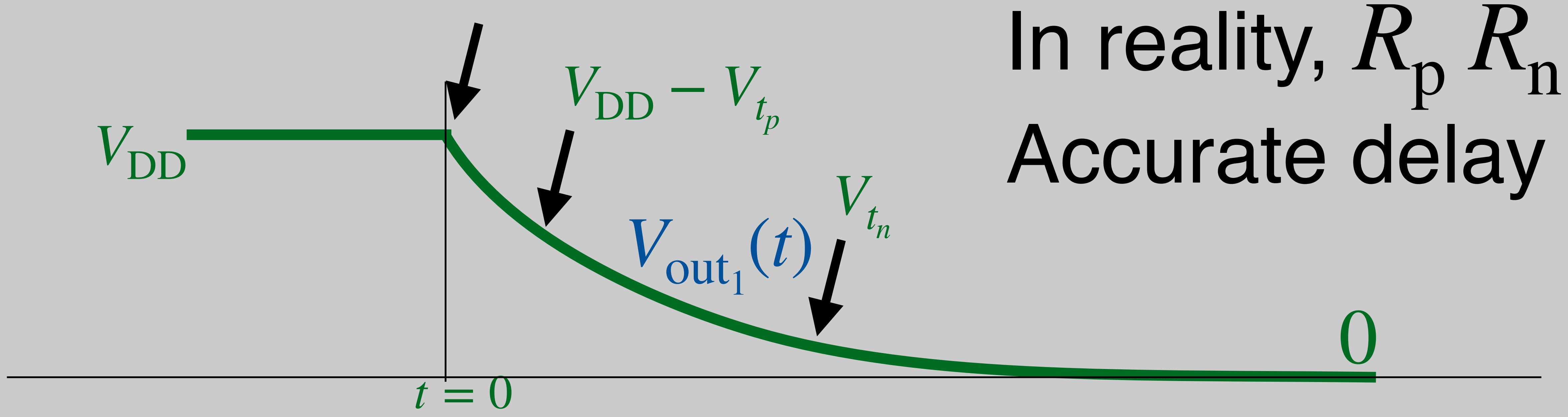
Realistic RC Delay

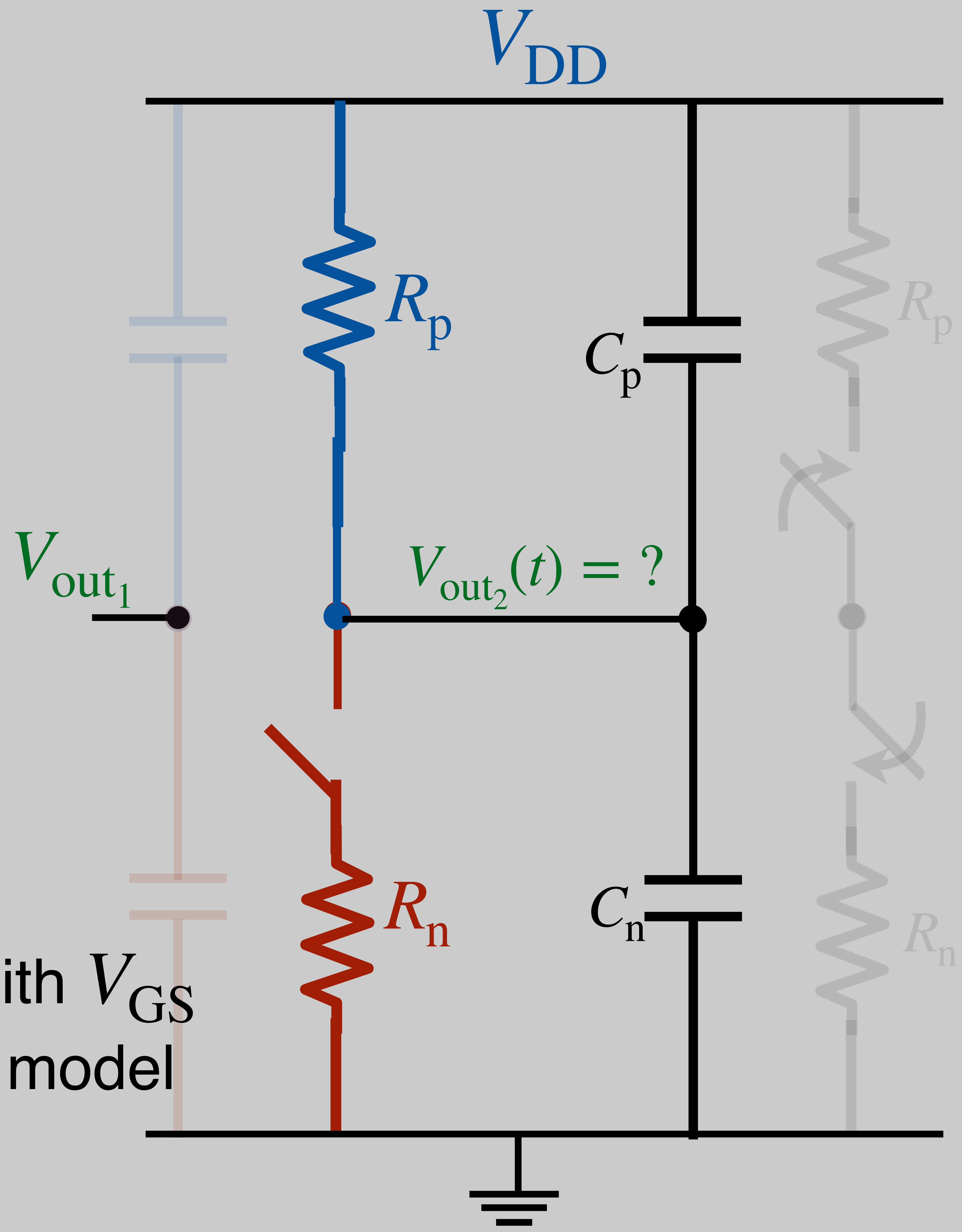




Realistic RC Delay





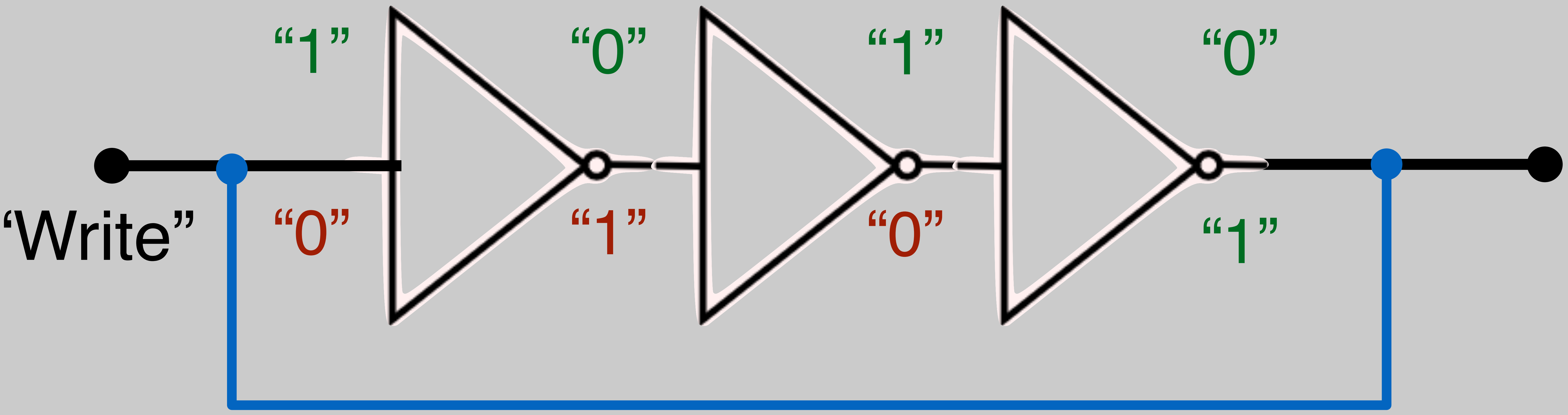


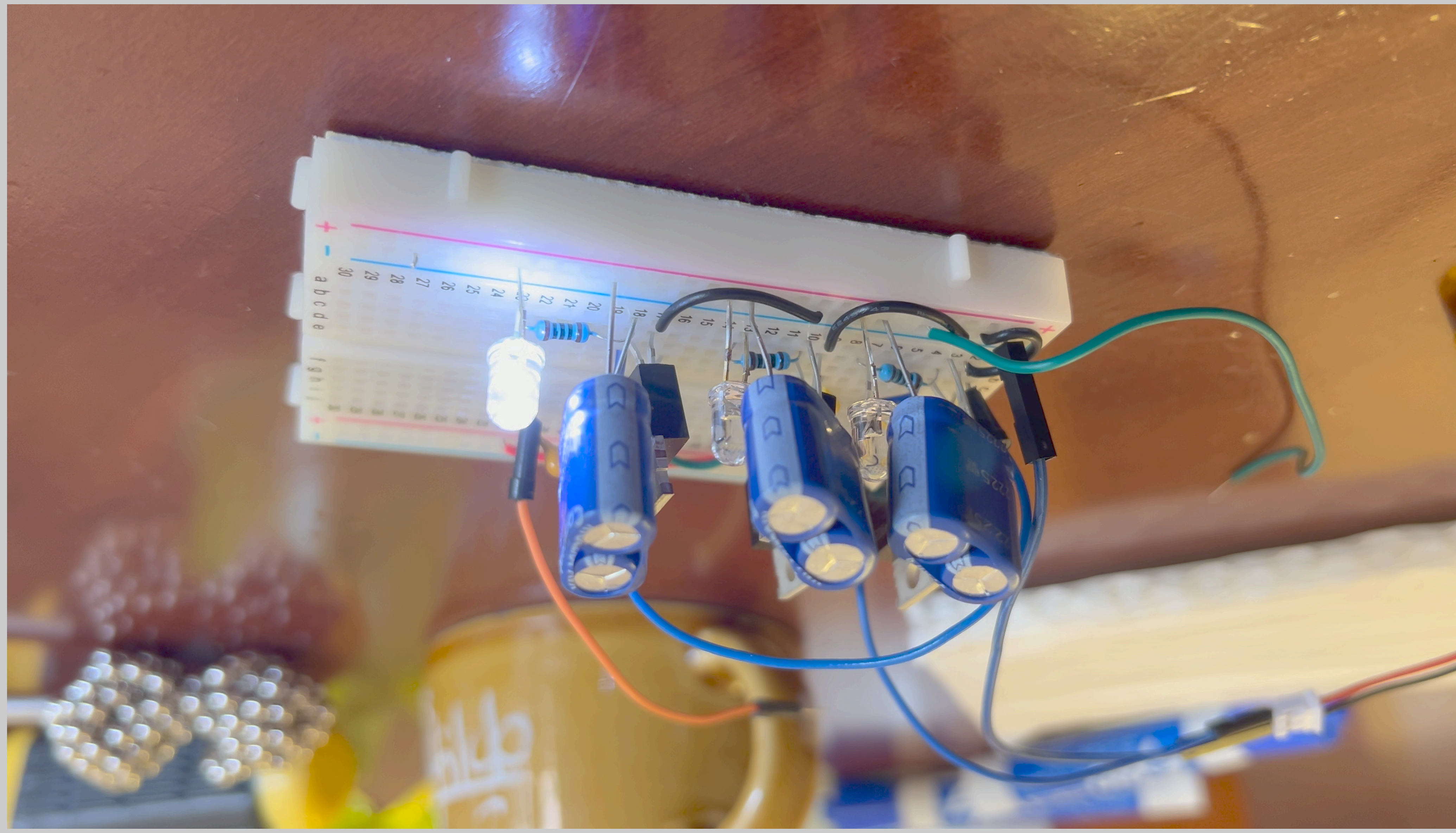
In reality, R_p R_n also change with V_{GS}

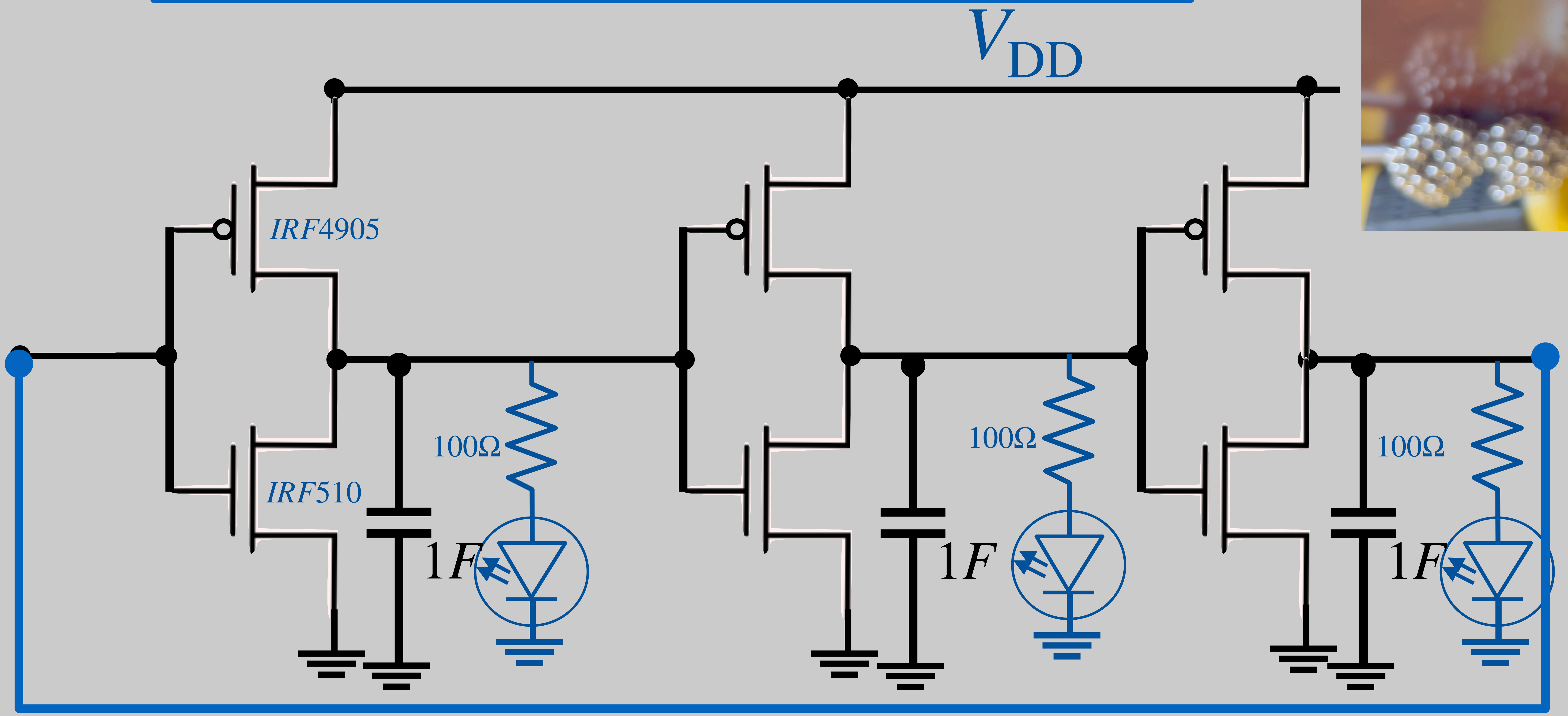
Accurate delay needs a better model

Demo

Inconsistant!







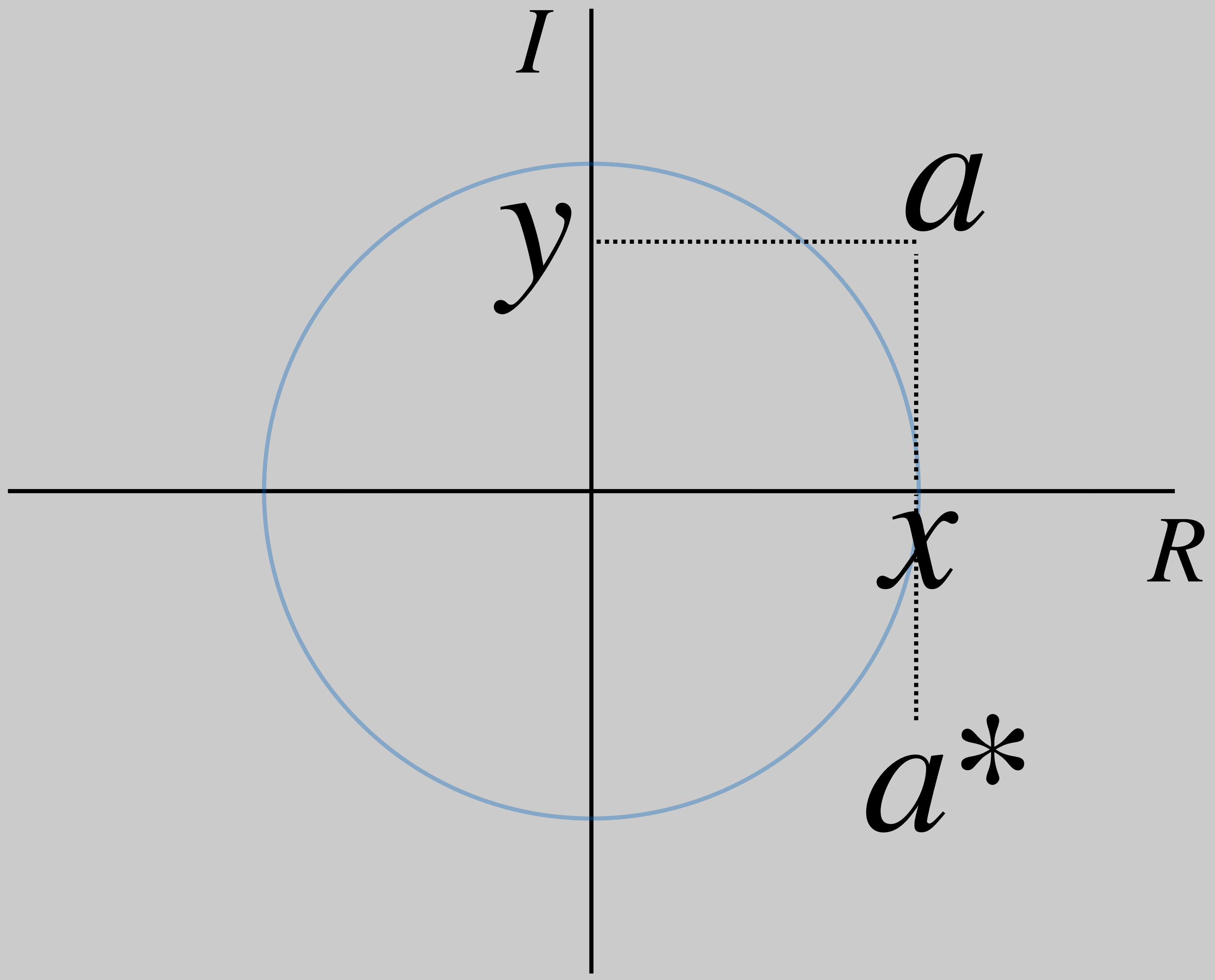
Complex Numbers Review

$$a = x + jy \quad | j = \sqrt{-1}$$

$$a^* = x - jy$$

$$\mathscr{Re}\{a\} = \frac{1}{2}(a + a^*) = x$$

$$\mathscr{Im}\{a\} = \frac{1}{2j}(a - a^*) = y$$



Euler Formula

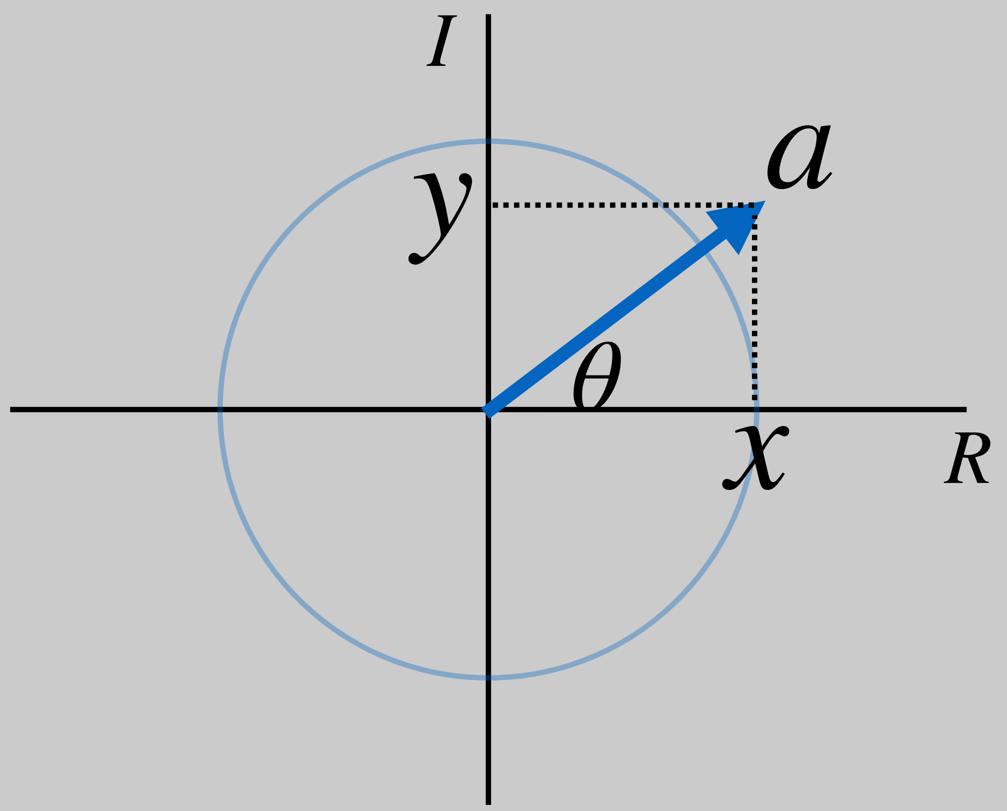
$$a = x + jy$$

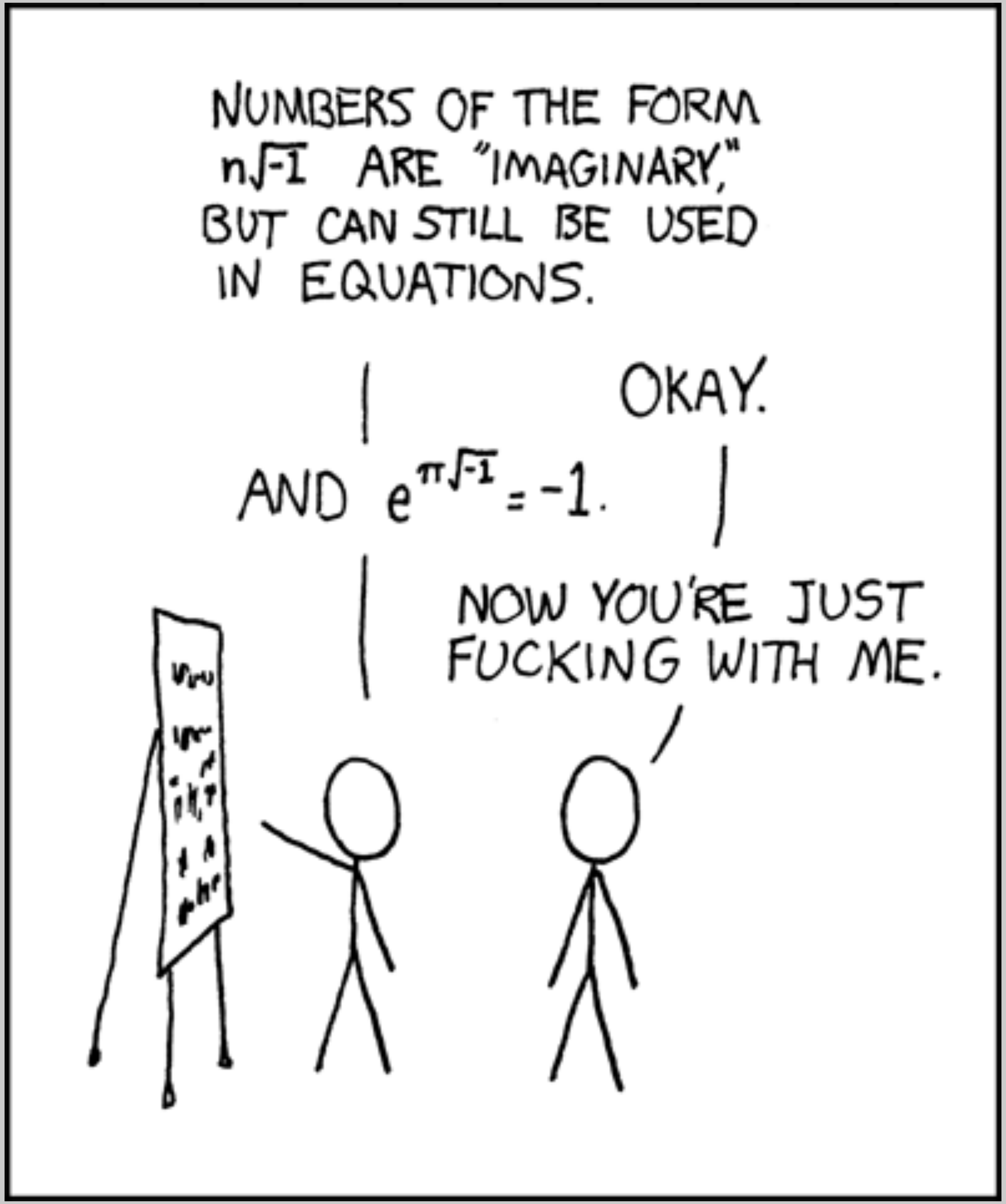
$$aa^* = |a|^2 = |x|^2 + |y|^2$$

$$\theta = \angle(a) = \text{atan}(\frac{y}{x})$$

Euler:

$$a = |a|e^{j\theta}$$
$$= |a|(\cos(\theta) + j\sin(\theta))$$





Euler Formula

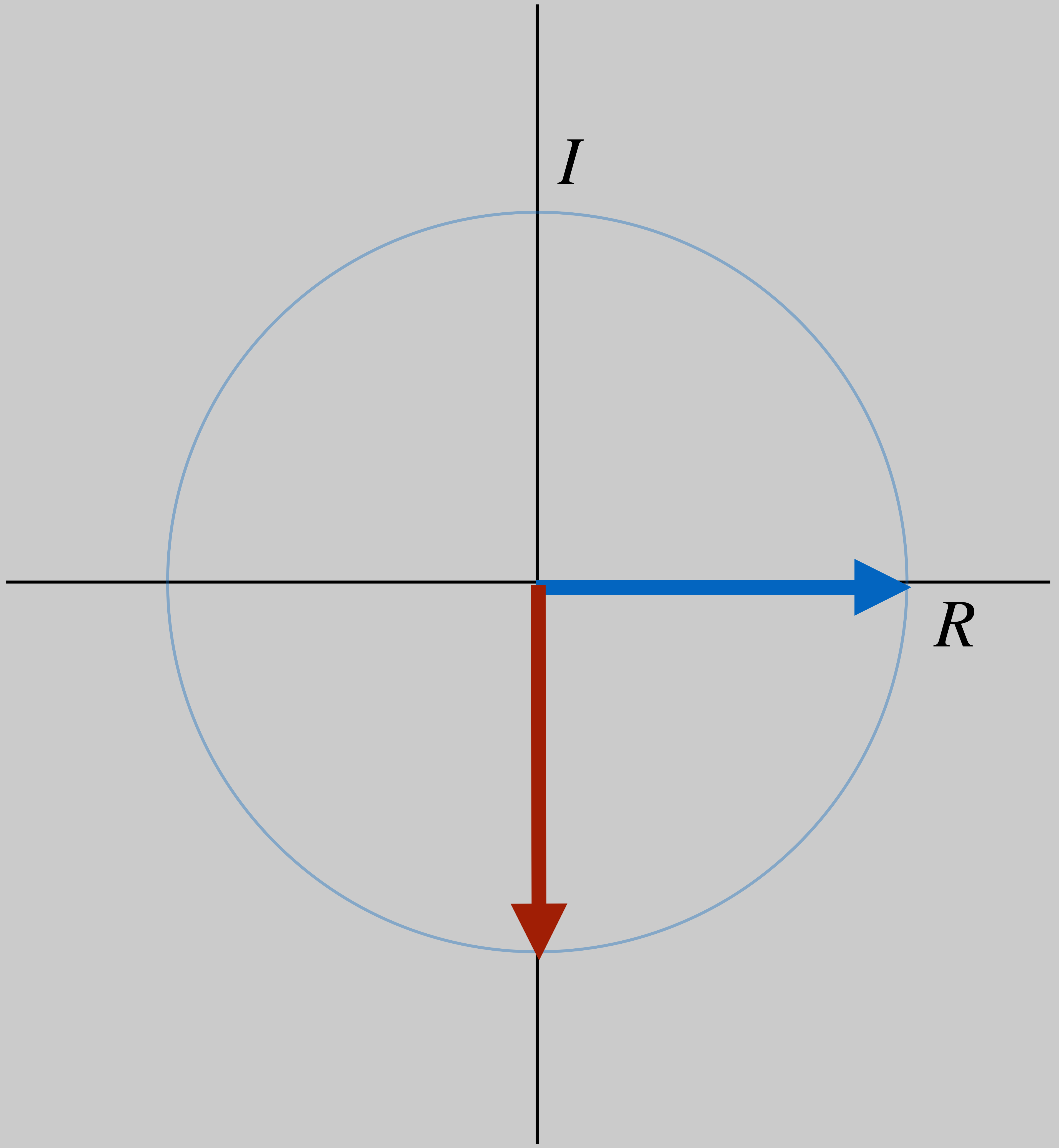
Euler: $a = |a|e^{j\theta}$

$$= |a|(\cos(\theta) + j\sin(\theta))$$

$$|e^{j\theta}| = \cos^2(\theta) + \sin^2(\theta) = 1$$

$$\mathscr{R}e\{e^{j\omega t}\} = \cos(\omega t)$$

$$\mathscr{R}e\{e^{j(\omega t - \pi/2)}\} = \sin(\omega t)$$



Euler Formula

Euler: $a = |a| e^{j\theta}$

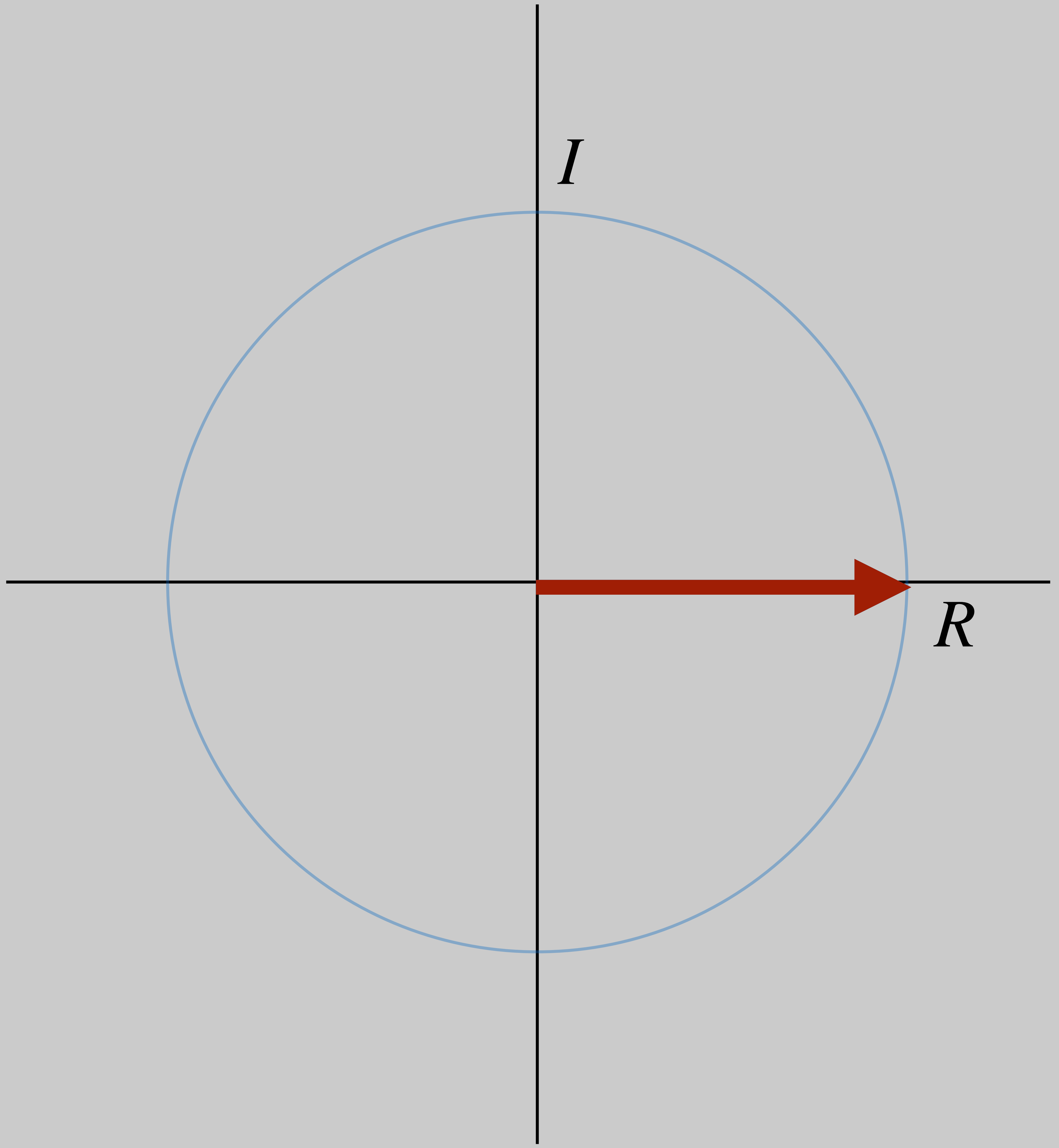
$$= |a| (\cos(\theta) + j \sin(\theta))$$

$$|e^{j\theta}| = \cos^2(\theta) + \sin^2(\theta) = 1$$

$$\mathcal{R}e\{e^{j\omega t}\} = \cos(\omega t)$$

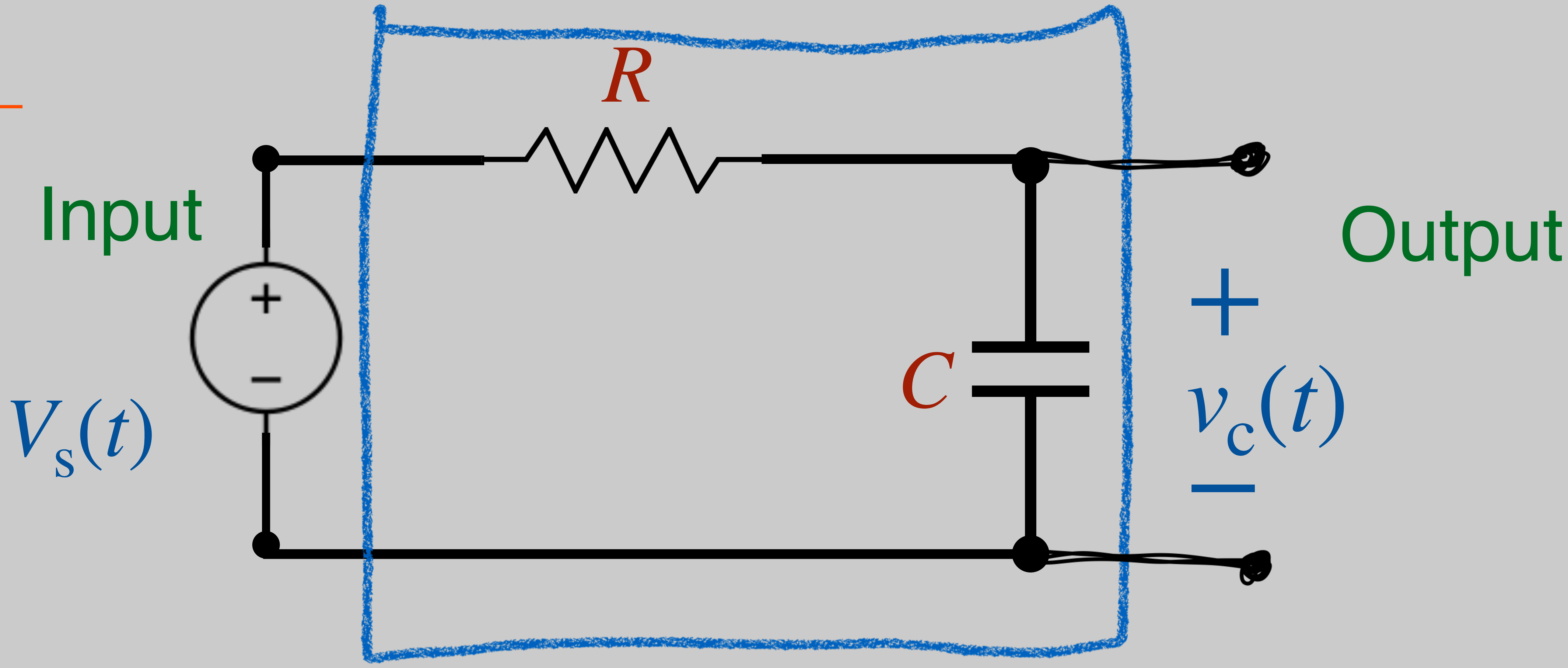
$$\mathcal{R}e\{e^{j(\omega t - \pi/2)}\} = \sin(\omega t)$$

$$\frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) = \cos(\omega t)$$



Sinusoidal Response

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_s(t)$$



Example 5: Sinusoidal Input

$$V_s(t) = V_0 \cos(\omega t) \quad | t > 0$$

$$v_c(0) = 0$$

$$\Rightarrow \frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_0 \cos(\omega t)$$

Sinusoidal Response

General Solution:

$$v_c(t) = Ke^{-\frac{t}{RC}} + \frac{e^{-\frac{t}{RC}}}{RC}\int_{-\infty}^{t} V_s(\tau)e^{\frac{\tau}{RC}}d\tau$$

$$V_s(t) = V_0\cos(\omega t) \quad | \, t > 0$$

$$v_c(0) = 0$$

Approach:

Set $V_s(t) = V_0 e^{j\omega t} \quad | \, t > 0$

Solve for $v_c(t)$ as usual

Solution for $V_s(t) = V_0\cos(\omega t)$ is $\mathscr{Re}\{v_c(t)\}$

General Solution:

$$v_c(t) = Ke^{-\frac{t}{RC}} + \frac{e^{-\frac{t}{RC}}}{RC}\int_{-\infty}^{t} V_s(\tau)e^{\frac{\tau}{RC}}d\tau$$

$$V_s(t) = V_0 e^{j\omega t} \quad | t > 0$$

$$v_c(0) = 0$$

$$v_c(0) = 0 \quad \Rightarrow K = 0$$

$$v_c(t) = \frac{e^{-\frac{t}{RC}}}{RC}\int_{0}^{t} V_0 e^{j\omega\tau} e^{\frac{\tau}{RC}} d\tau$$

$$= V_0 \frac{e^{-\frac{t}{RC}}}{RC}\int_{0}^{t} e^{(j\omega+\frac{1}{RC})\tau}d\tau$$

$$= V_0 \frac{e^{-\frac{t}{RC}}}{RC}\frac{1}{j\omega+\frac{1}{RC}}\left[e^{(j\omega+\frac{1}{RC})\tau}\right]\Big|_0^t$$

$$= V_0 \frac{e^{-\frac{t}{RC}}}{j\omega RC + 1}\left(e^{(j\omega+\frac{1}{RC})t} - 1\right)$$

$$= \frac{V_0}{j\omega RC + 1}\left(e^{j\omega t} - e^{-\frac{t}{RC}}\right)$$

Sinusoidal Response

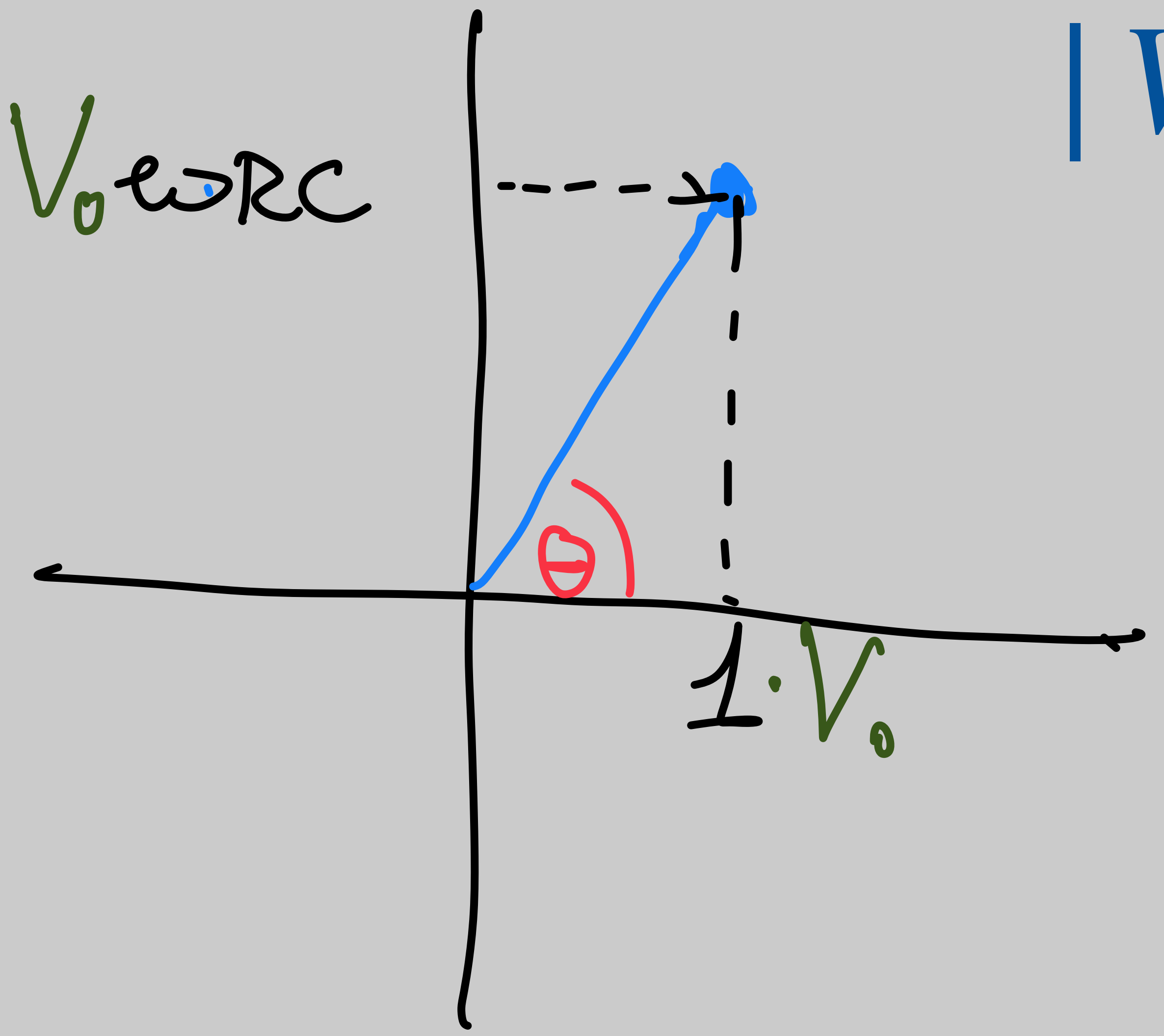
General Solution:

$$v_c(t) = \frac{V_0}{j\omega RC + 1}\left(e^{j\omega t} - e^{-\frac{t}{RC}}\right)$$

Transient

$t \gg RC$

$$v_c(t) = \frac{V_0}{j\omega RC + 1}e^{j\omega t}$$



$$|\,V_0(j\omega RC + 1)^{-1}\,| = |\,V_0\,|\left((\omega RC)^2 + 1\right)^{-1/2}$$

$$\theta = \angle(j\omega RC + 1)^{-1} = -\,\mathrm{atan}(\omega RC)$$

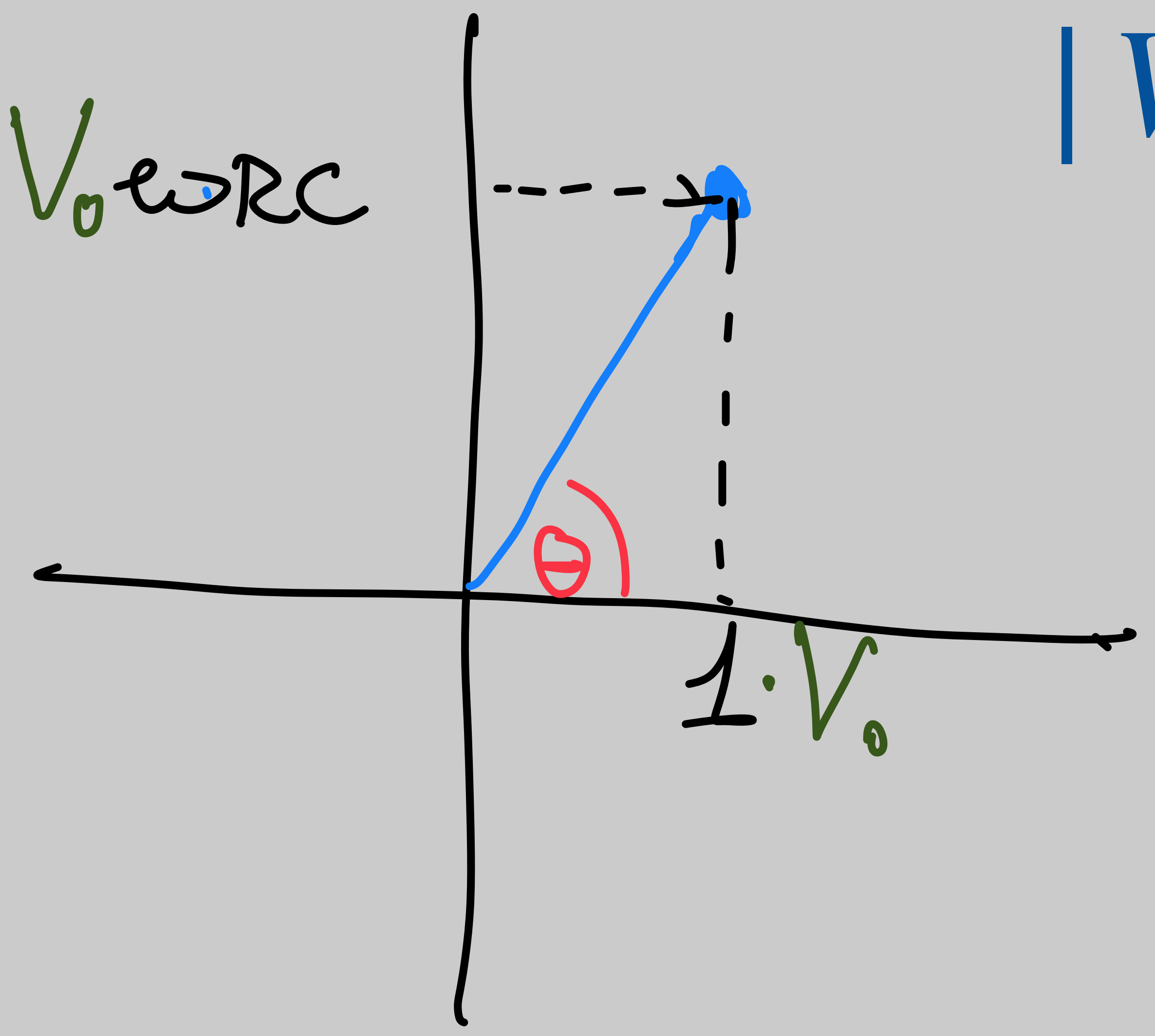
General Solution:

$$v_c(t) = \frac{V_0}{j\omega RC + 1}\left(e^{j\omega t} - e^{-\frac{t}{RC}}\right)$$

Transient

$t \gg RC$

$$v_c(t) = \frac{V_0}{j\omega RC + 1}e^{j\omega t}$$



$$|V_0(j\omega RC + 1)^{-1}| = |V_0|\left((\omega RC)^2 + 1\right)^{-1/2}$$

$$\theta = \angle(j\omega RC + 1)^{-1} = -\operatorname{atan}(\omega RC)$$

$$v_c(t) = \frac{V_0}{\sqrt{(\omega RC)^2 + 1}}e^{j(\omega t + \theta)}$$

Sinusoidal Response

$$V_s(t) = V_0 e^{\omega t} \quad | t > 0$$
$$v_c(0) = 0$$

$$\Rightarrow v_c(t) = \frac{V_0}{\sqrt{(\omega RC)^2 + 1}} e^{j(\omega t + \theta)}$$

$$V_s(t) = V_0 \cos(\omega t) \quad | t > 0$$
$$v_c(0) = 0$$

$$\Rightarrow v_c(t) = \frac{V_0}{\sqrt{(\omega RC)^2 + 1}} \cos(\omega t + \theta)$$

$$\omega = 0 \qquad \Rightarrow v_c(t) = V_0$$

$$\omega RC = 1 \qquad \Rightarrow v_c(t) = \frac{1}{\sqrt{2}} V_0 \cos(\omega t + \theta)$$

$$\omega RC \gg 1 \qquad \Rightarrow v_c(t) \approx \frac{V_0}{\omega RC} \cos(\omega t + \theta)$$

Low-pass, attenuating high freq!