EE16B
Designing Information Devices and Systems II

Lecture 2A
Inductors and RL Circuit

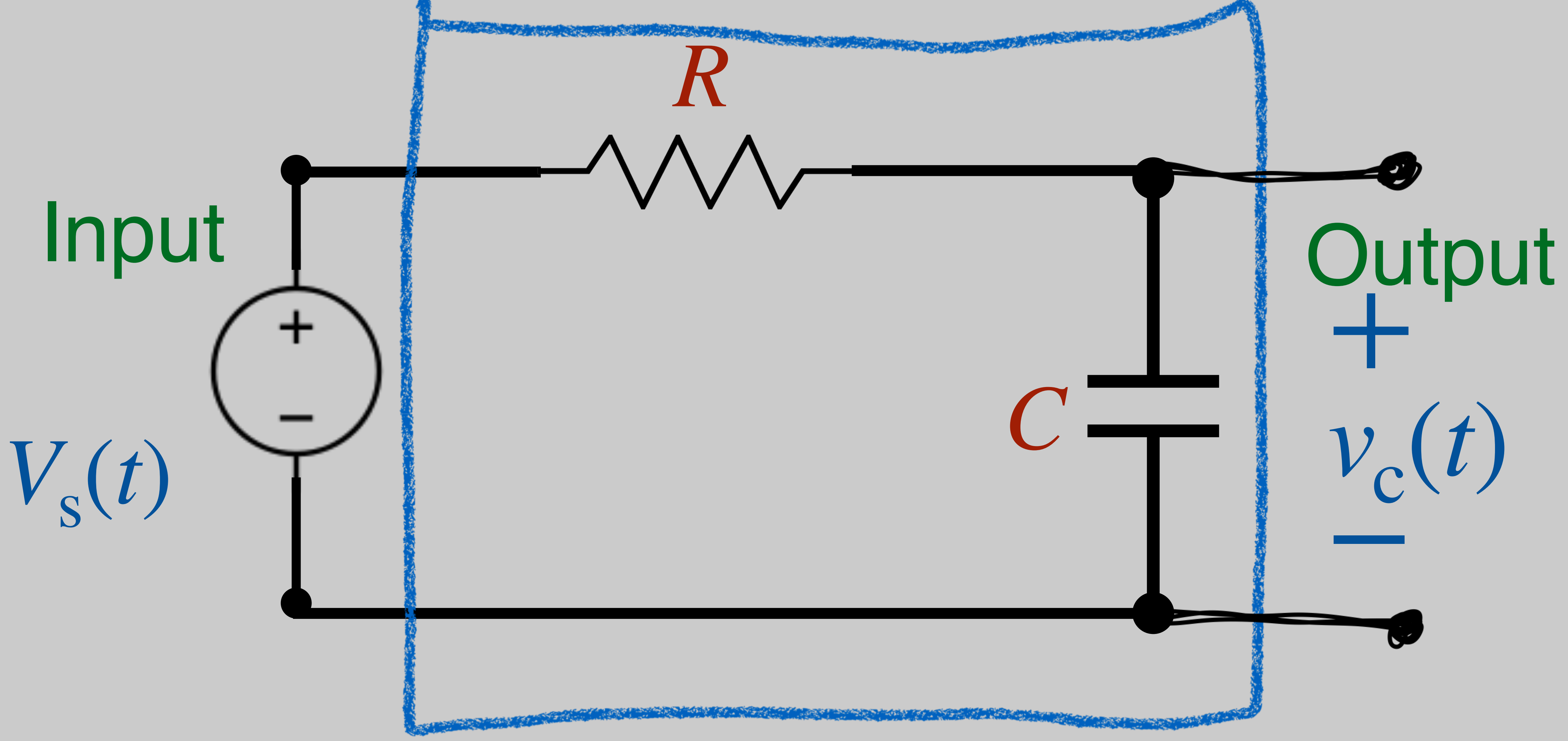
Announcements

- Last time:
  - Transient behavior of RC circuits
  - Solving 1st Order Diff. Eq.
    - Homogenious - exponential decay
    - Inhomogenious - asymptotic exponential recovery
    - Step response
    - Pulse response
- Today:
  - Inductance, inductors, transformers
  - Solving diff. EQ for arbitrary inputs

Natural Response of RC circuits

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_s(t)$$

$$\Rightarrow \frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = 0$$

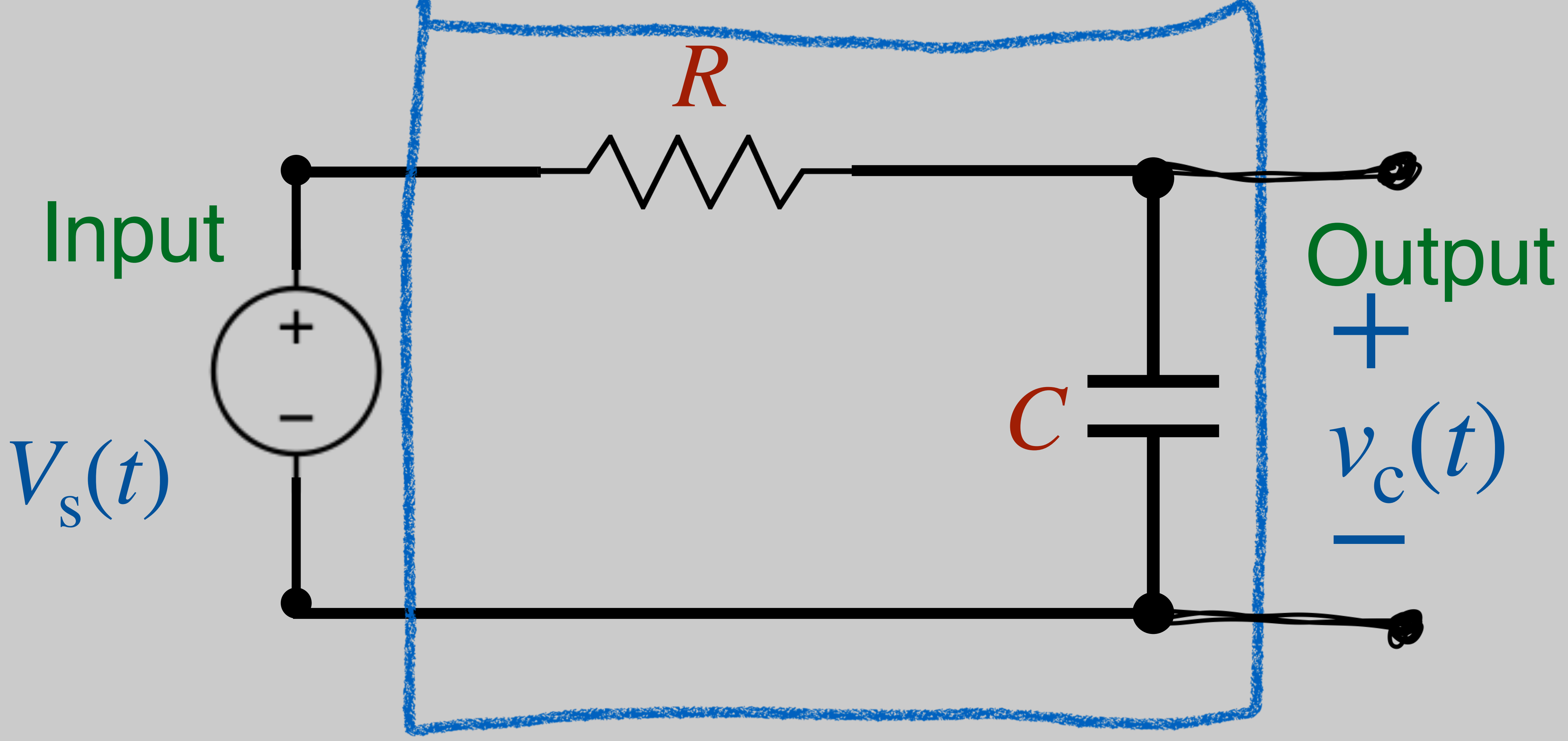


Solution: $v_c(t) = V_0 e^{-\frac{t}{RC}}$

\

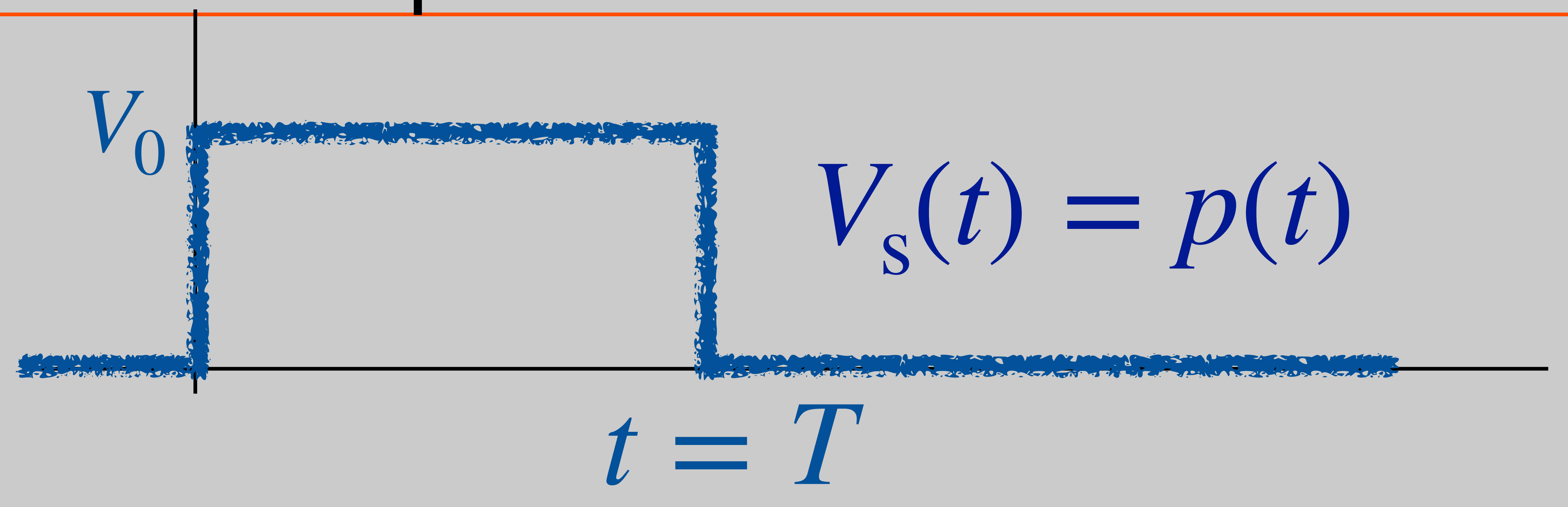
$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_s(t)$$

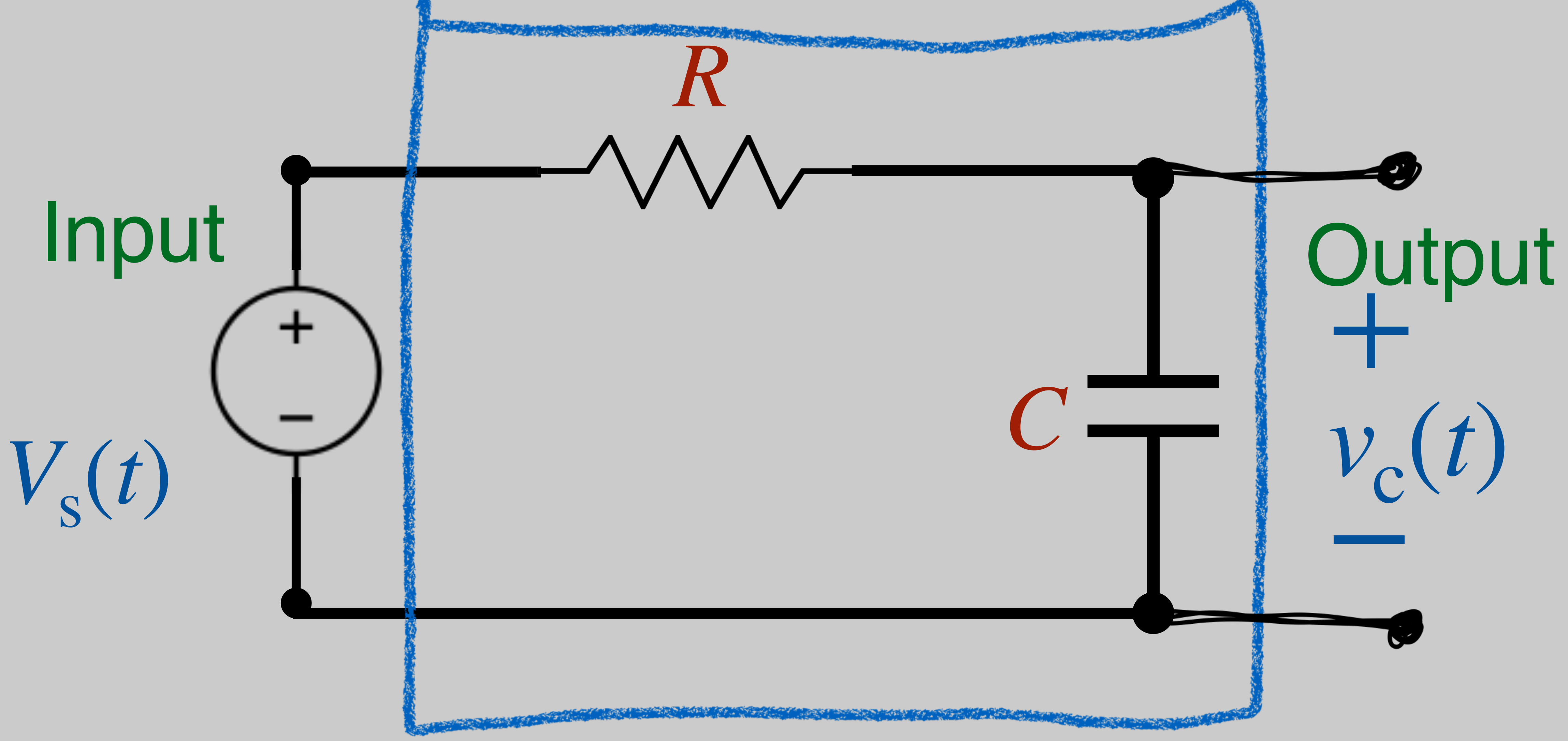
$$\Rightarrow \frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_0$$



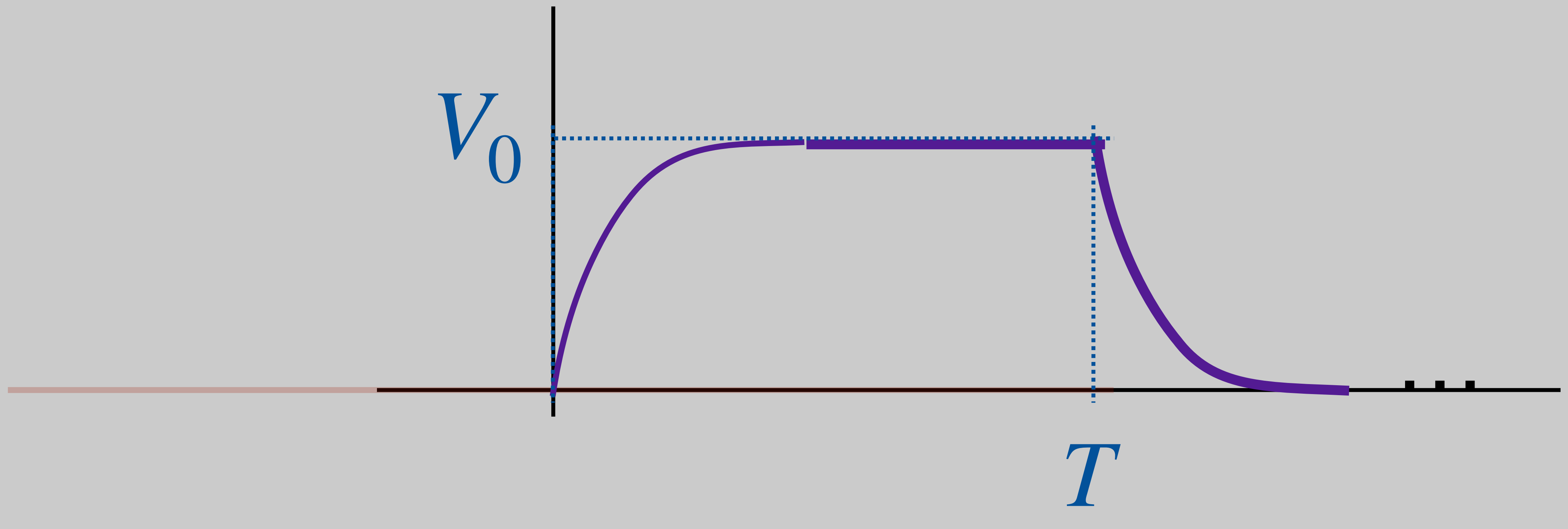
Solution: $v_c = V_0\left(1 - e^{-\frac{t-t_0}{RC}}\right)$

Pulse Response of RC circuits

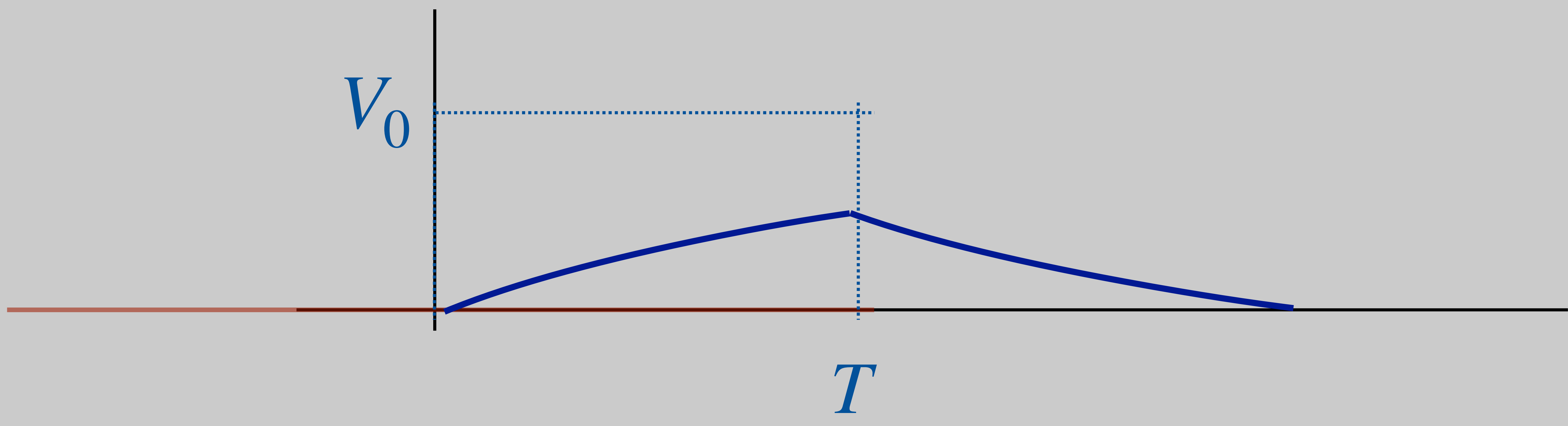




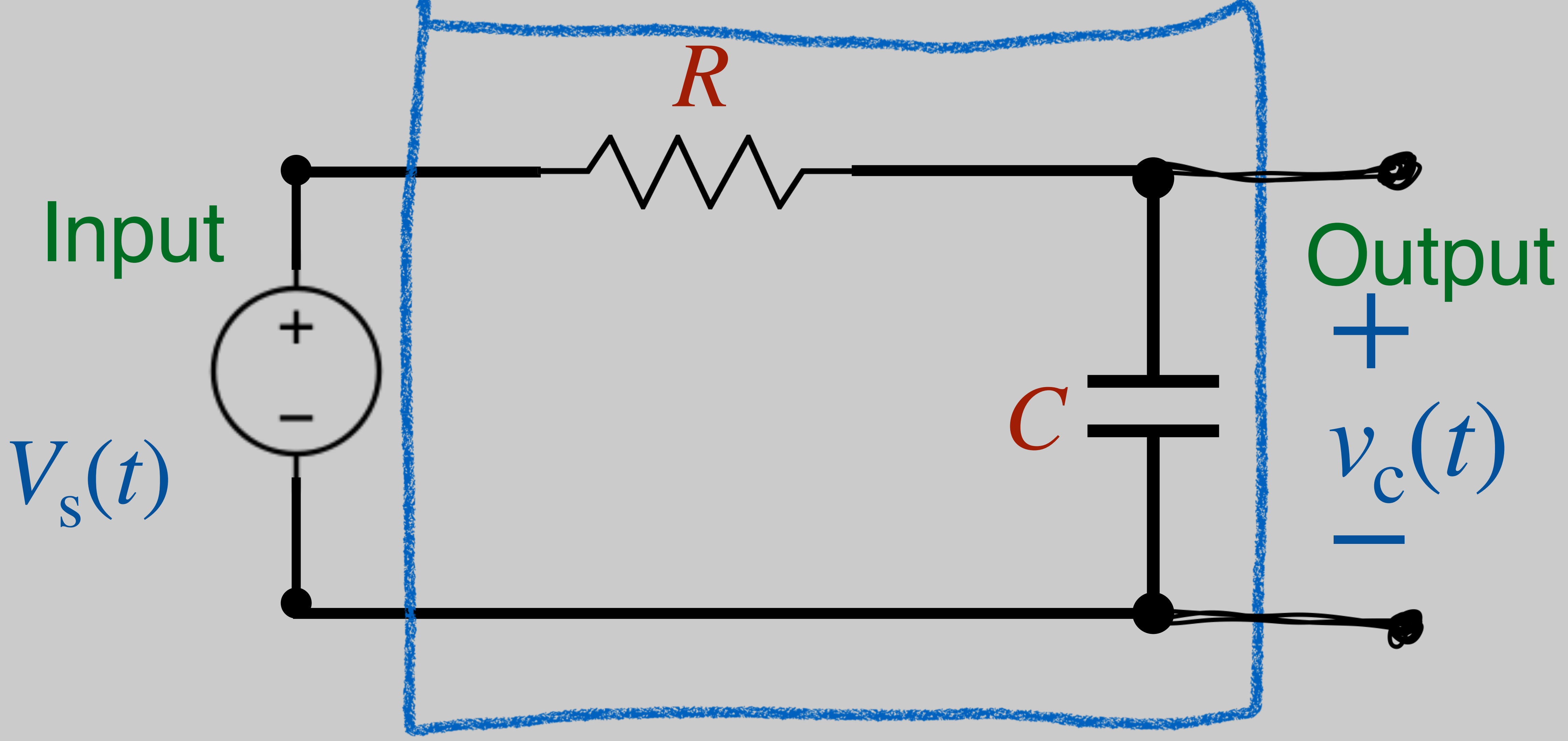
"Fast" Circuit $T \gg RC$



Slow Circuit $T \ll RC$



Pulse Response of RC circuits



① $V_{c1} = V_{c0} e^{-\frac{T}{RC}}$

② $V_{c0} = V_{c1} e^{-\frac{T}{RC}} + V_0\left(1 - e^{-\frac{T}{RC}}\right)$

Pulse Response of RC circuits

$$① \quad V_{c1} = V_{c0} e^{-\frac{T}{RC}}$$

$$② \quad V_{c0} = V_{c1} e^{-\frac{T}{RC}} + V_0\left(1 - e^{-\frac{T}{RC}}\right)$$

$$V_{c0} = V_{c0} e^{-\frac{2T}{RC}} + V_0\left(1 - e^{-\frac{T}{RC}}\right)$$

$$V_{c0} = V_0 \frac{\left(1 - e^{-\frac{T}{RC}}\right)}{\left(1 - e^{-\frac{2T}{RC}}\right)}$$

Pulse Response of RC circuits

① $V_{c1} = V_{c0} e^{-\frac{T}{RC}}$

② $V_{c0} = V_{c1} e^{-\frac{T}{RC}} + V_0\left(1 - e^{-\frac{T}{RC}}\right)$

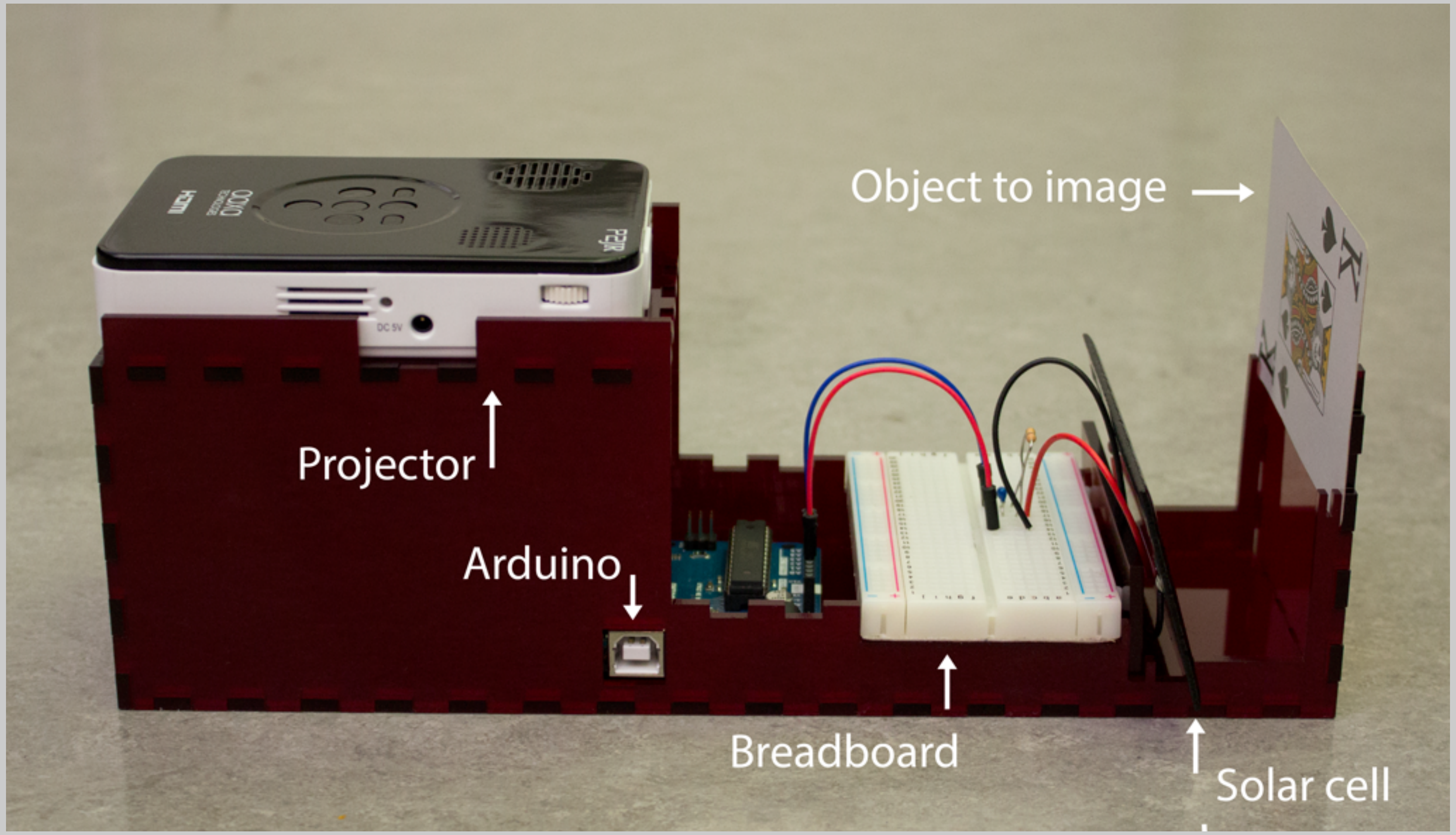
$V_{c0} = V_{c0} e^{-\frac{2T}{RC}} + V_0\left(1 - e^{-\frac{T}{RC}}\right)$

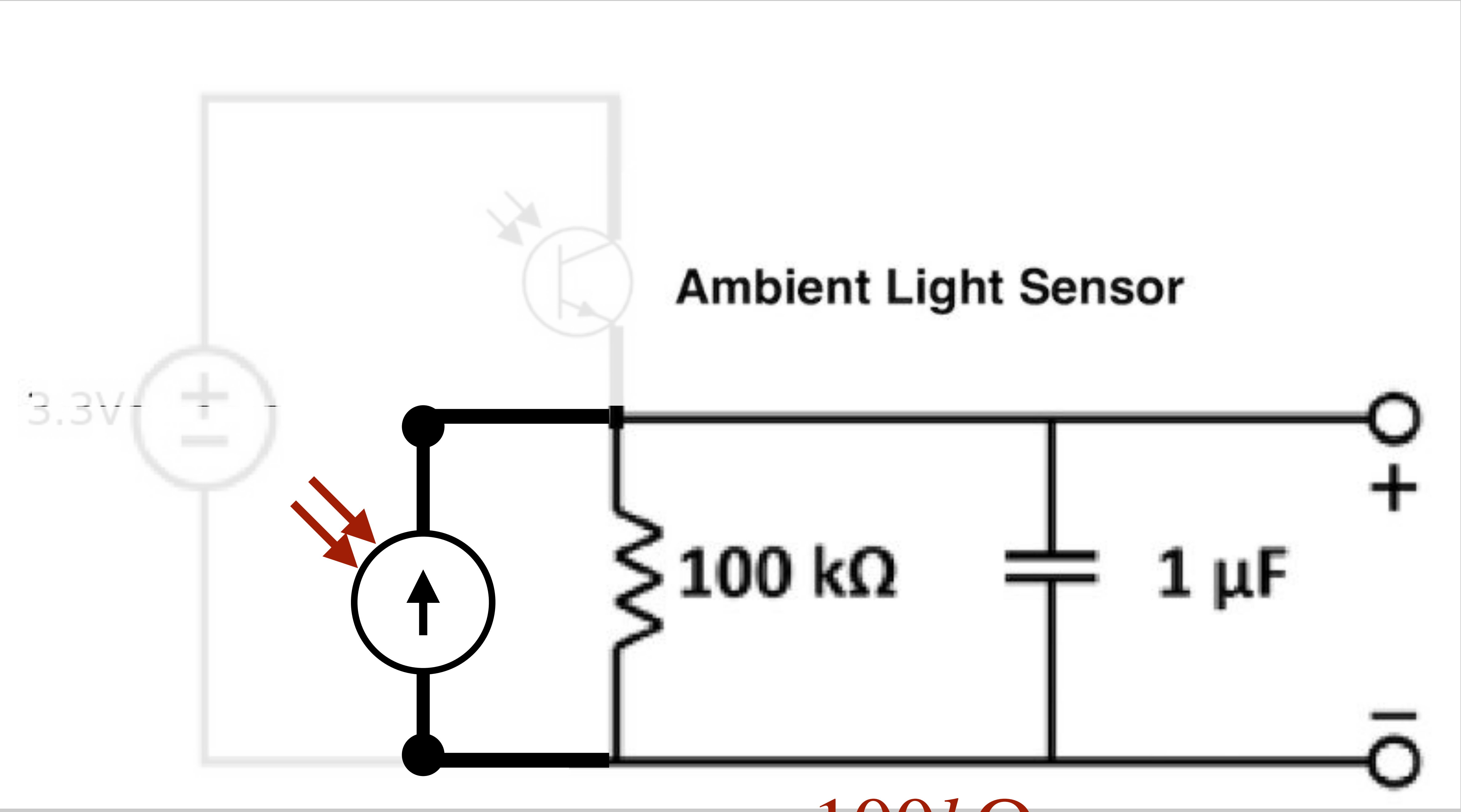
$$V_{c0} = V_0 \frac{\left(1 - e^{-\frac{T}{RC}}\right)}{\left(1 - e^{-\frac{2T}{RC}}\right)}$$

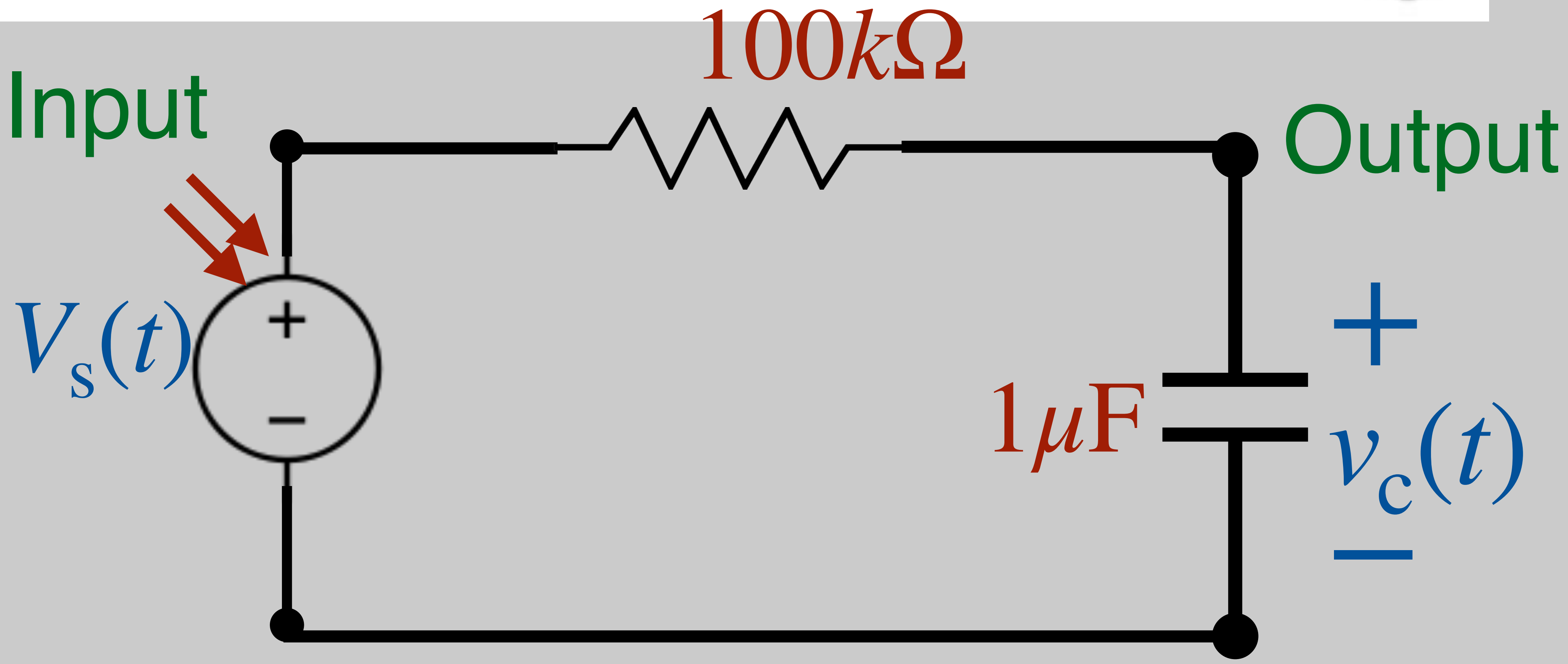
$\lim_{\frac{T}{RC} \to 0} V_{c0} = \frac{1}{2} V_0$

$\Delta = V_{c0} - V_{c1} = V_{c0}\left(1 - e^{-\frac{T}{RC}}\right)$

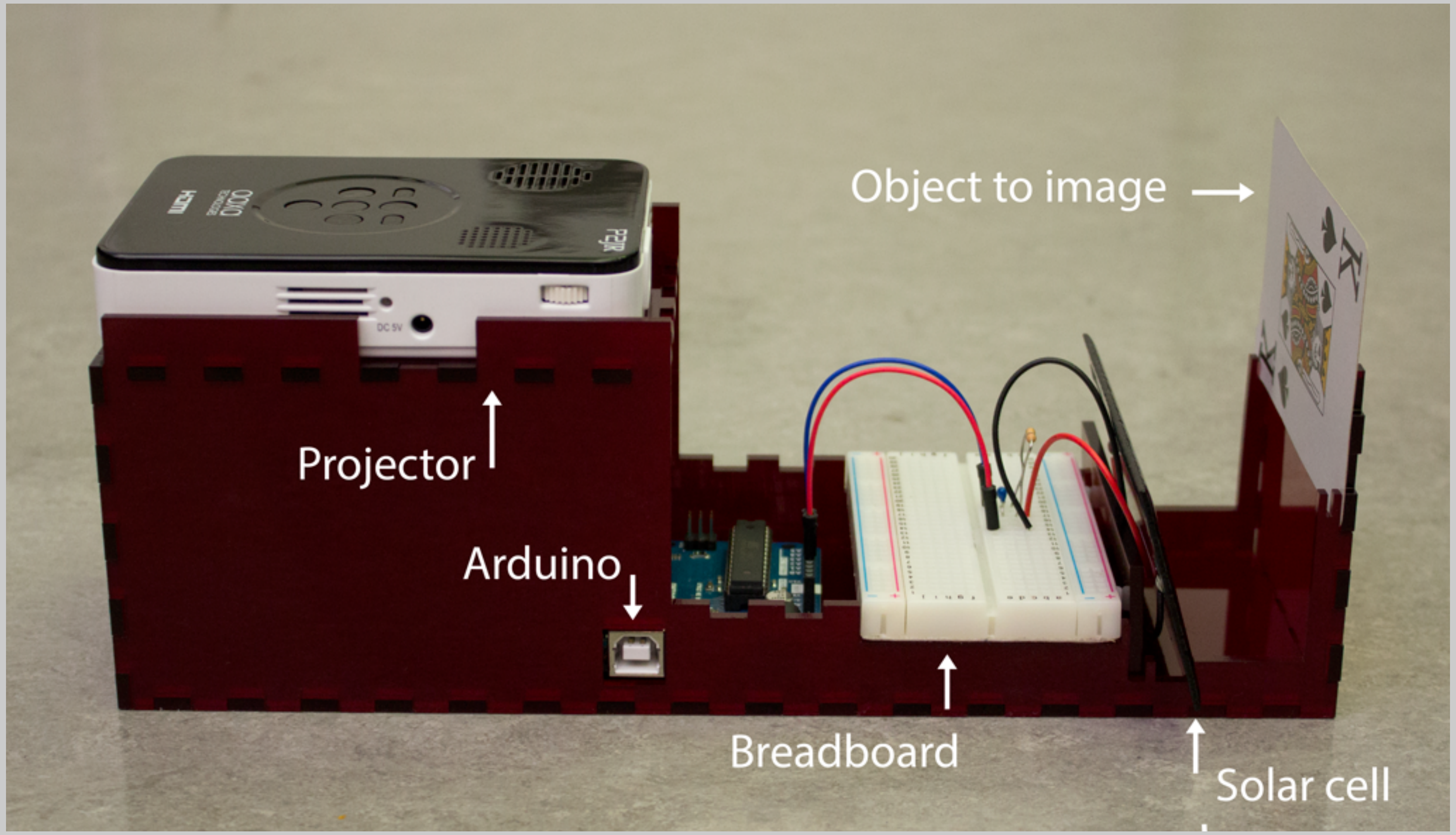
EECS16A Lab1

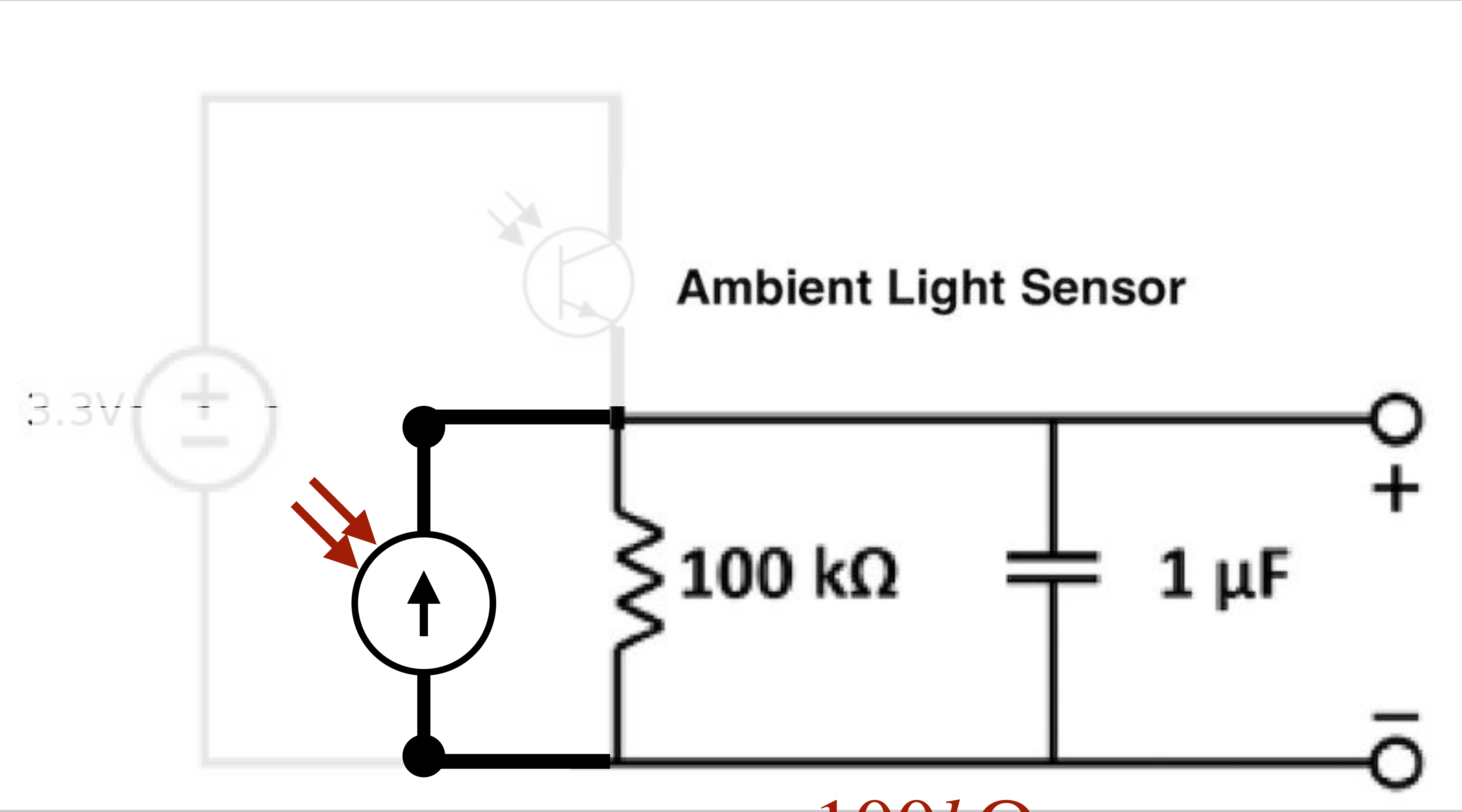


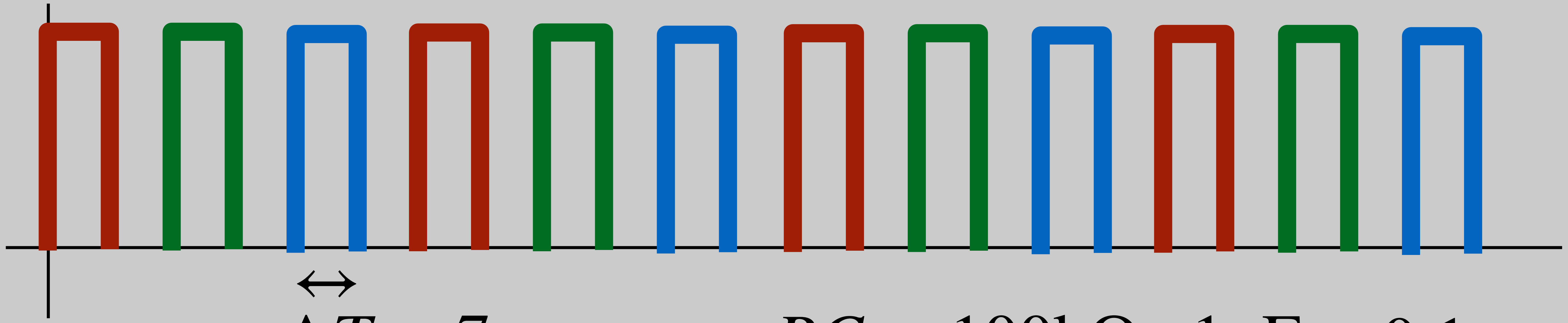




EECS16A Lab1

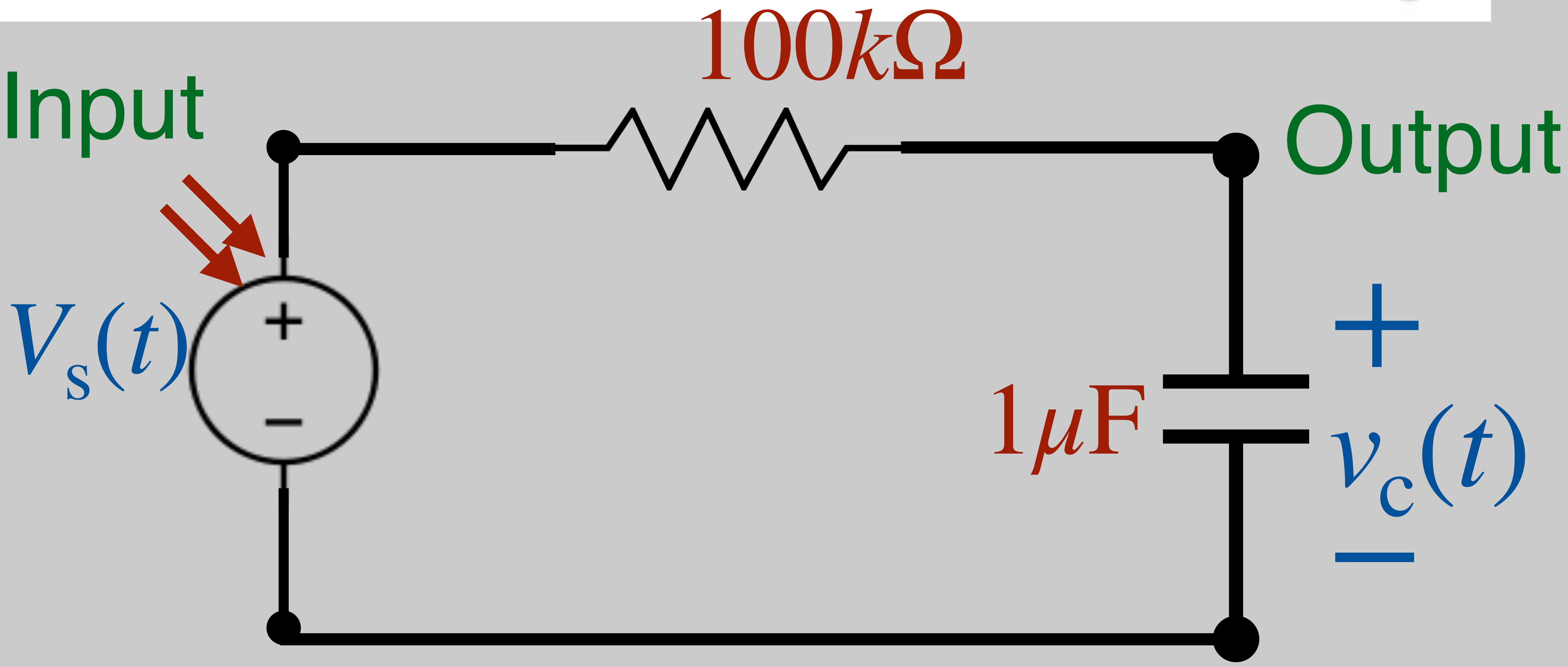




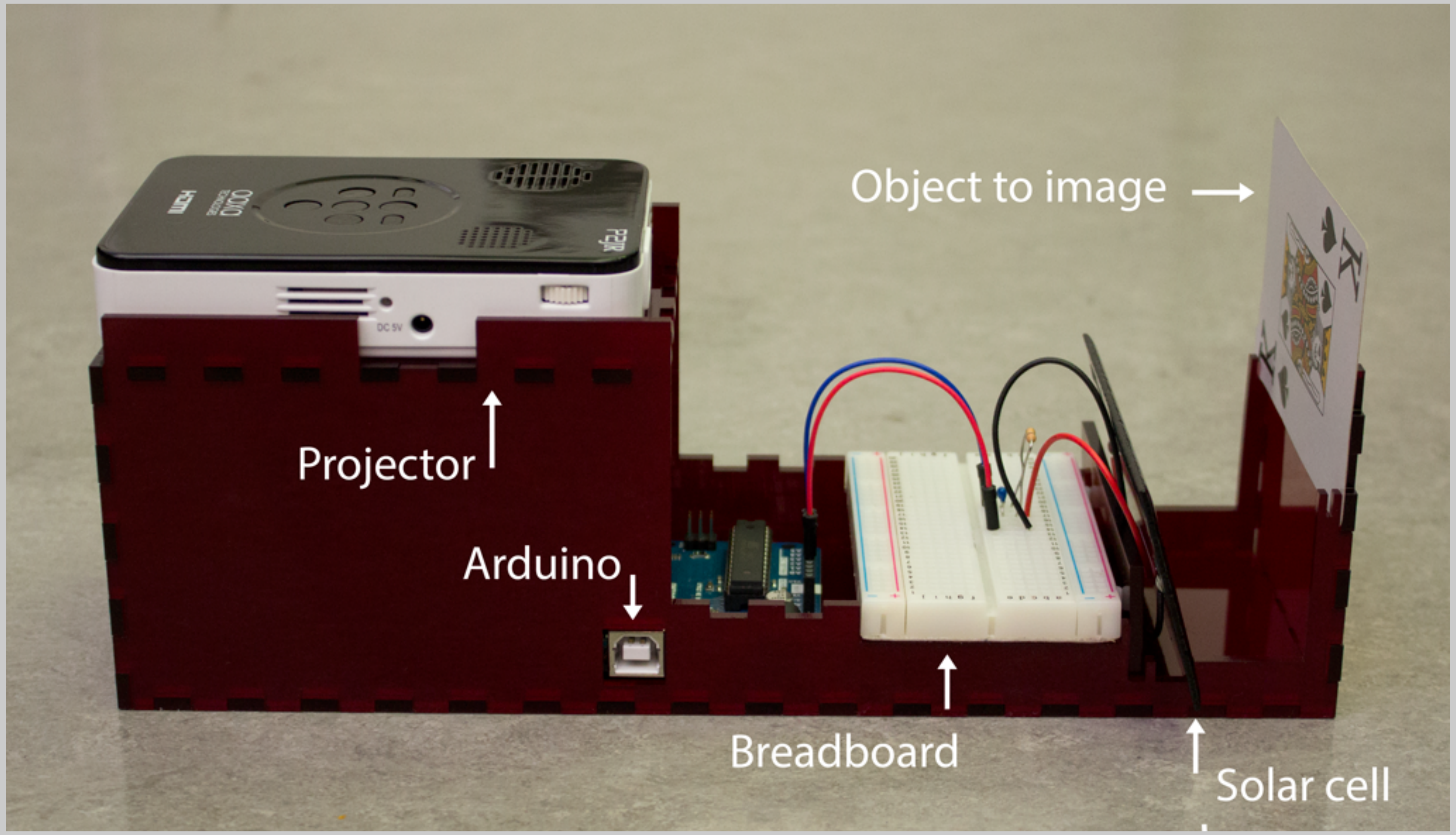


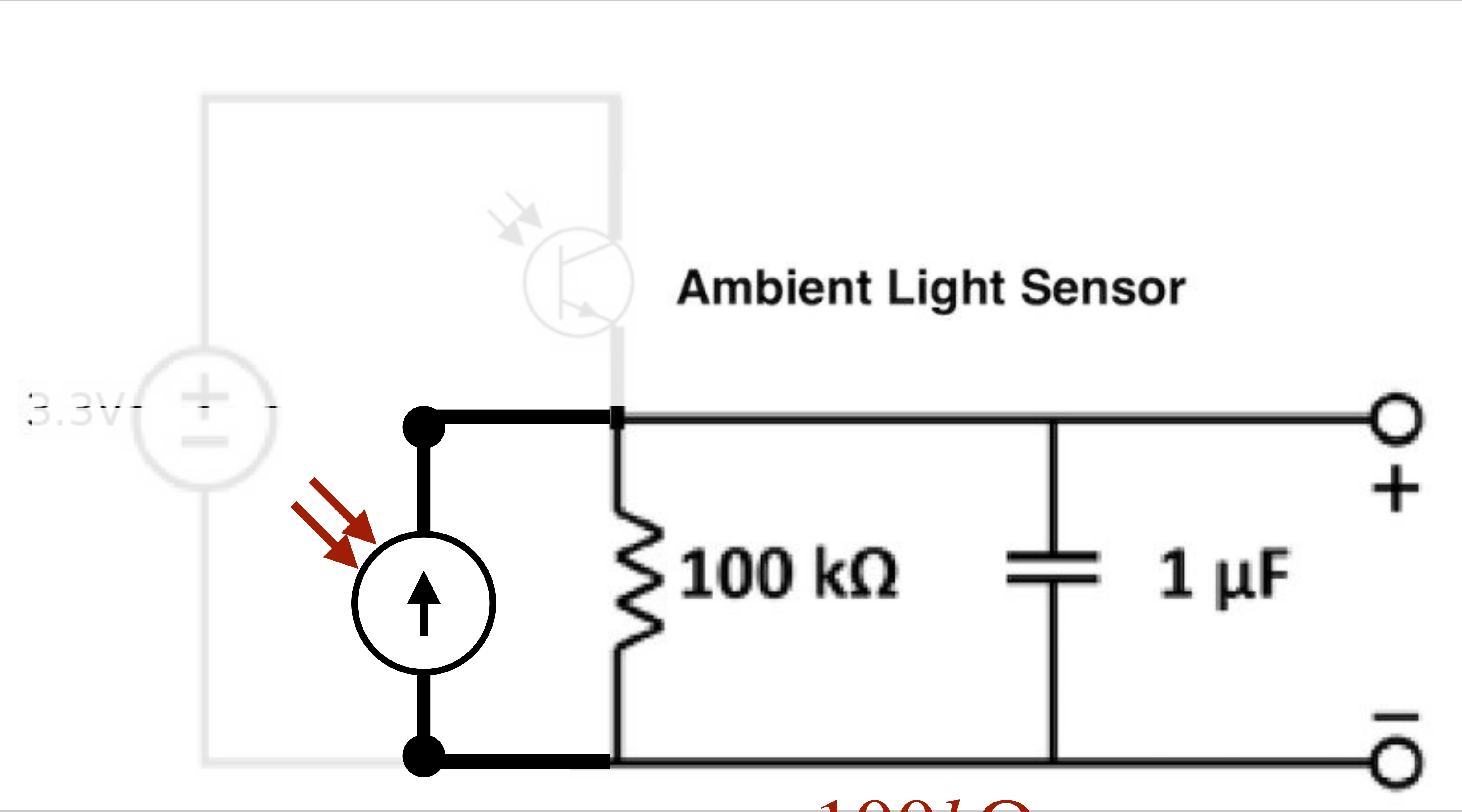
$\Delta T = 7\text{ms}$

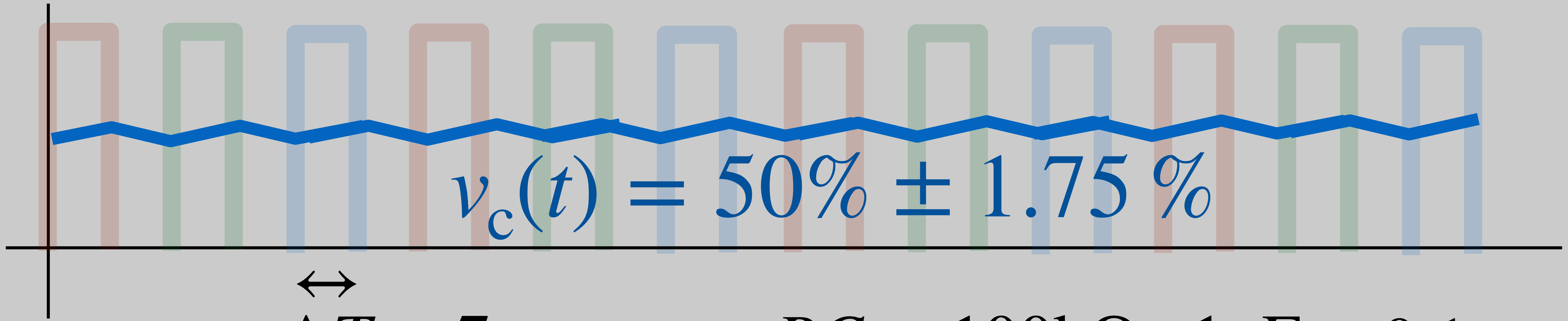
$RC = 100\text{k}\Omega \cdot 1\mu\text{F} = 0.1\text{s}$

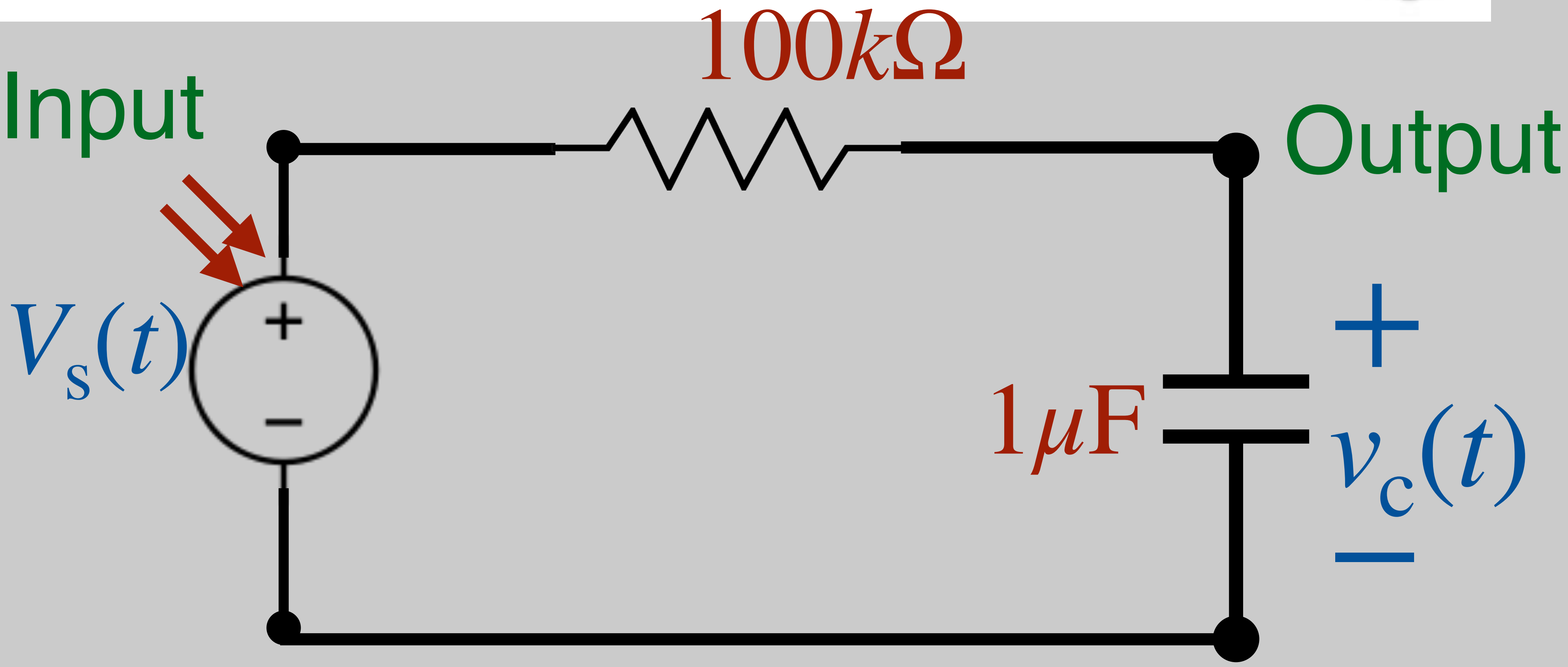


EECS16A Lab1









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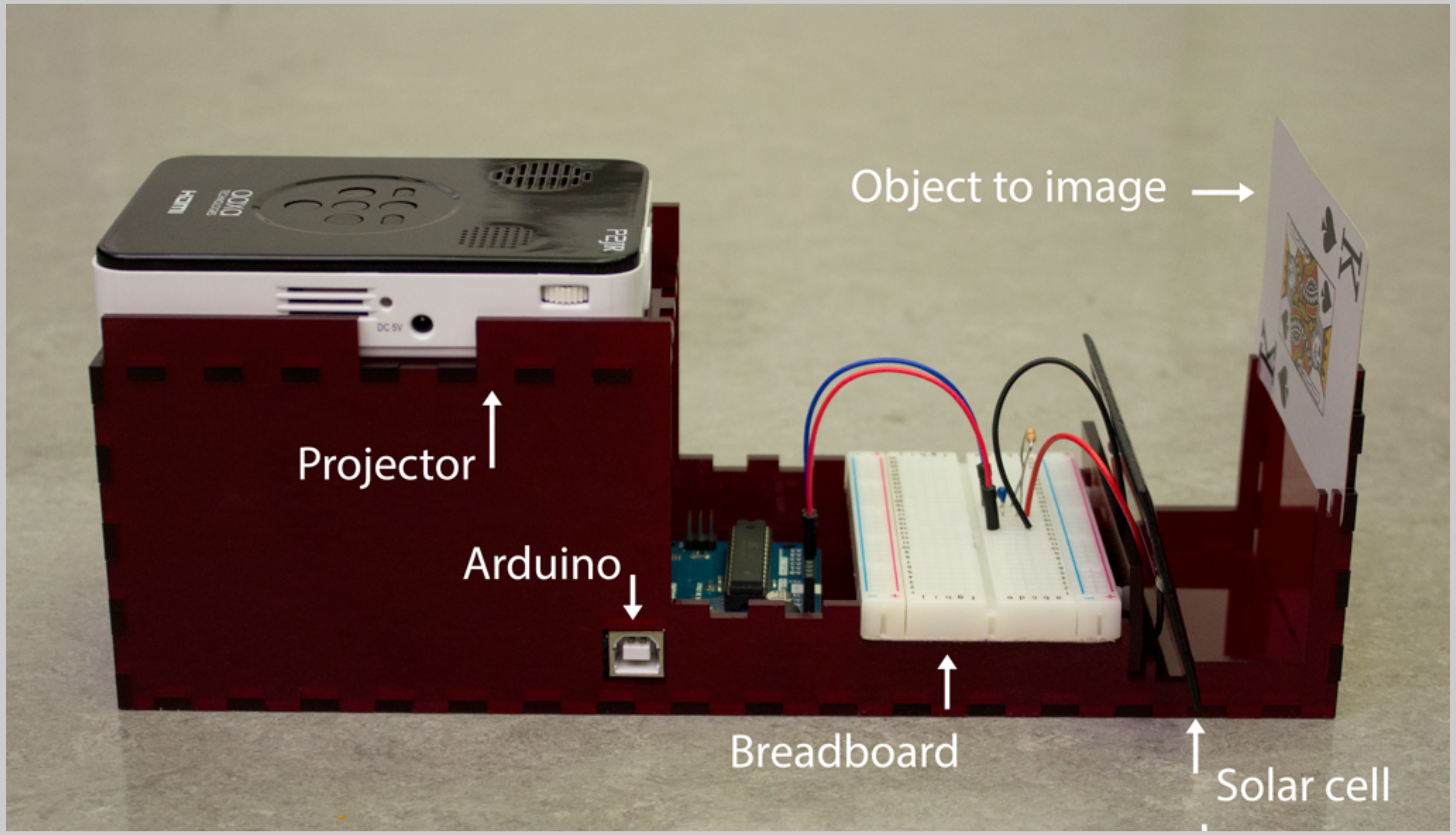
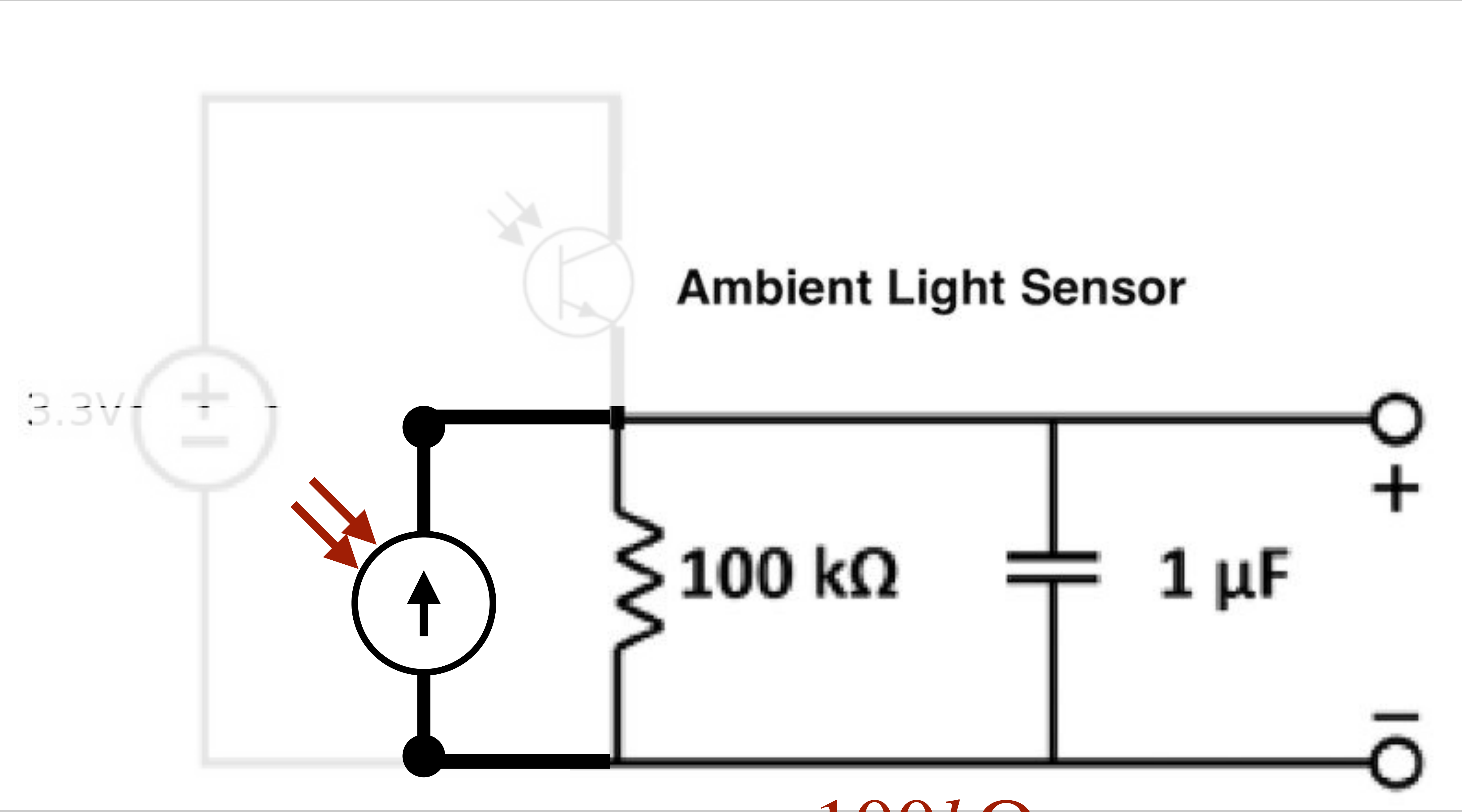
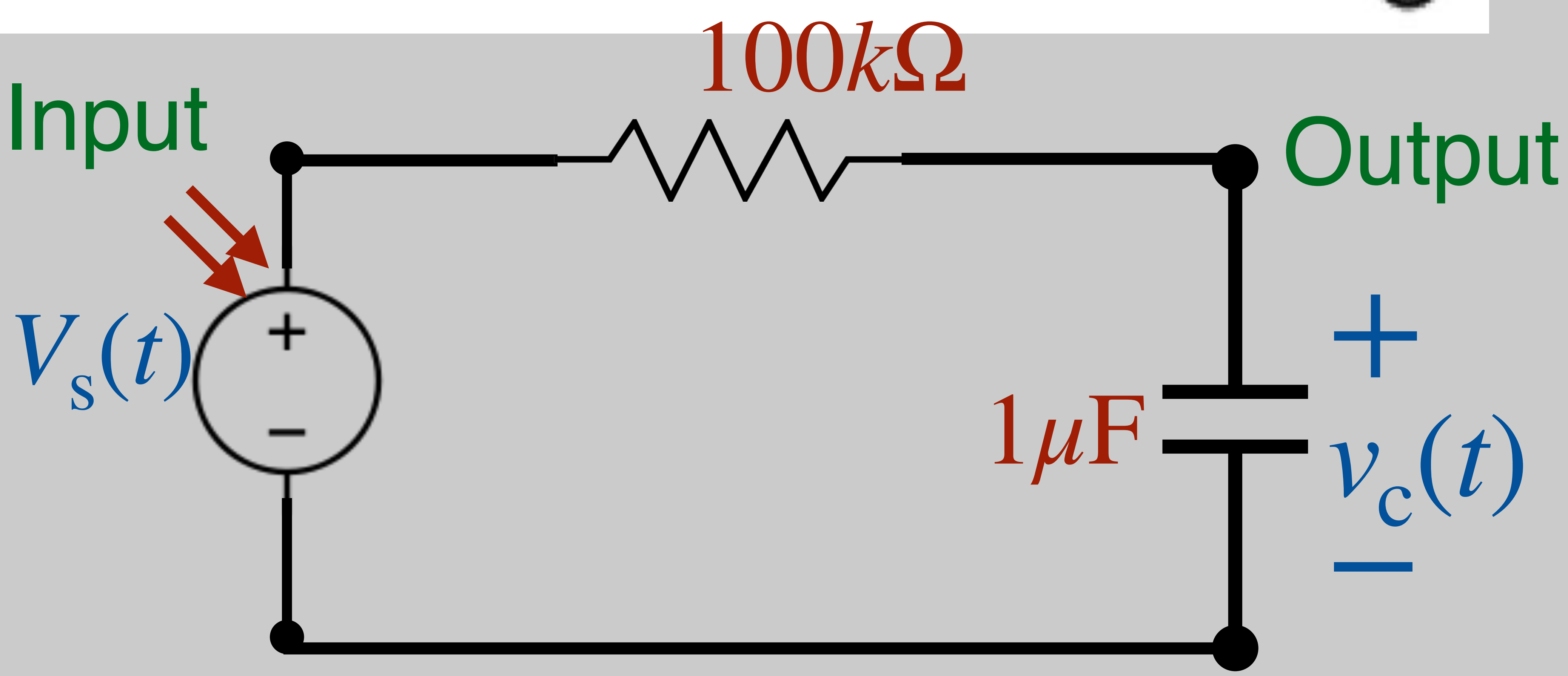
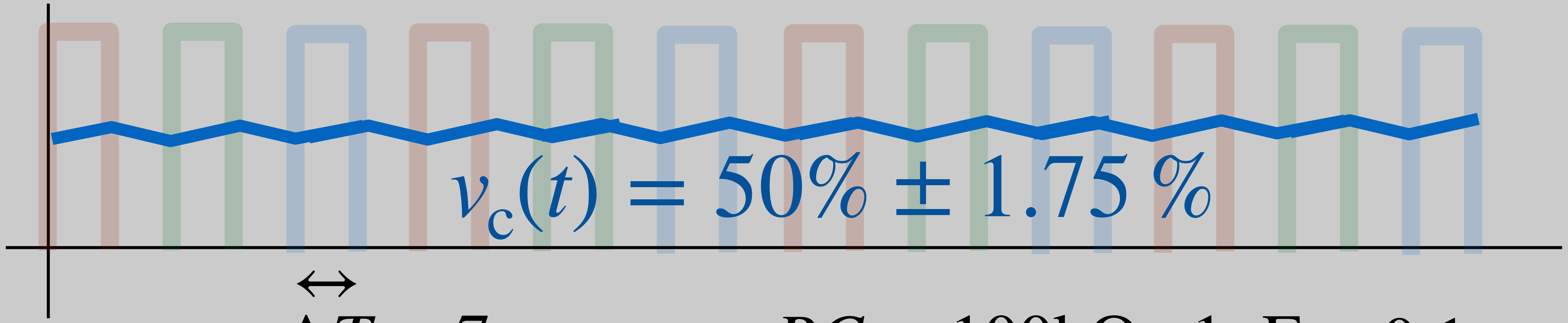
EECS16A Lab1

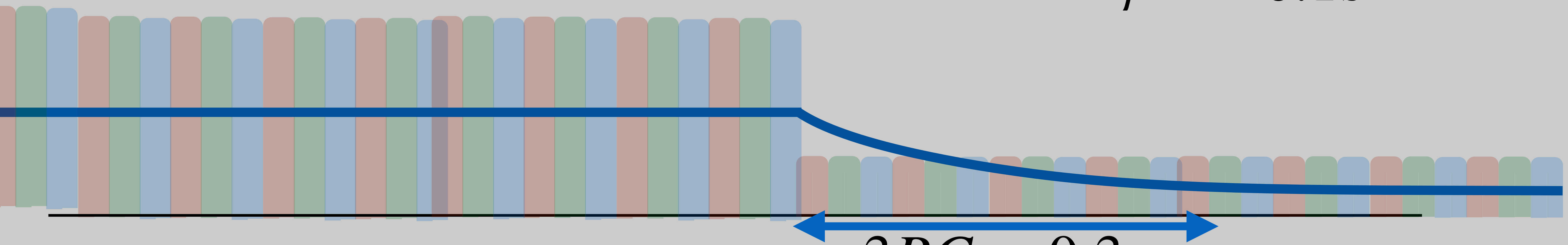








$\Delta T = 7\text{ms}$ $\qquad RC = 100\text{k}\Omega \cdot 1\mu\text{F} = 0.1\text{s}$



$3RC = 0.3\text{s}$

Inductance

- Capacity to store energy in a magnetic field
- Units: Henry [H]
- Components nH (10^{-9}), uH (10^{-6}), mH (10^{-3})
- Magnets, power transformers > 1H

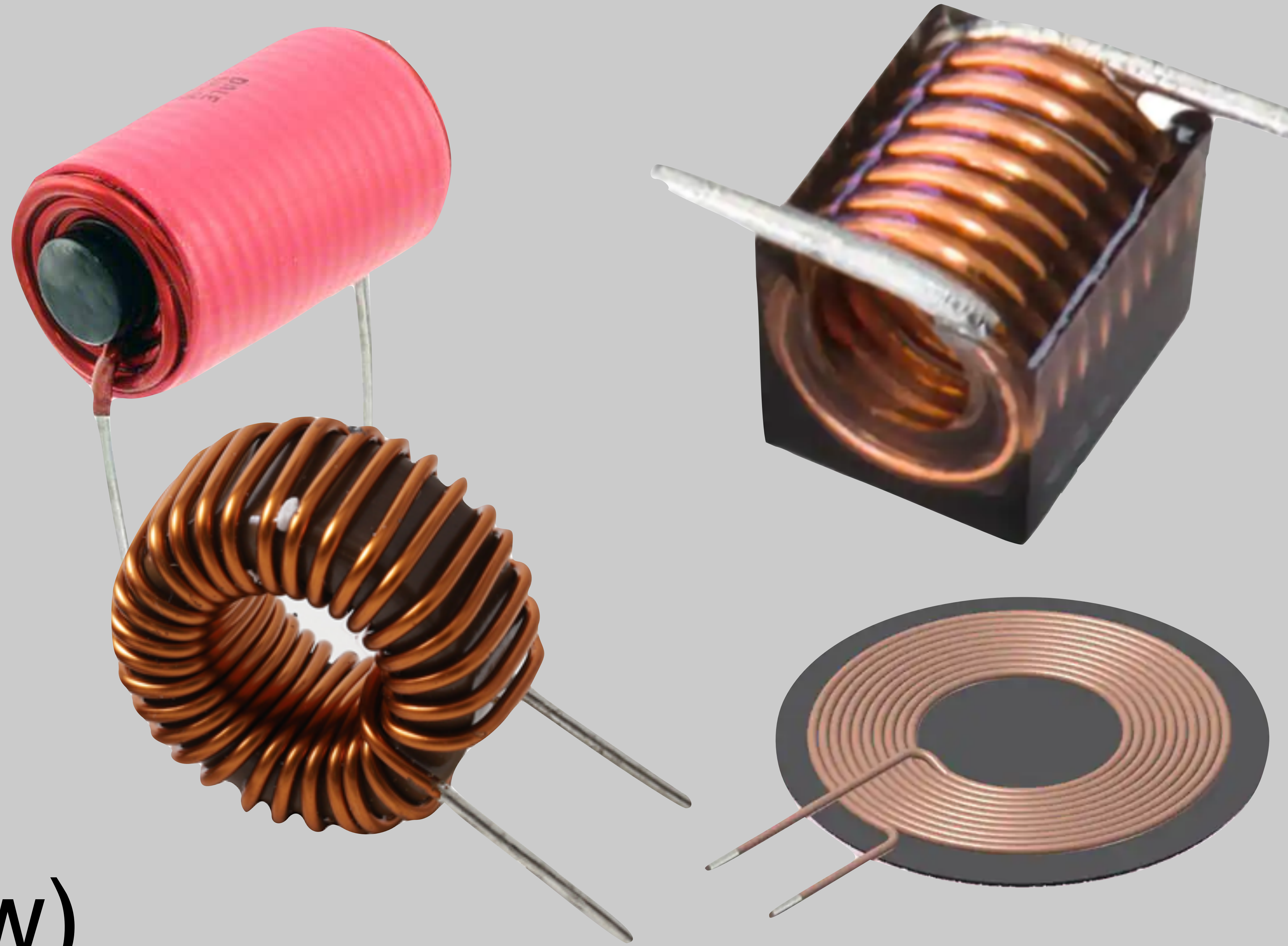
Inductors

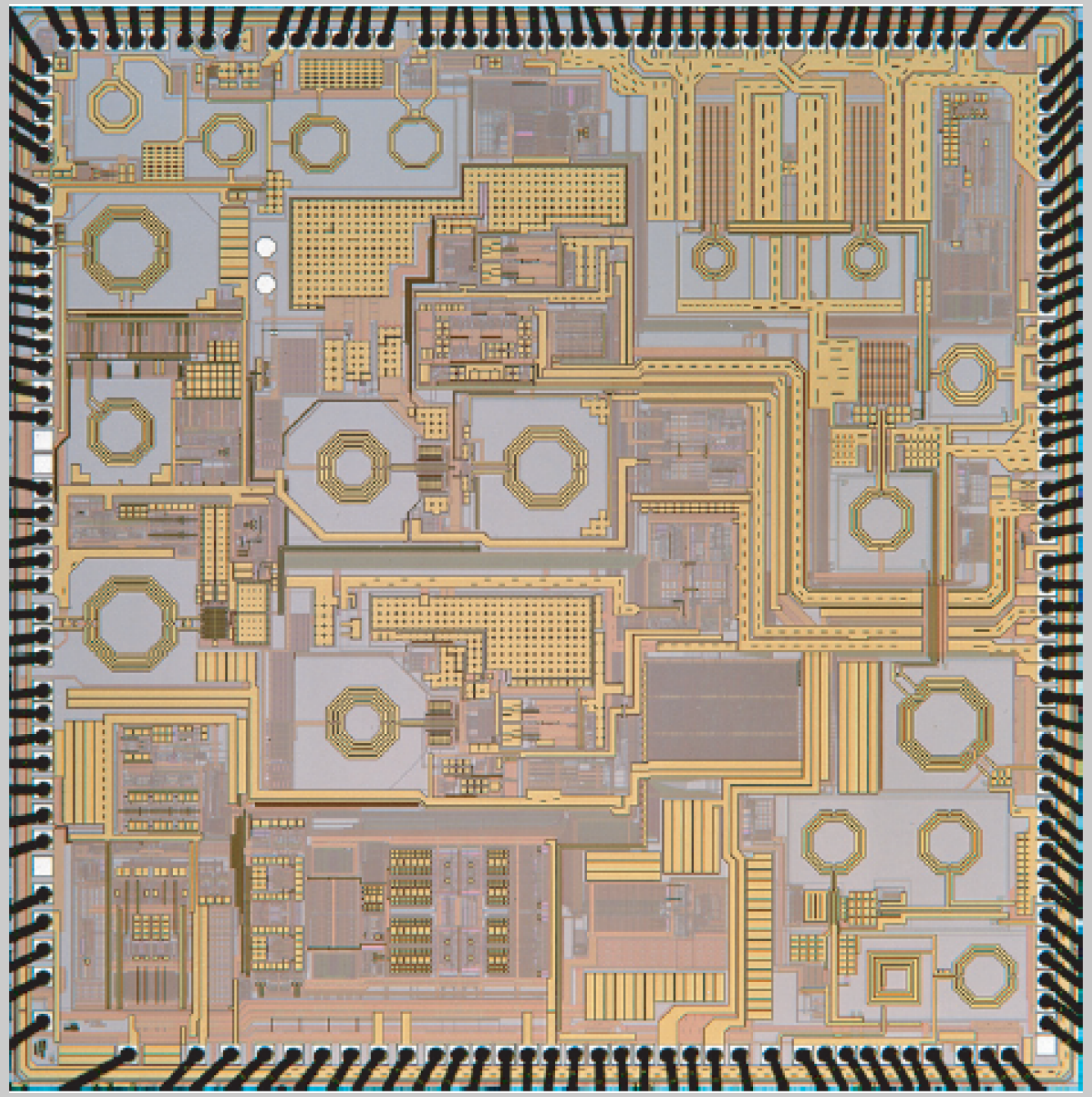
- Many designs and uses
- Current produces a magnetic flux:

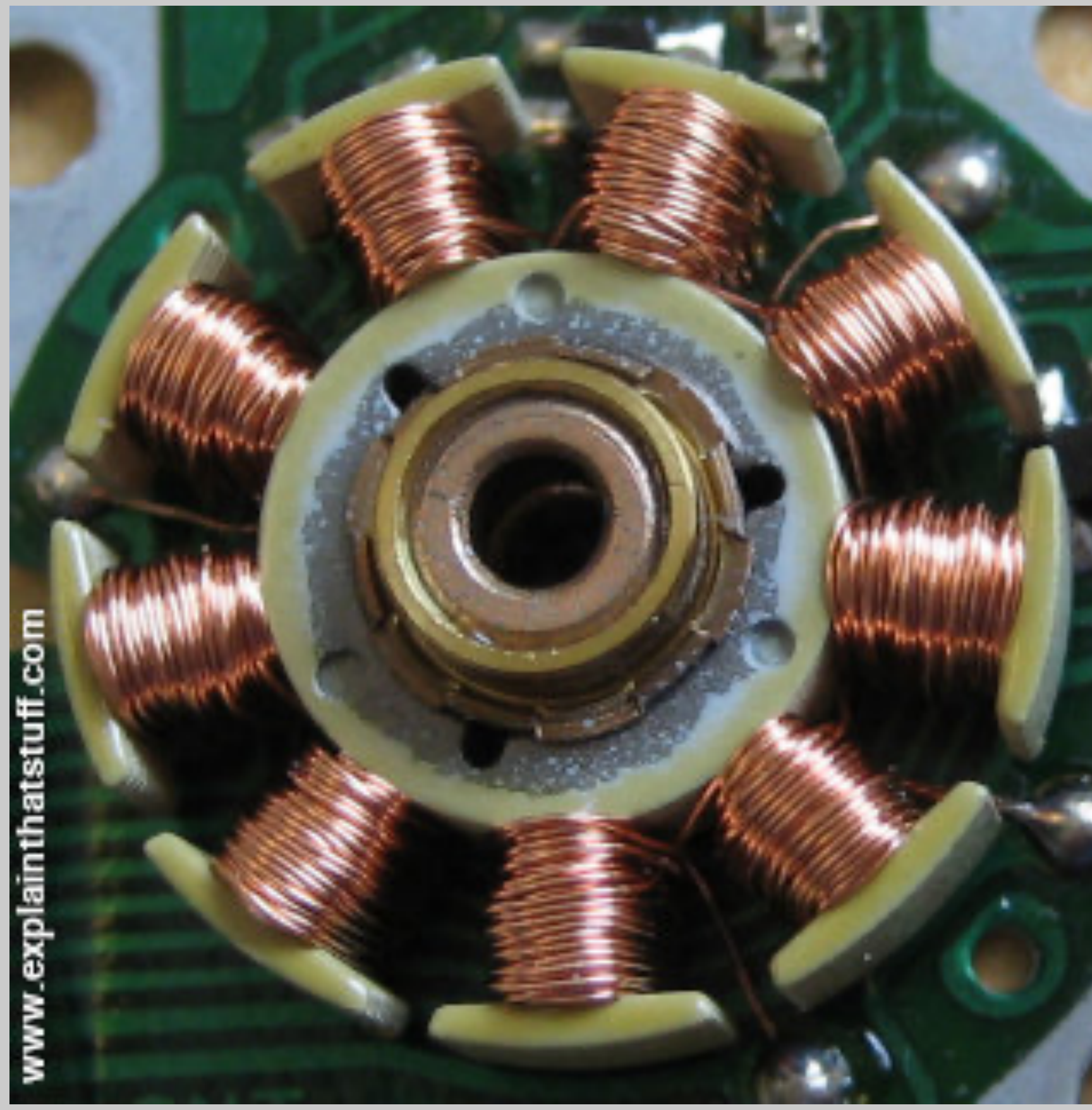
$$\Phi_B = Li$$

- Change in flux induces voltage

$$v = \frac{d\Phi_B}{dt} \text{ (Faraday's Law)}$$





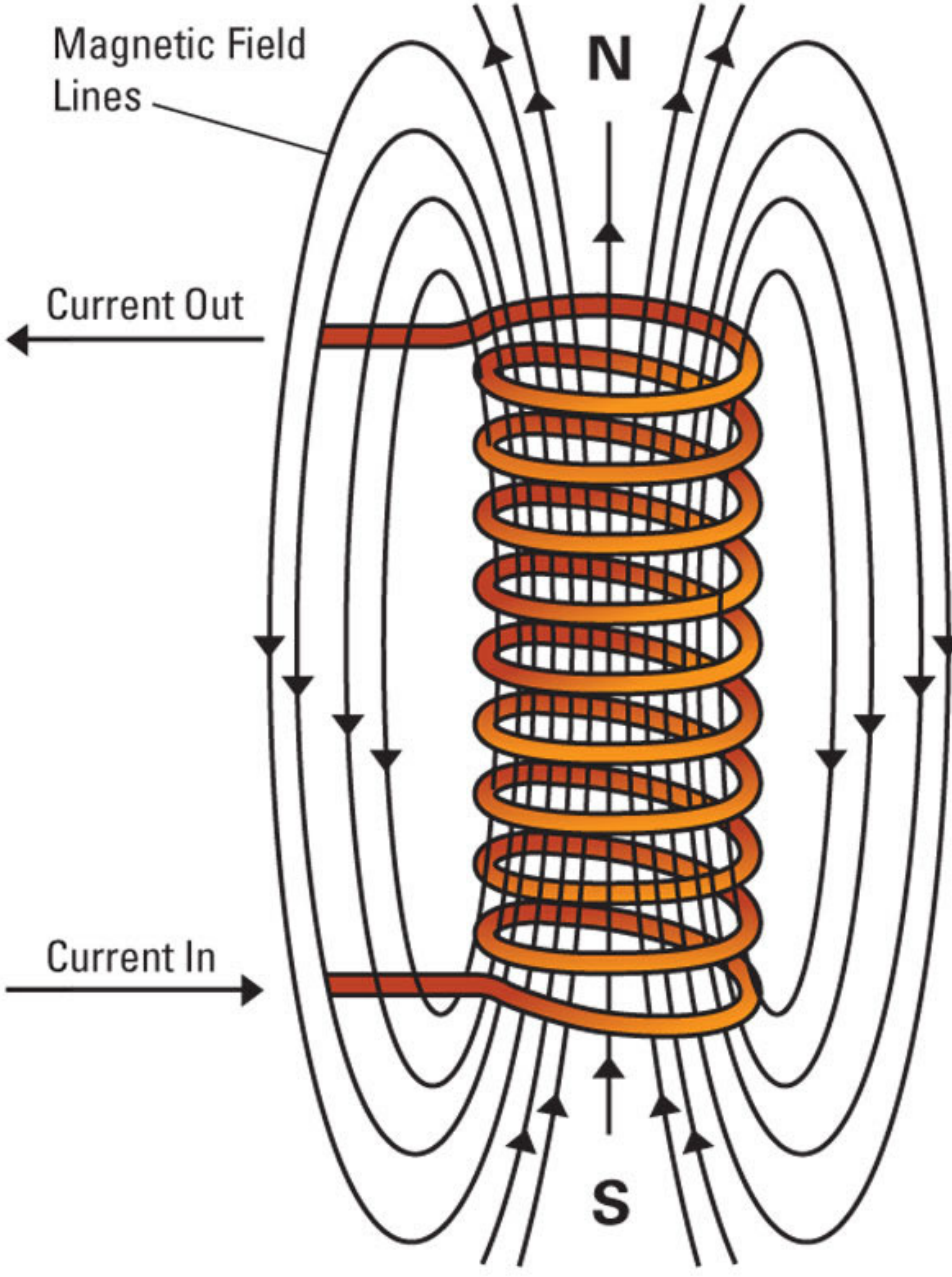


- For N-turn solenoid, area A and length l

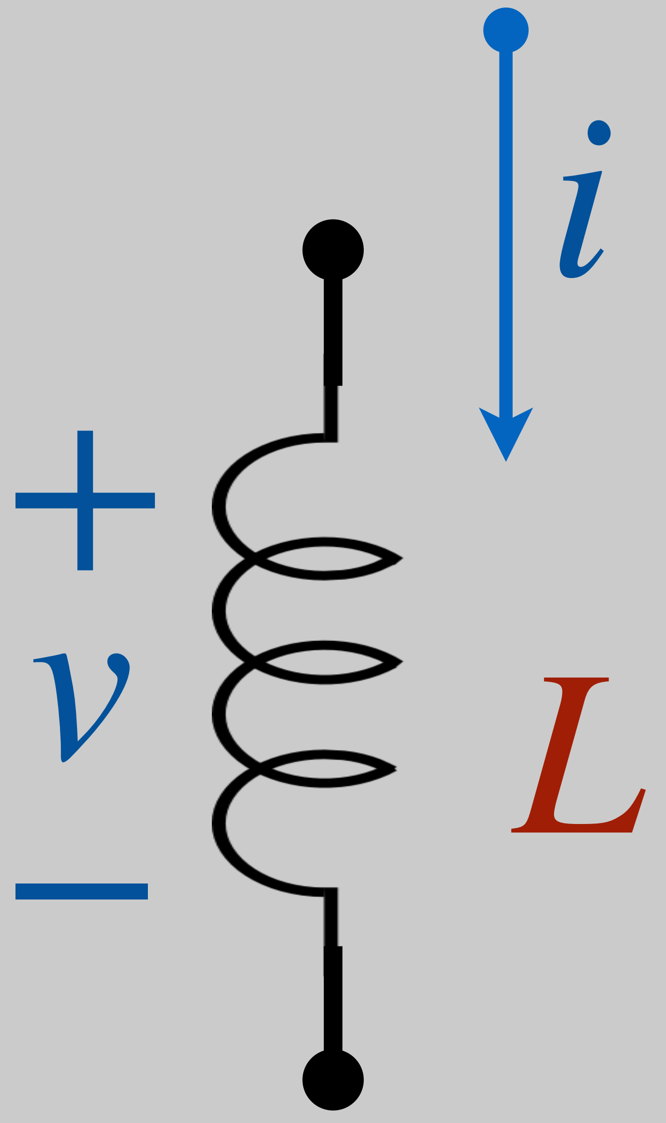
$$L = \frac{\mu N^2 A}{l}$$

magnetic permeability: $\mu = \mu_0 \mu_r$

$$\mu_0 = 4\pi \times 10^{-7}$$
$$\mu_r \approx 1 \quad \text{air}$$
$$\mu_r \approx 350 - 20000 \quad \text{Ferrite}$$




SIEMENS
MAGNETOM Skyra



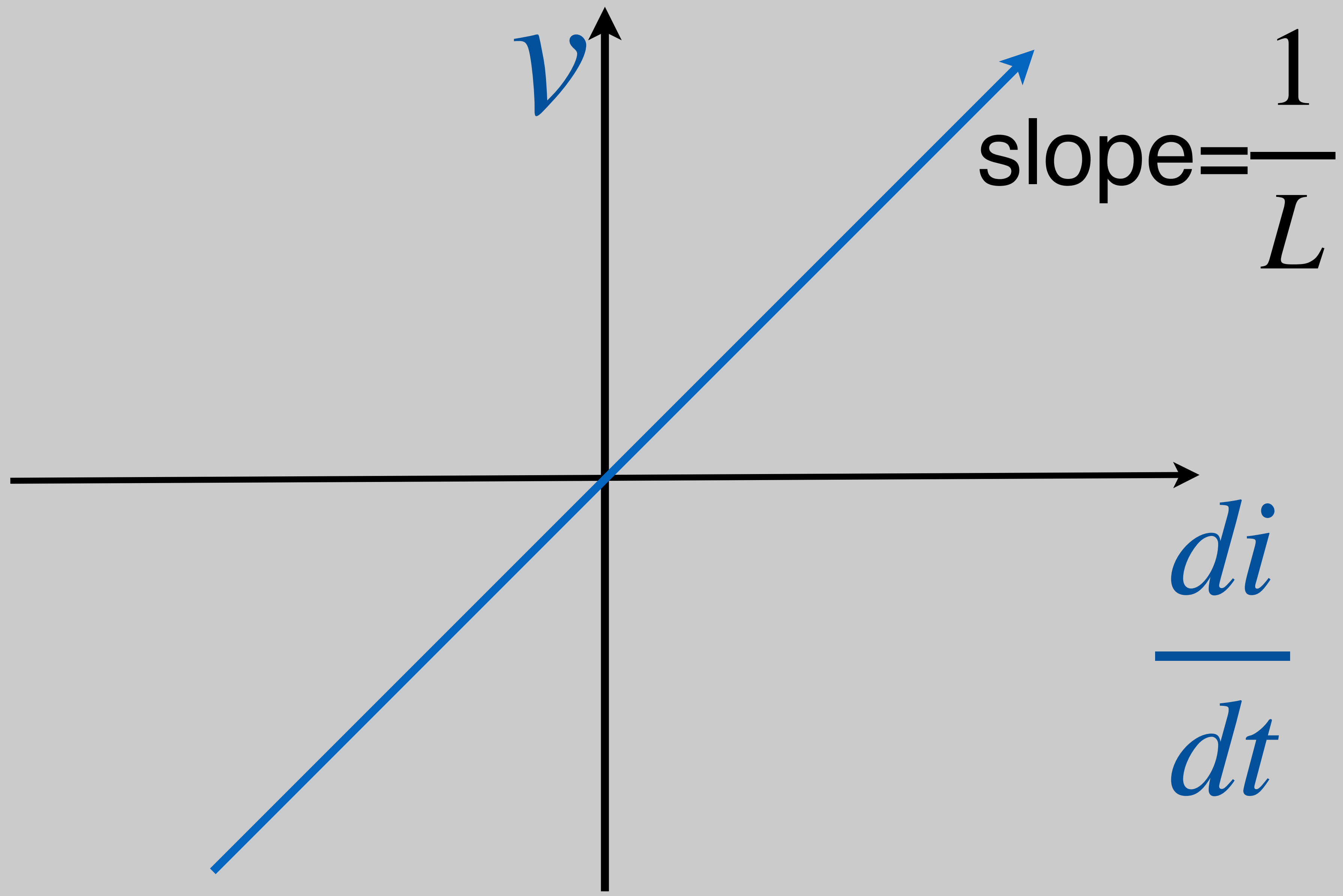
$$\Phi_B = L \cdot i$$

Time varying current:

$$L\frac{di}{dt} = \frac{d\Phi_B}{dt} = v(t)$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)(\tau)d\tau + i(t_0)$$

Memory!!!



- Inductors store energy in the magnetic field
- Power $p(t)$:

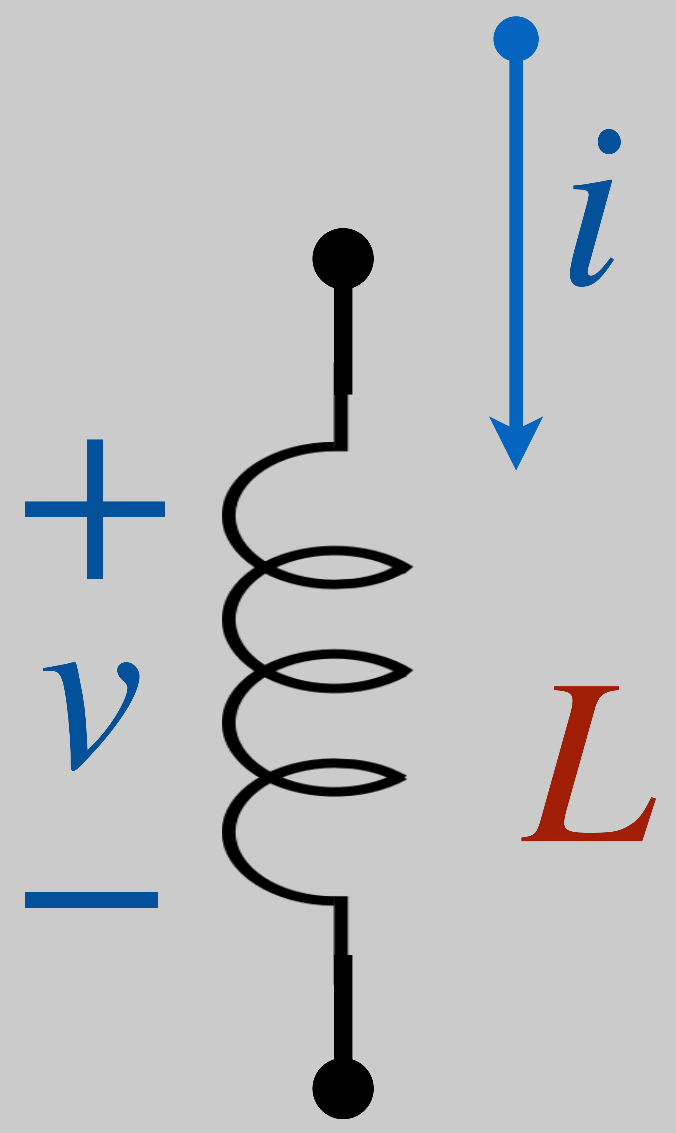
$$p(t) = i(t) \cdot v(t)$$

$$= Li(t)\frac{di(t)}{dt}$$

$$p(t)dt = Li(t)di(t)$$

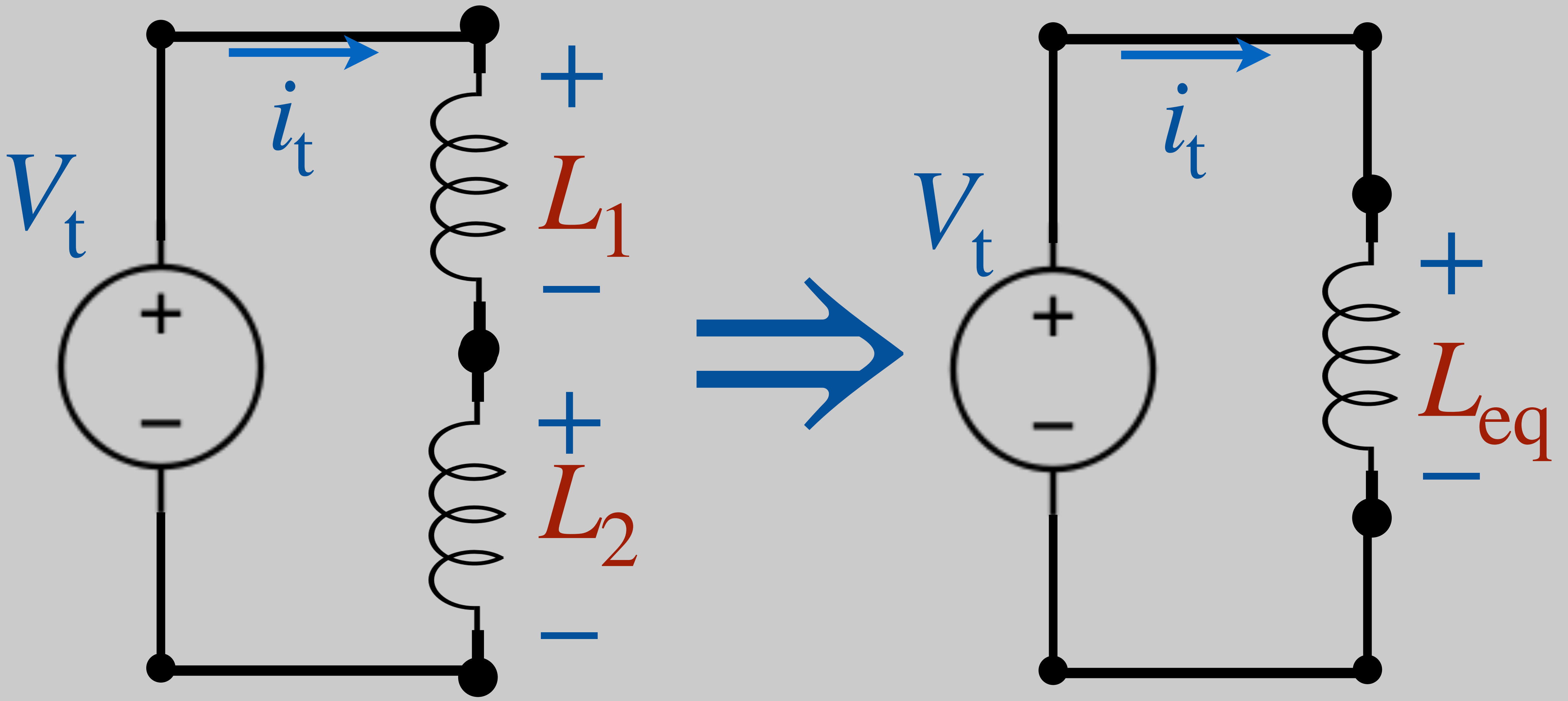
- Energy w is integration of power:

$$w = L\int_0^I idi \quad = L\frac{i^2}{2}$$



Equivalent Inductance

~~Parallel~~ Series Inductors:



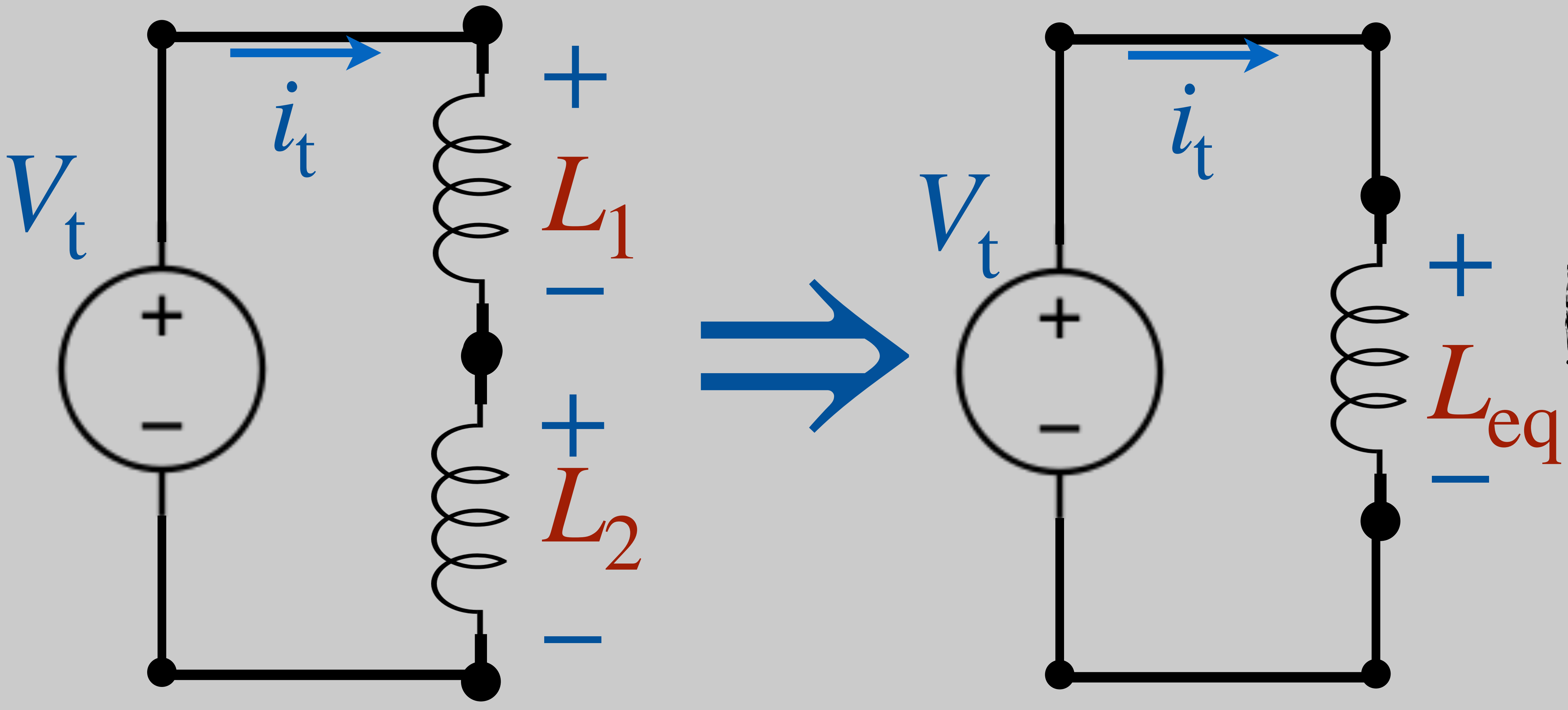
$$V_t = L_1\frac{di_t}{dt} + L_2\frac{di_t}{dt} = (L_1 + L_2)\frac{di_t}{dt}$$

$$L_{eq} = L_1 + L_2$$

~~Series~~ Parallel Inductors:

Equivalent Inductance

~~Parallel~~ Series Inductors:

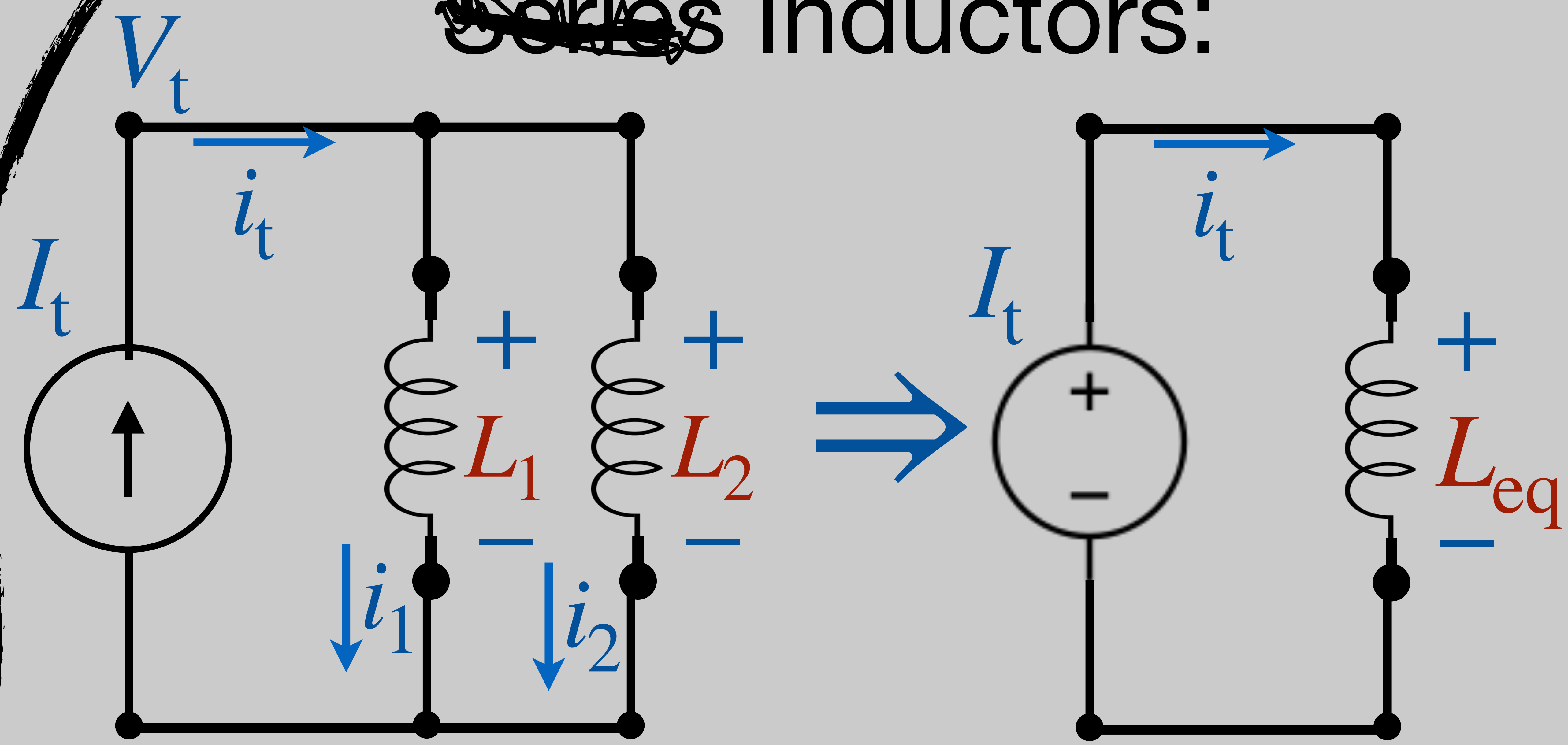


$$V_t = L_1\frac{di_t}{dt} + L_2\frac{di_t}{dt} = (L_1 + L_2)\frac{di_t}{dt}$$

$$L_{eq} = L_1 + L_2$$

Same as resistors!

~~Series~~ Parallel Inductors:



$$L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt} = V_t$$

$$i_t = i_1 + i_2 \quad \Rightarrow \frac{di_t}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{V_t}{L_{eq}} = \frac{V_t}{L_1} + \frac{V_t}{L_2} \quad \Rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Summary

Capacitors

$$i(t) = C\frac{dv(t)}{dt}$$

$$w = \frac{1}{2}Cv^2$$

- Capacitors resist instantaneous change in voltage
- Current can change instantaneously
- Open circuit in steady-state

$$C_{\text{par}} = C_1 + C_2$$

$$\frac{1}{C_{\text{ser}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Inductors

$$v(t) = L\frac{di(t)}{dt}$$

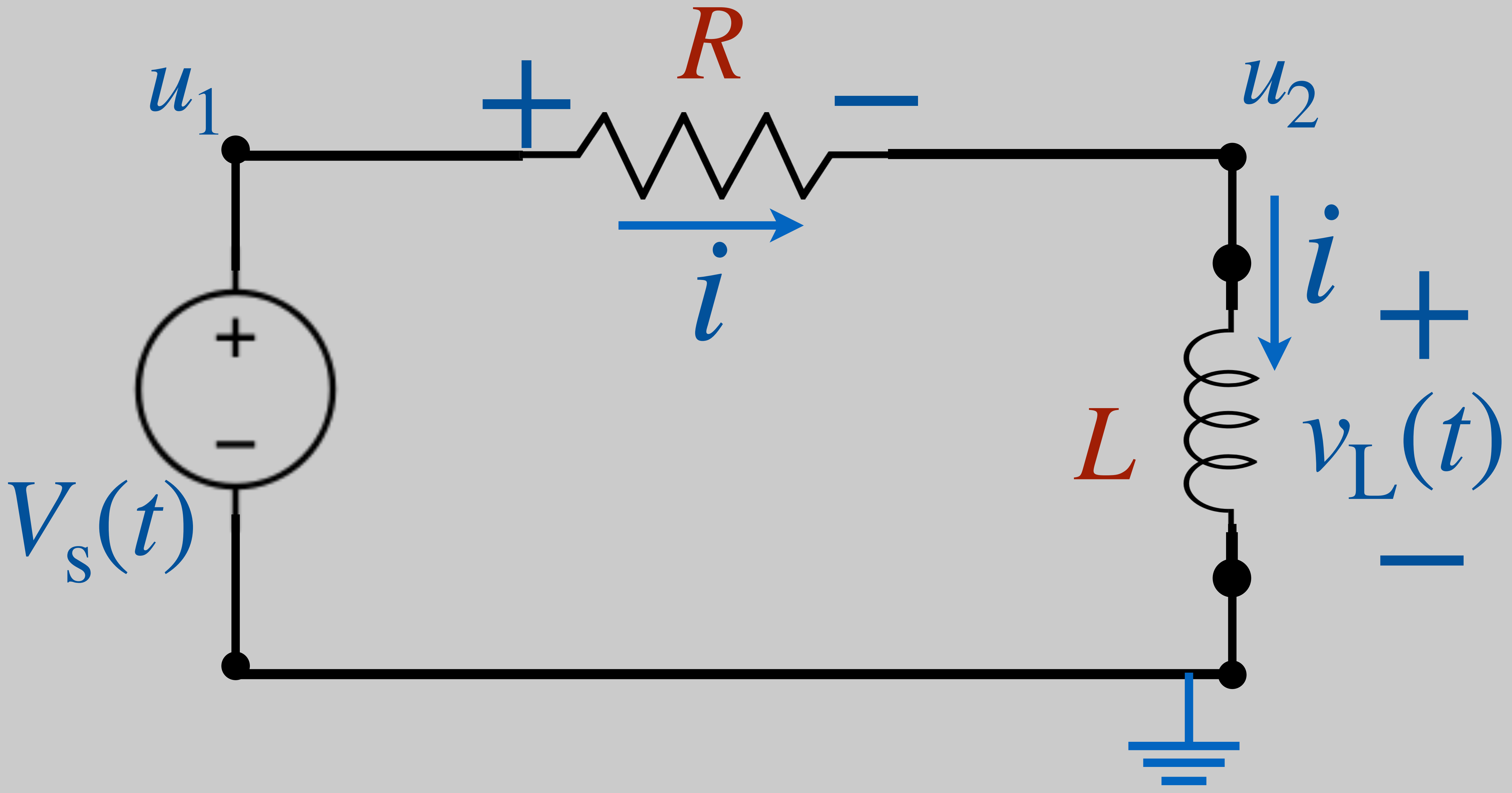
$$w = \frac{1}{2}Li^2$$

- Inductors resist instantaneous change in current
- Voltage can change instantaneously
- Short circuit in steady-state

$$\frac{1}{L_{\text{par}}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$L_{\text{ser}} = L_1 + L_2$$

RL Circuits



$$u_1 = V_s$$

$$u_2 = v_L$$

$$V_s - v_L = iR$$

$$v_L = L\frac{di}{dt}$$

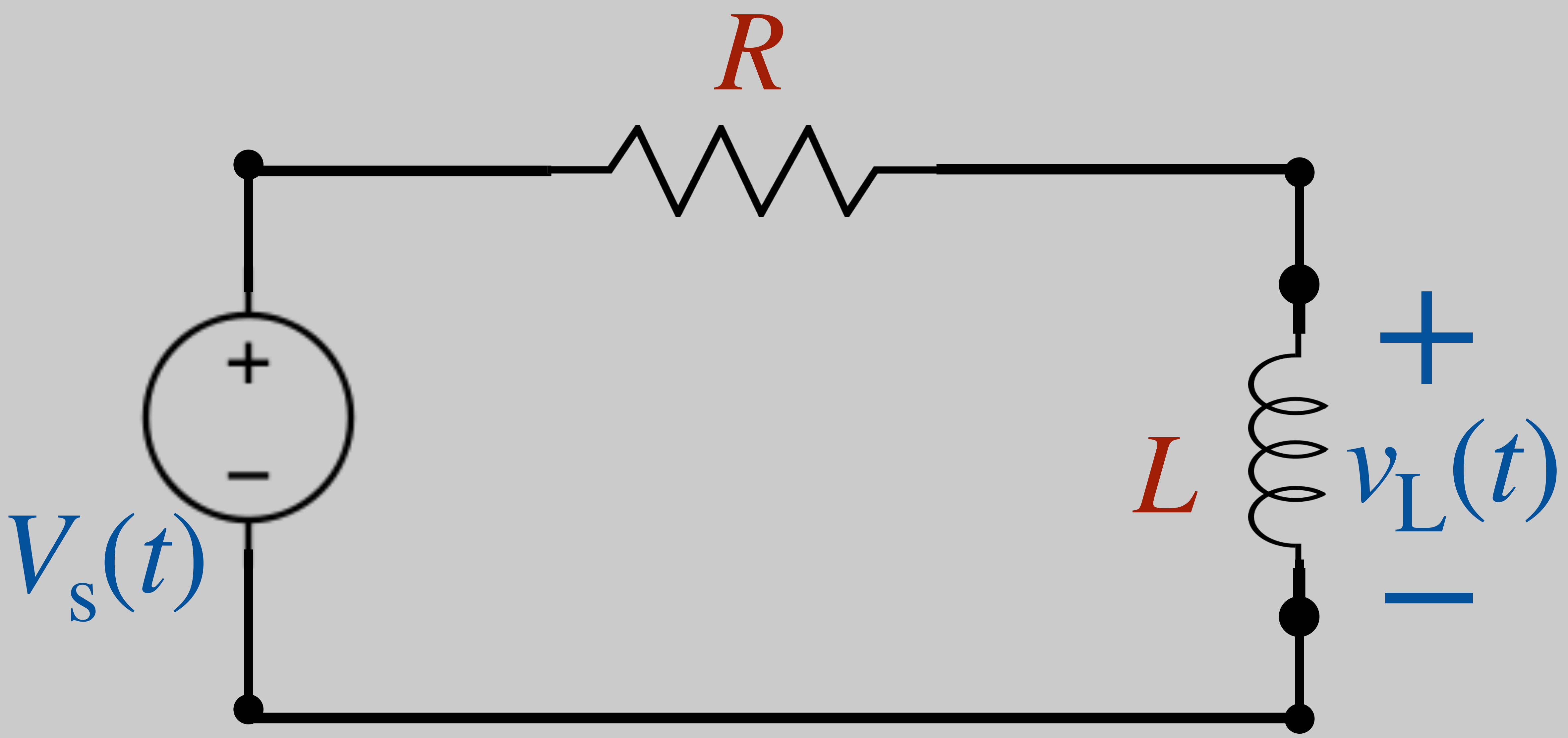
$$V_s - L\frac{di}{dt} = iR$$

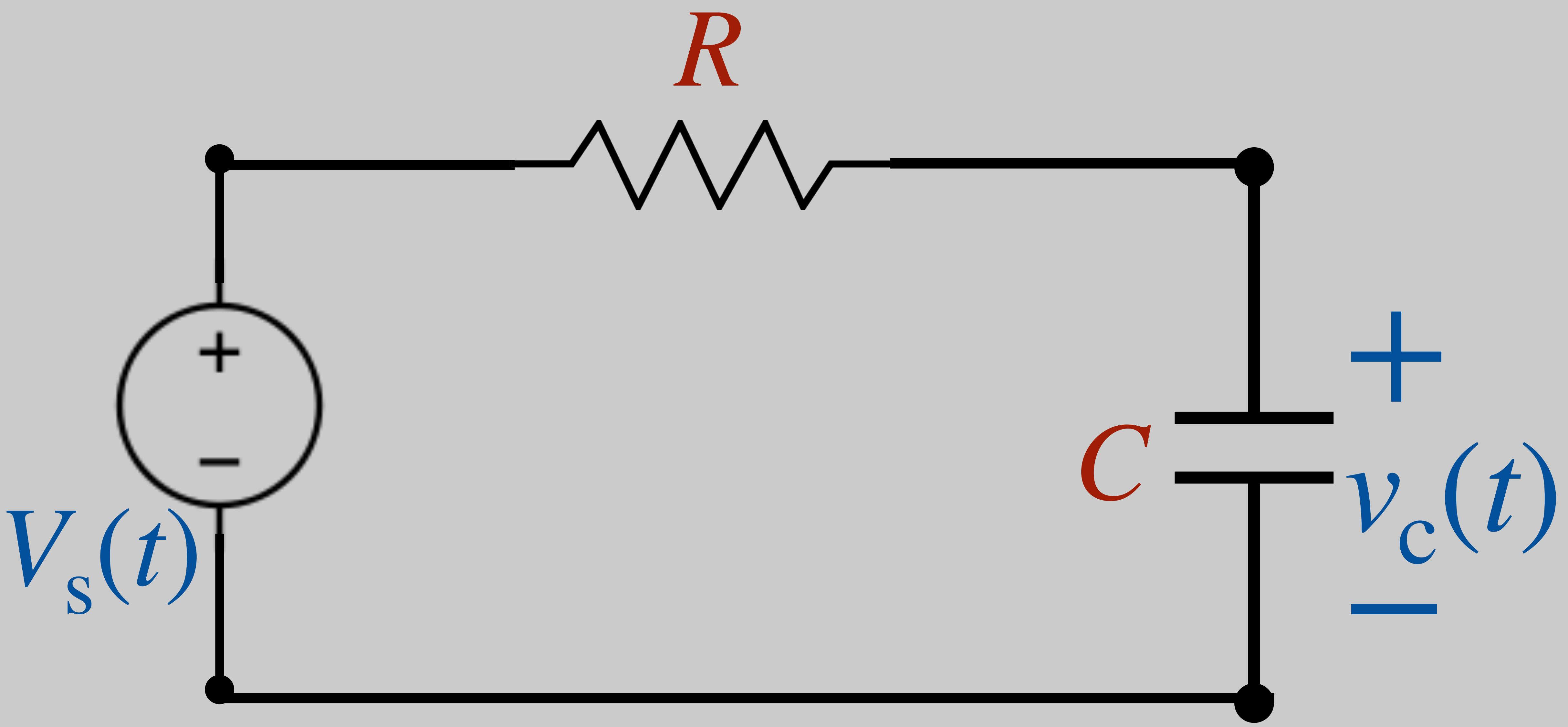
Constant coefficients

1st order diff. Eq.

$$L\frac{di}{dt} + Ri = V_s \quad \Rightarrow \frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{V_s(t)}{L}$$

RL Circuits



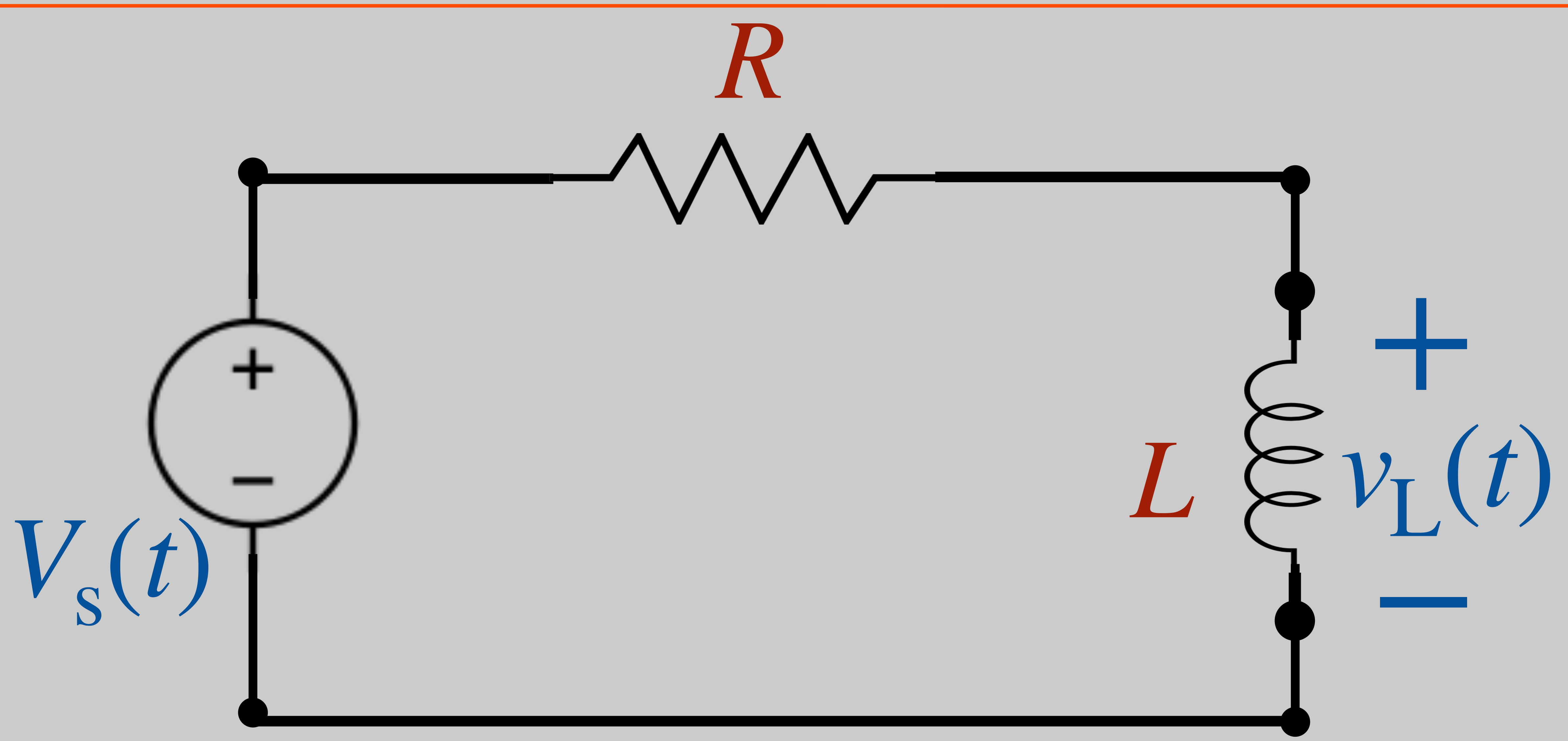


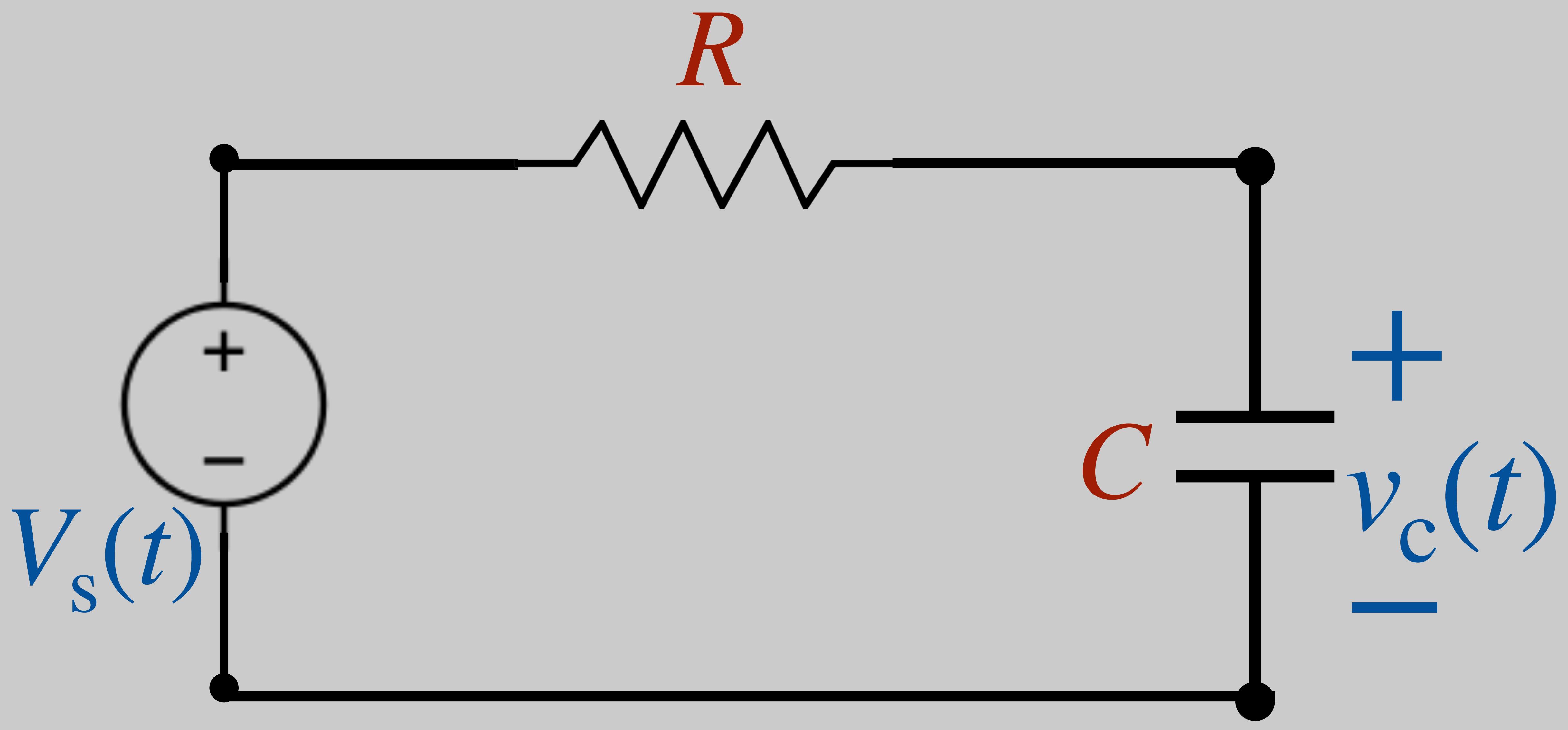
$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V_s(t)}{L}$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} V_s(t)$$

$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{R}{L} \frac{V_s(t)}{R}$$

RL Circuits





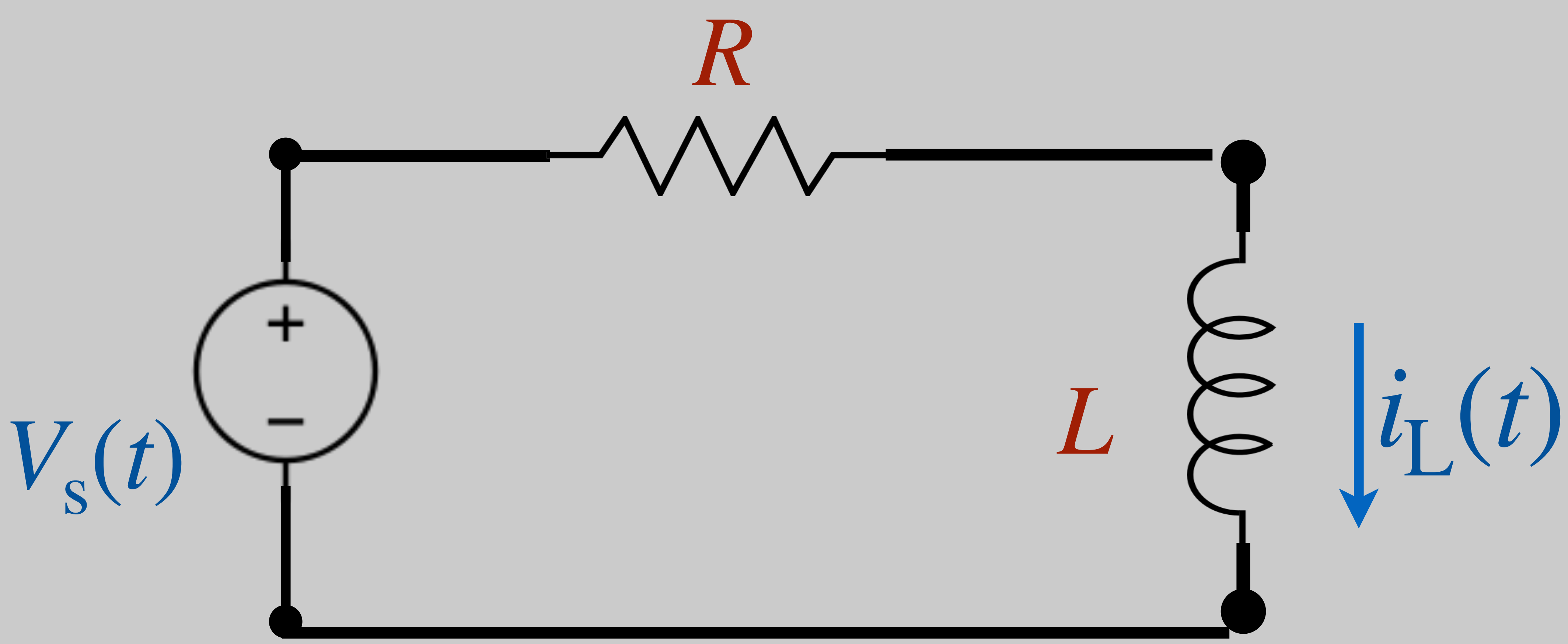
$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{R}{L}\frac{V_s(t)}{R} \quad \Leftrightarrow \quad \frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} V_s(t)$$

$$\tau = \frac{L}{R} \qquad\qquad \tau = RC$$

$$\frac{di(t)}{dt} + \tau^{-1} i(t) = \tau^{-1} \frac{V_s(t)}{R} \qquad\qquad \frac{dv_c(t)}{dt} + \tau^{-1} v_c(t) = \tau^{-1} V_s(t)$$

Step Response of LC circuits

$$\frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{R}{L}\frac{V_0(t)}{R}$$



Example: Step-Response

$$V_s(t) = V_0 \quad | t > t_0$$

$$i_L(t_0) = 0$$

$$v_L = V_0 - iR$$

$$i_L = \frac{V_0}{R}\left(1 - e^{-L\frac{t-t_0}{R}}\right)$$

$$v_L = V_0 - V_0\left(1 - e^{-L\frac{t-t_0}{R}}\right)$$

$$\Rightarrow v_L = V_0 e^{-L\frac{t-t_0}{R}}$$

Step Response of LC circuits

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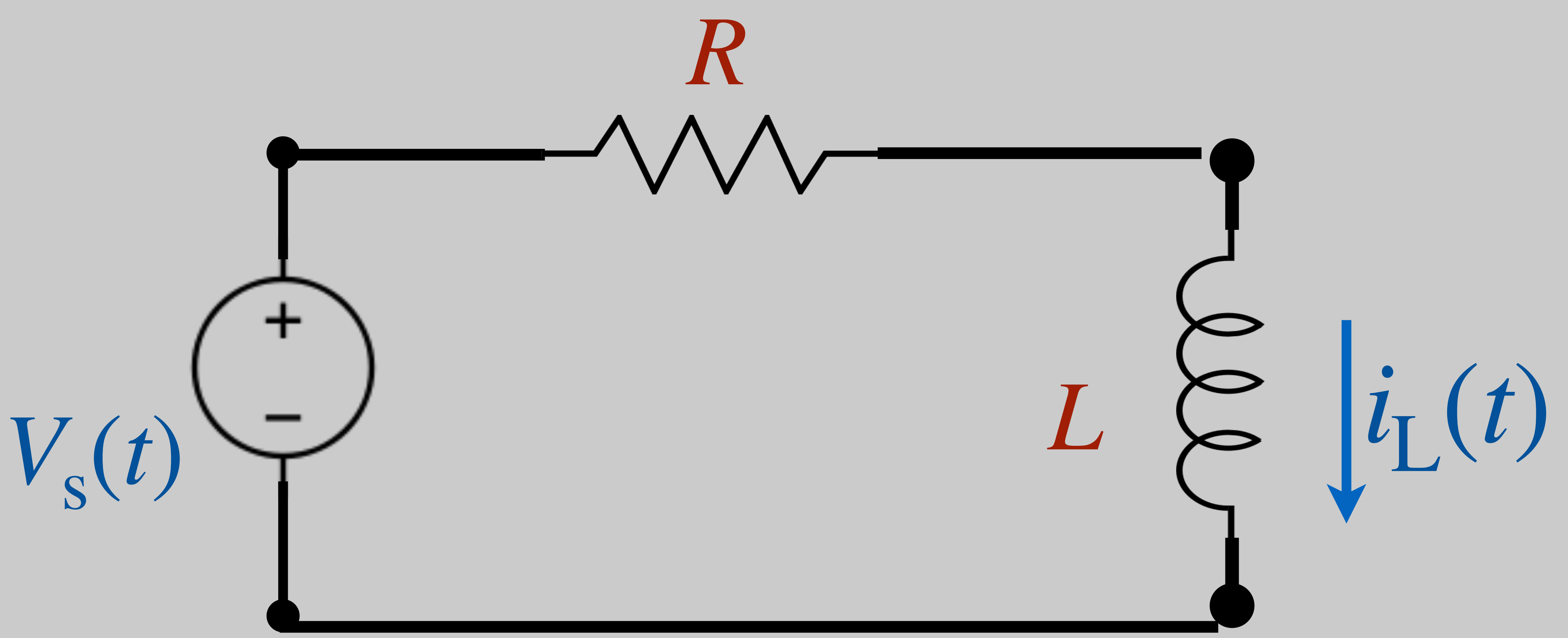


Example: Step-Response

$V_s(t) = V_0 \quad | t > t_0$

$i_L(t_0) = 0$

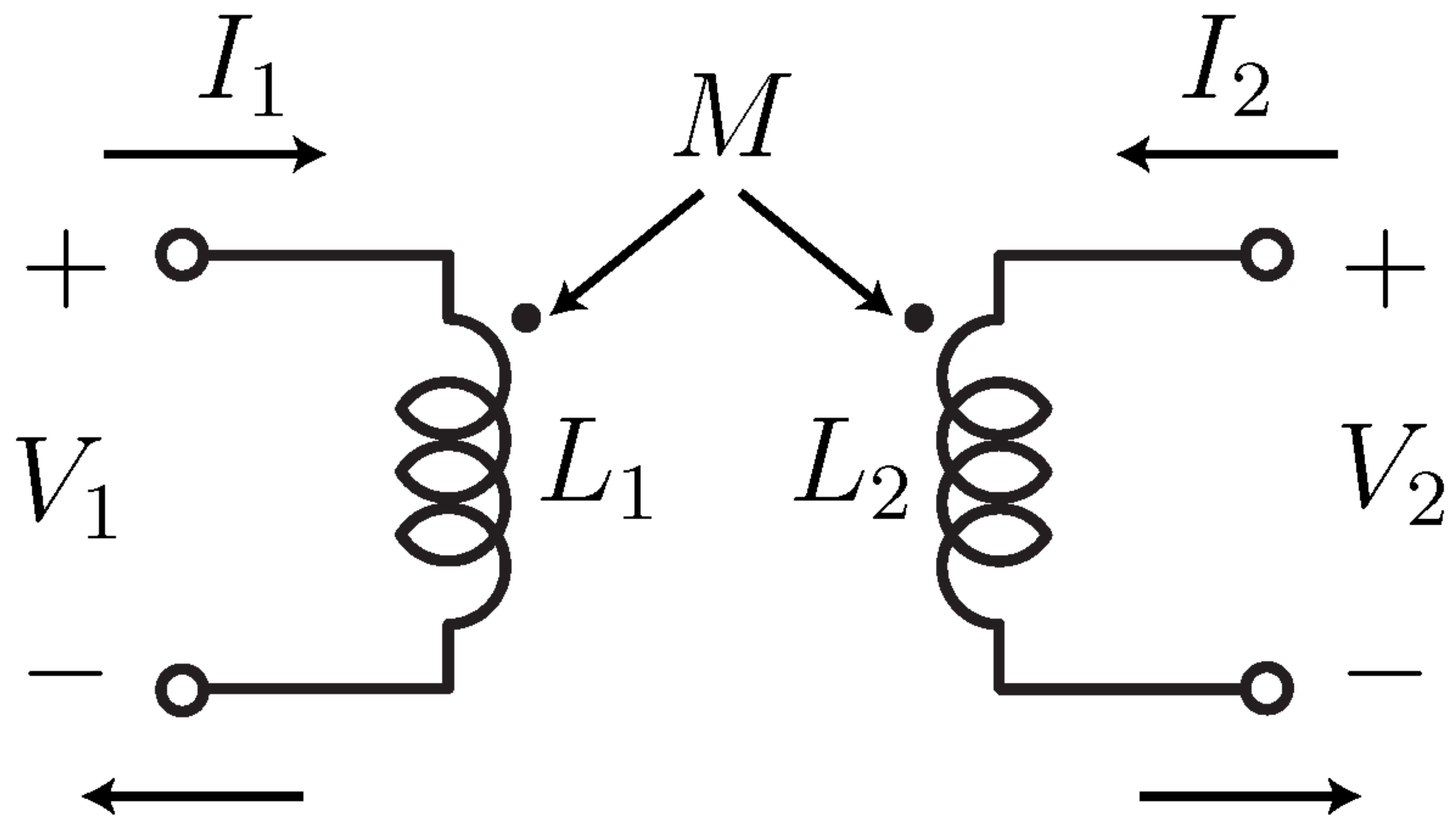
$v_L = V_0 - iR$

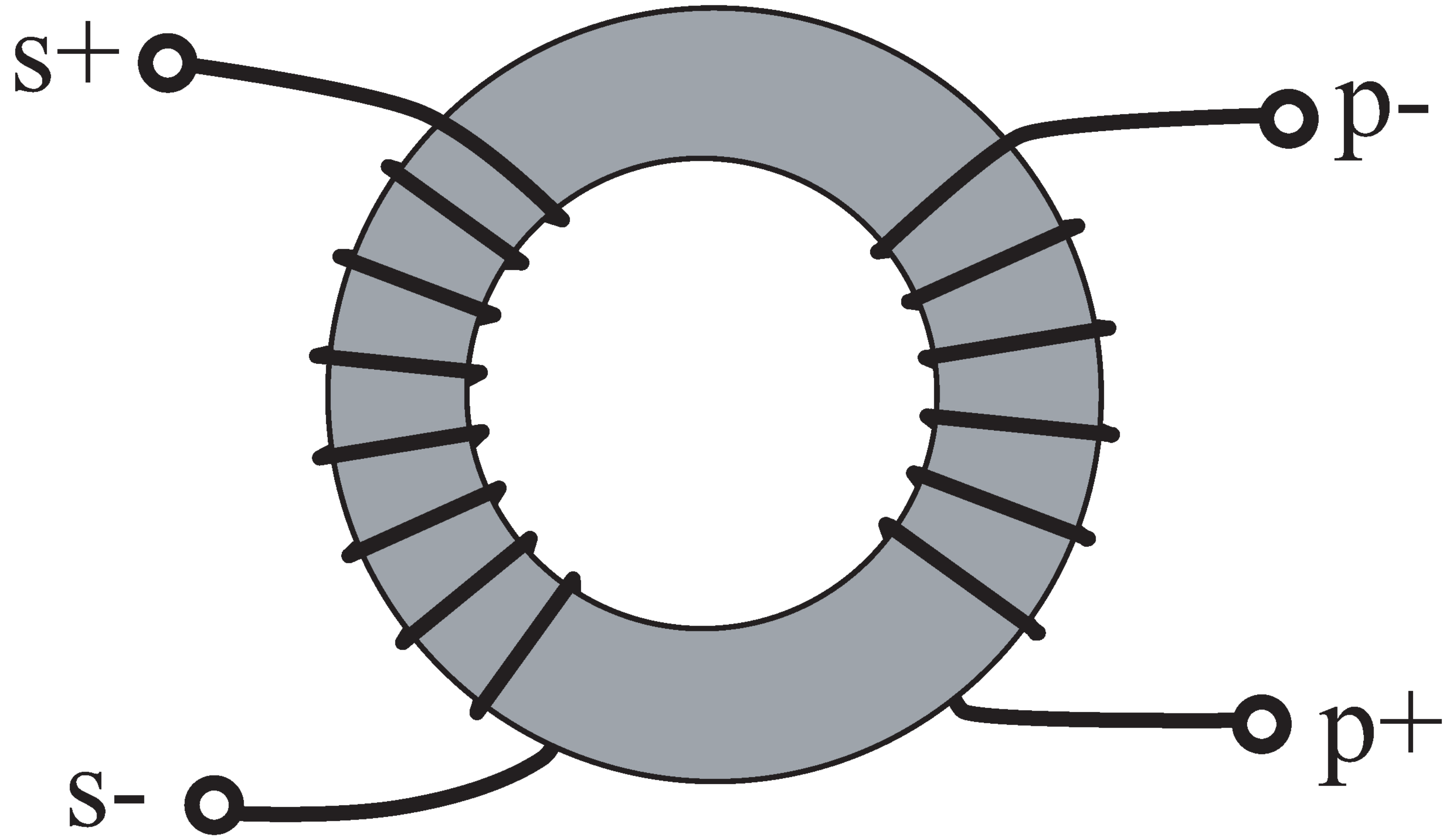
$$i_L = \frac{V_0}{R}\left(1 - e^{-L\frac{t-t_0}{R}}\right)$$

$$v_L = V_0 - V_0\left(1 - e^{-L\frac{t-t_0}{R}}\right)$$

$$\Rightarrow v_L = V_0 e^{-L\frac{t-t_0}{R}}$$

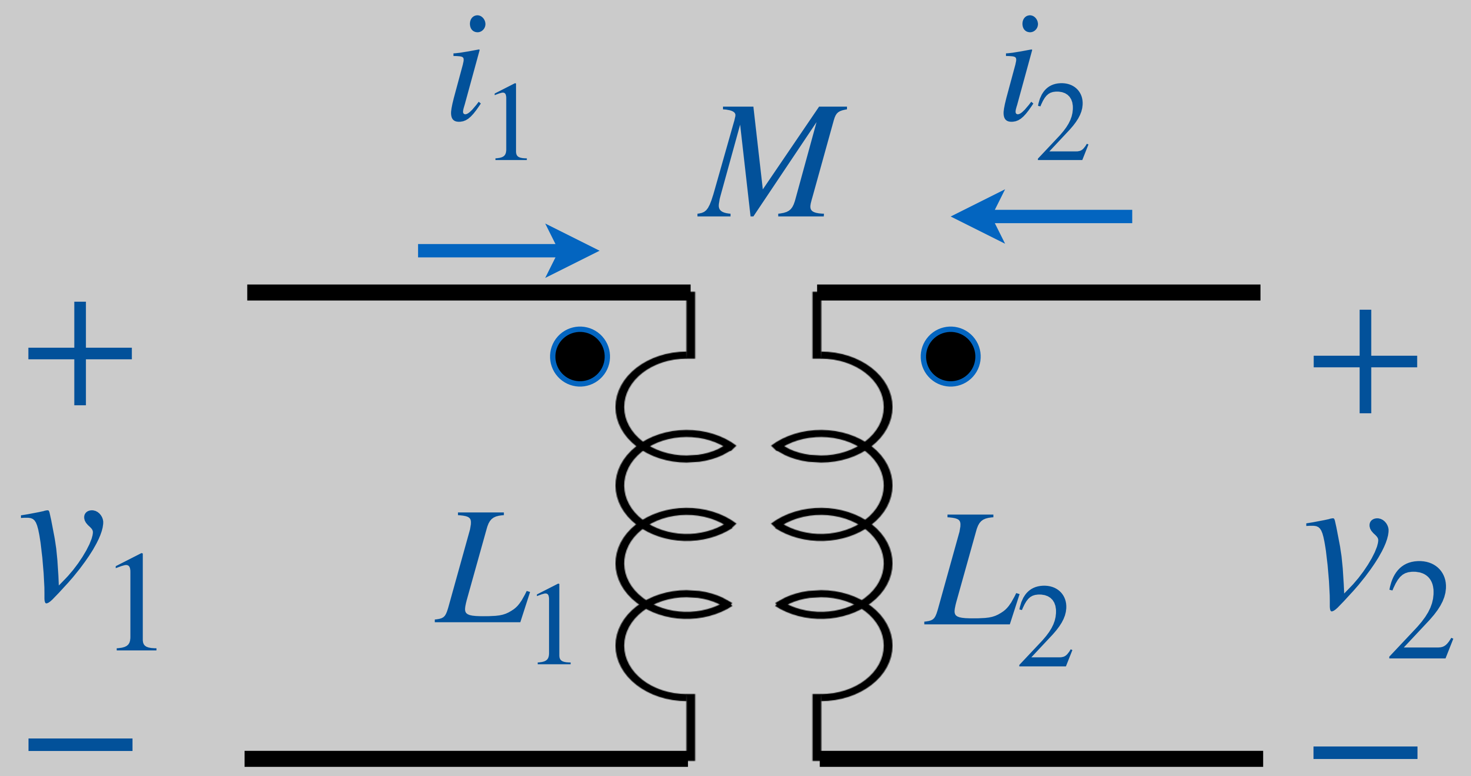
- Occurs when two windings are arranged such flux generated in one runs through the other
- Change in current in one winding, causes voltage in the other

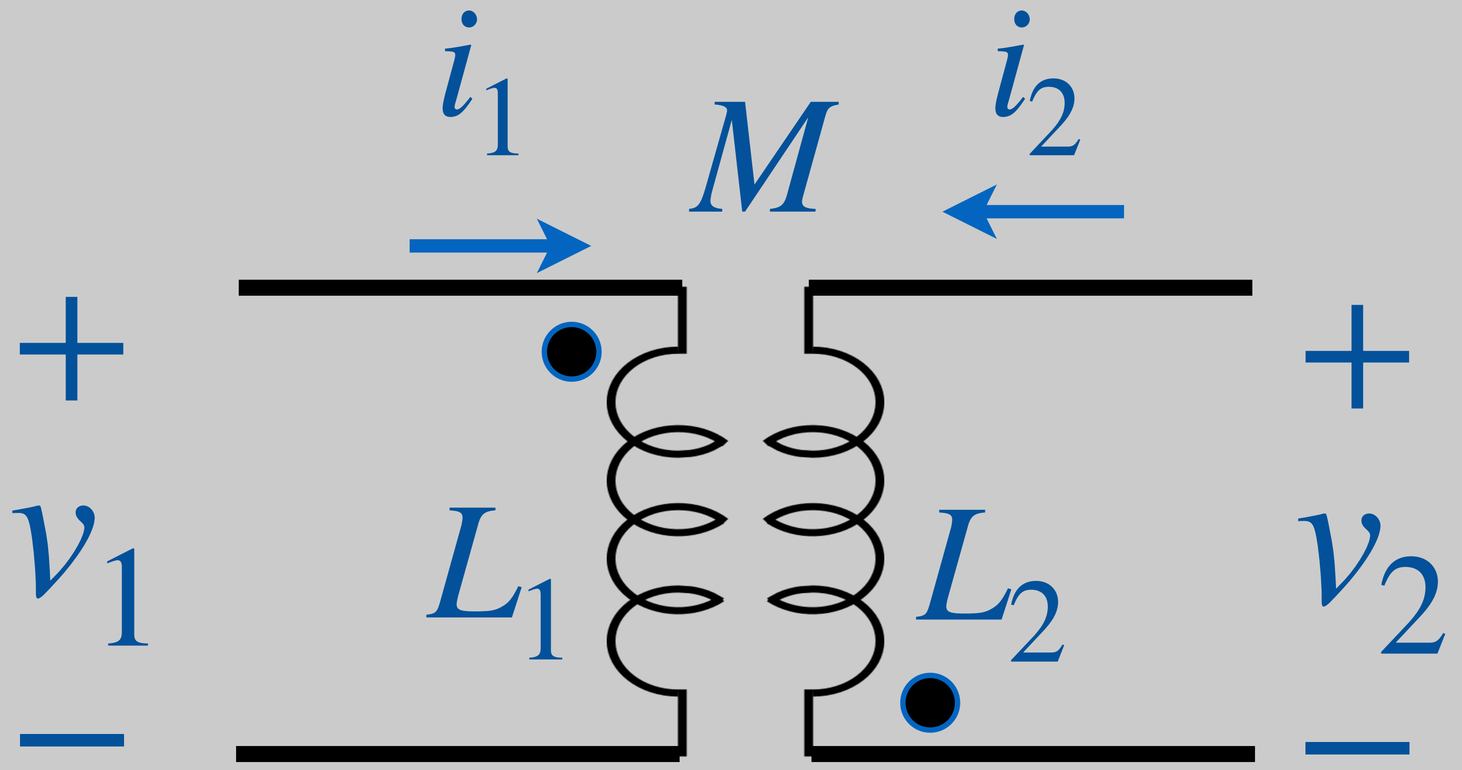




Used in transformers, motors and generators

- Dot indicates polarity of coupling





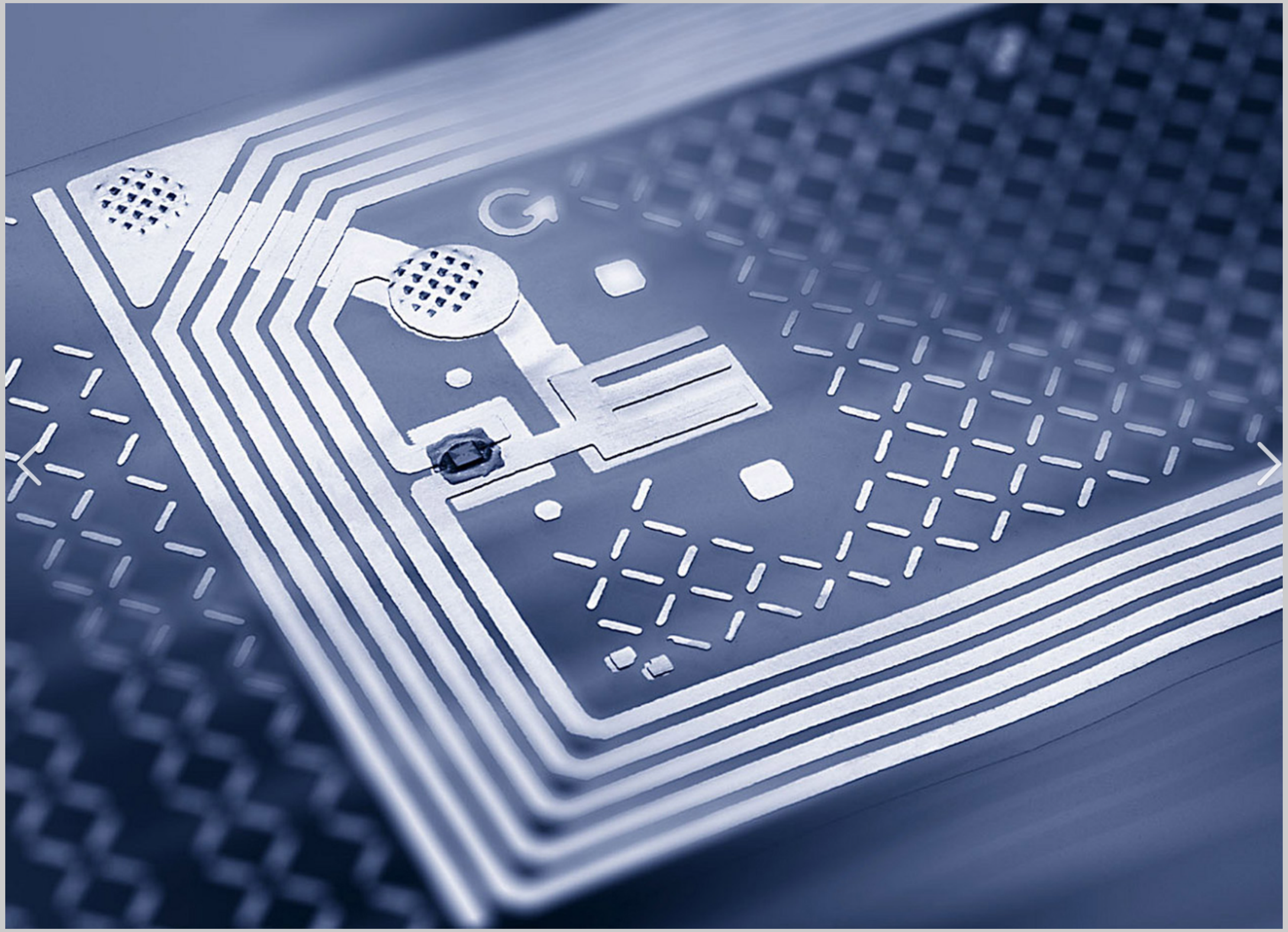
$$v_1 = L1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

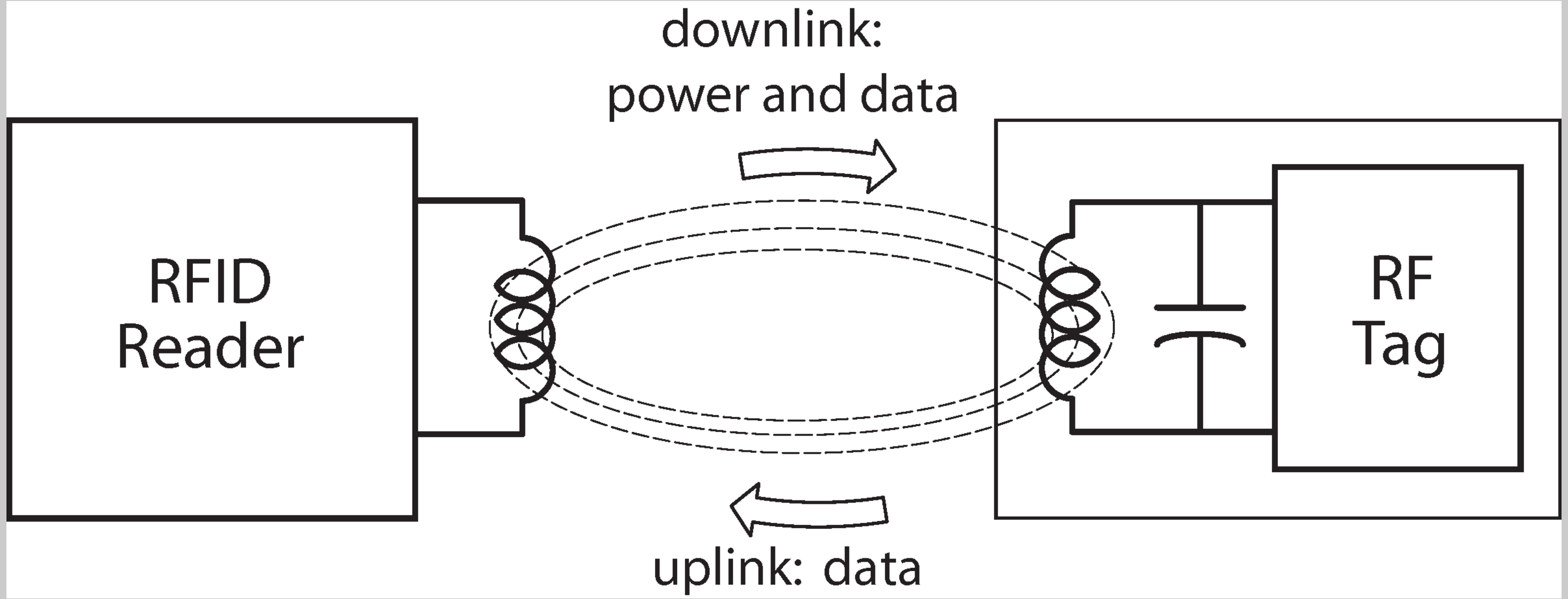
$$v_2 = M\frac{di_1}{dt} + L2\frac{di_2}{dt}$$

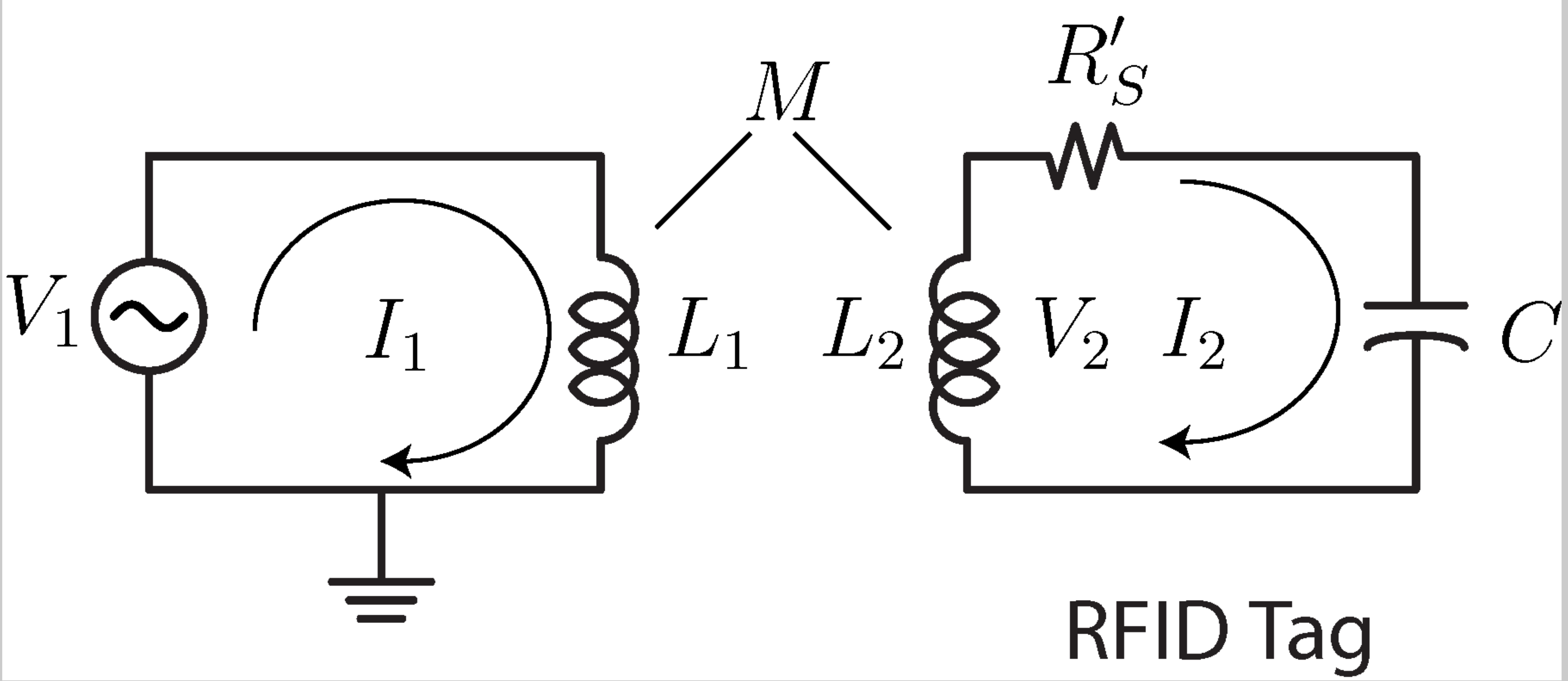
$$v_1 = L1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$v_2 = -M\frac{di_1}{dt} + L2\frac{di_2}{dt}$$

RFID: Transformer at a distance!



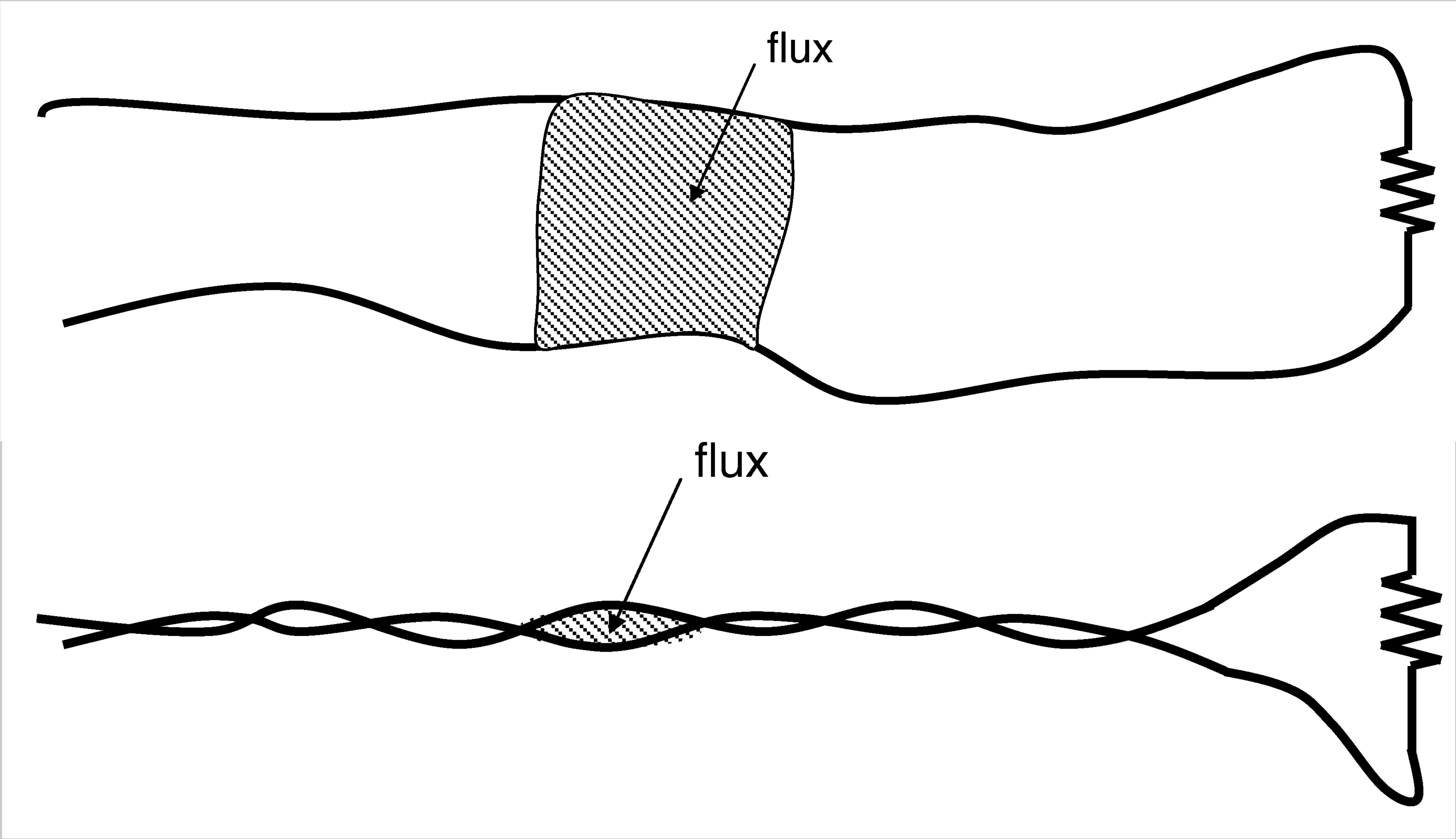


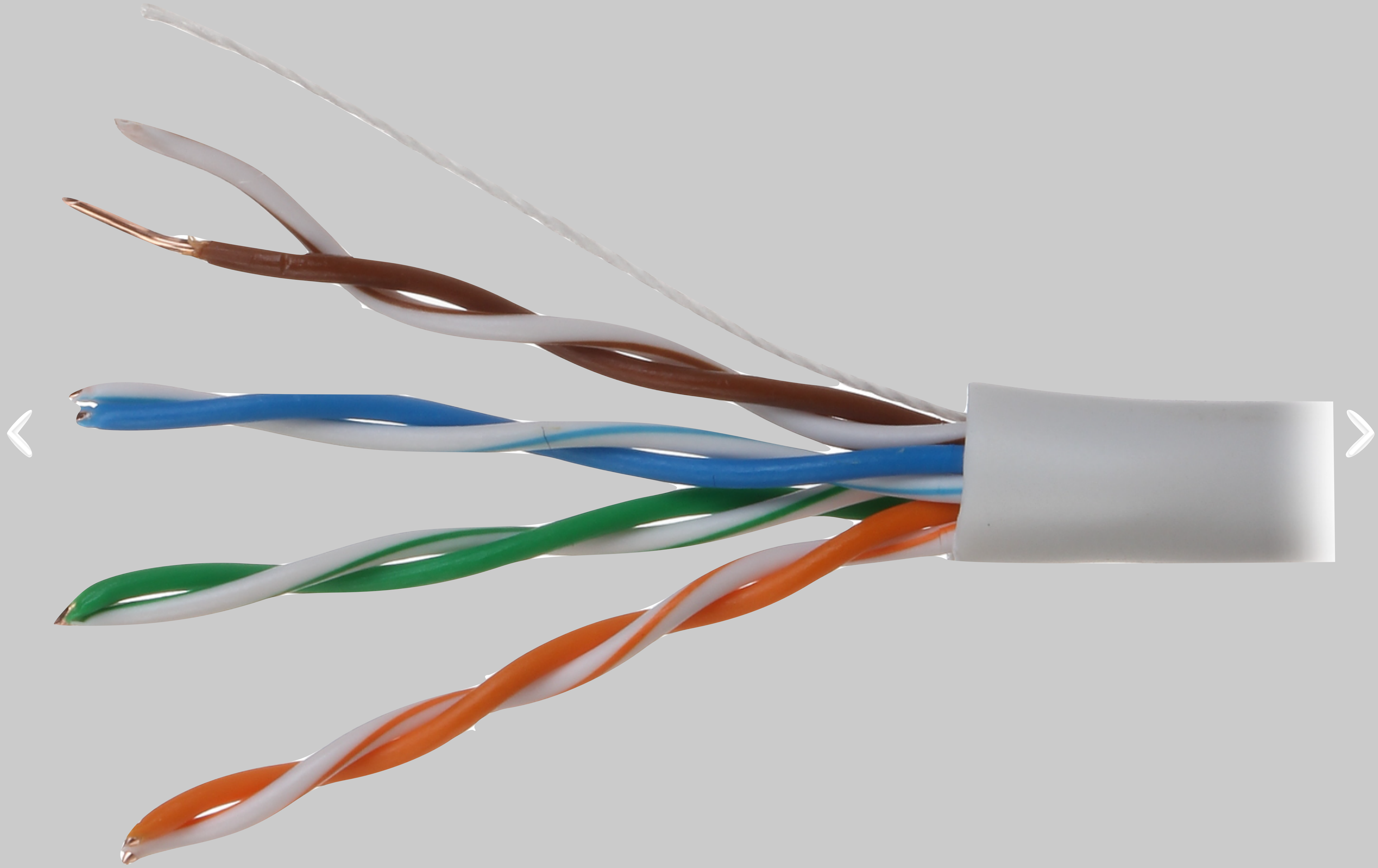


- Card Keys, contactless payment, wireless charging

Why twist wires?

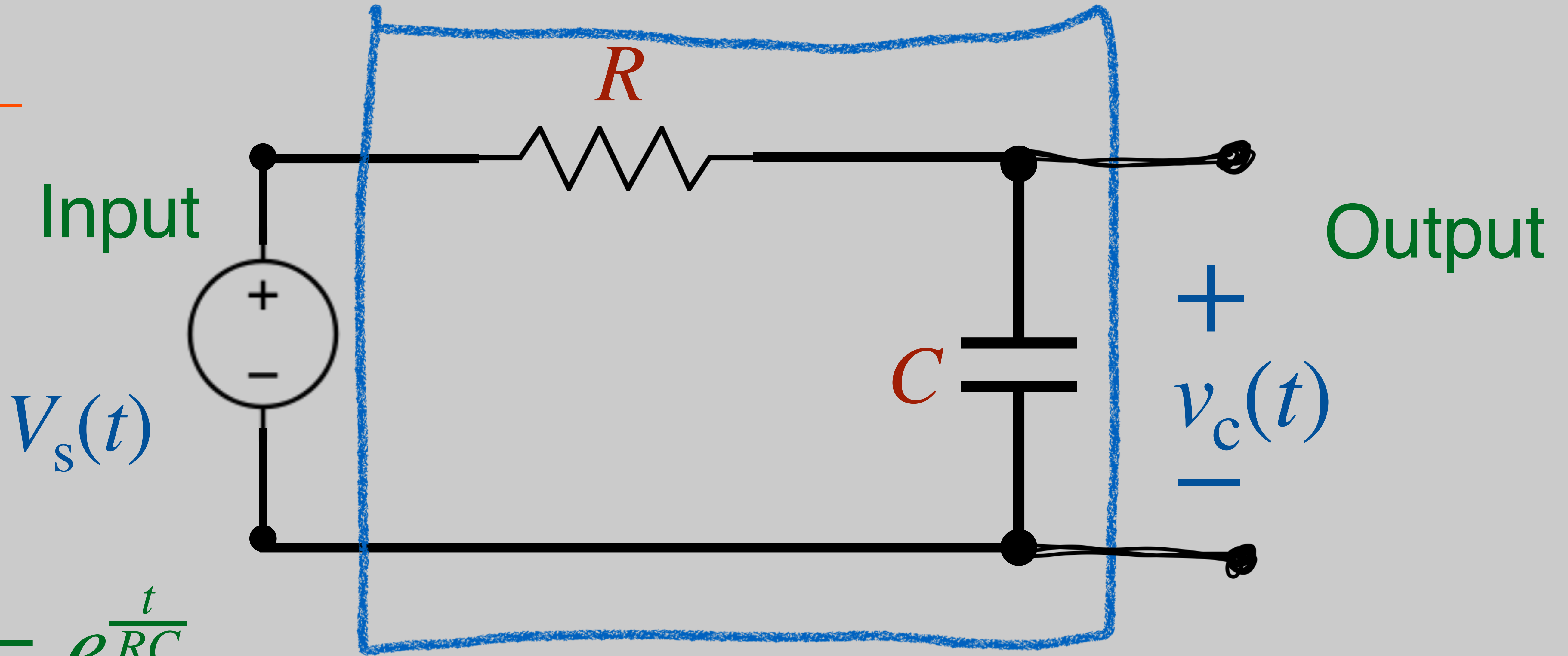
- Any loop can capture flux
- Captures interference through mutual inductance





RC Circuits - General Response

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_s(t)$$



Method of integrating factor. Use: $m(t) = e^{\frac{t}{RC}}$

$$v_c'm + \frac{1}{RC}v_c m = \frac{1}{RC}V_s m$$

$$(v_c m)' = \frac{1}{RC}V_s m$$

$$v_c(t)m(t) = \frac{1}{RC}\int_{-\infty}^{t} V_s(\tau)m(\tau)d\tau + K$$

Since: $m(t) = e^{\frac{t}{RC}}$

$$m' = \frac{1}{RC}e^{\frac{t}{RC}} = \frac{1}{RC}m$$

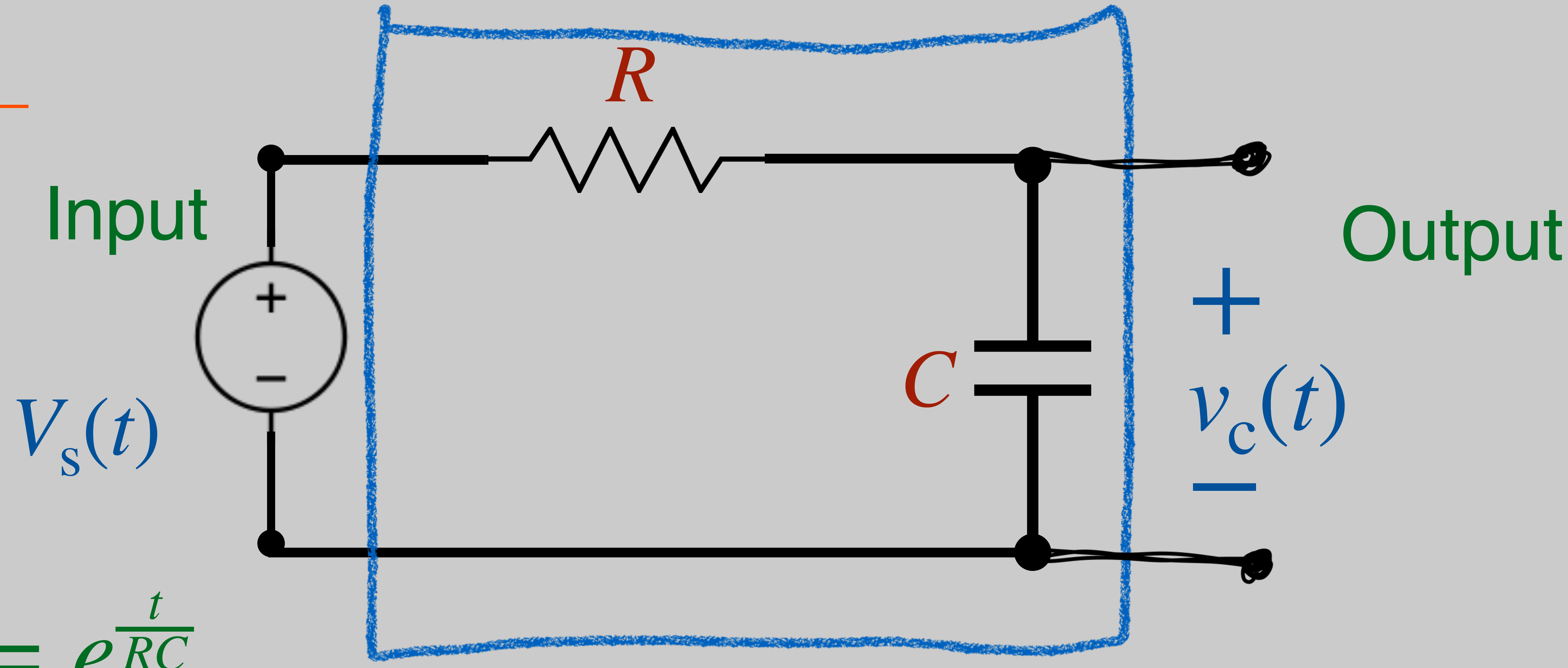
Also,

recall: $(v_c m)' = v_c'm + v_c m'$

$$= v_c'm + \frac{1}{RC}v_c m$$

RC Circuits - General Response

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_s(t)$$



Method of integrating factor. Use: $m(t) = e^{\frac{t}{RC}}$

$$v_c'm + \frac{1}{RC}v_cm = \frac{1}{RC}V_sm$$

$$(v_cm)' = \frac{1}{RC}V_sm$$

$$v_c(t)m(t) = \frac{1}{RC}\int_{-\infty}^{t} V_s(\tau)m(\tau)d\tau + K$$

$$v_c(t)e^{\frac{t}{RC}} = K + \frac{1}{RC}\int_{-\infty}^{t} V_s(\tau)e^{\frac{\tau}{RC}}d\tau$$

$$v_c(t) = Ke^{-\frac{t}{RC}} + \frac{e^{-\frac{t}{RC}}}{RC}\int_{-\infty}^{t} V_s(\tau)e^{\frac{\tau}{RC}}d\tau$$

Homogeneous

Particular