EE16B
Designing Information Devices and Systems II

Lecture 1B
Transient RC Circuit, Diff EQ

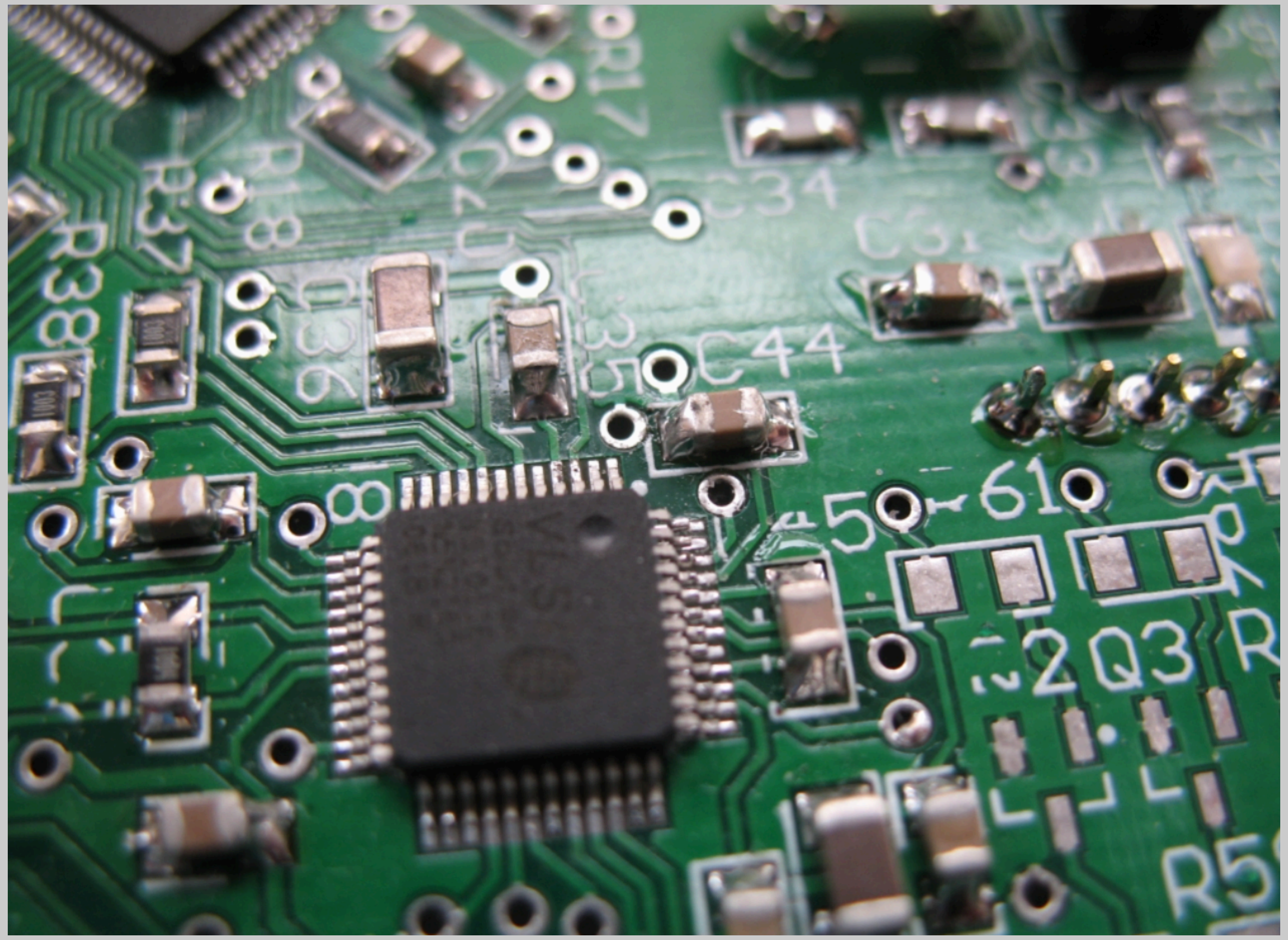
Announcements

- Last time:
  - Intro
  - Review of EECS16A

- Announcements:
  - Pre-Lab 0 & 1 are posted and due Monday (Junha says: no longer than 30min for both)

EECS16A Review

- Intentional capacitors:
  - Essential components in modern electronics
  - Energy storage, filtering, memory



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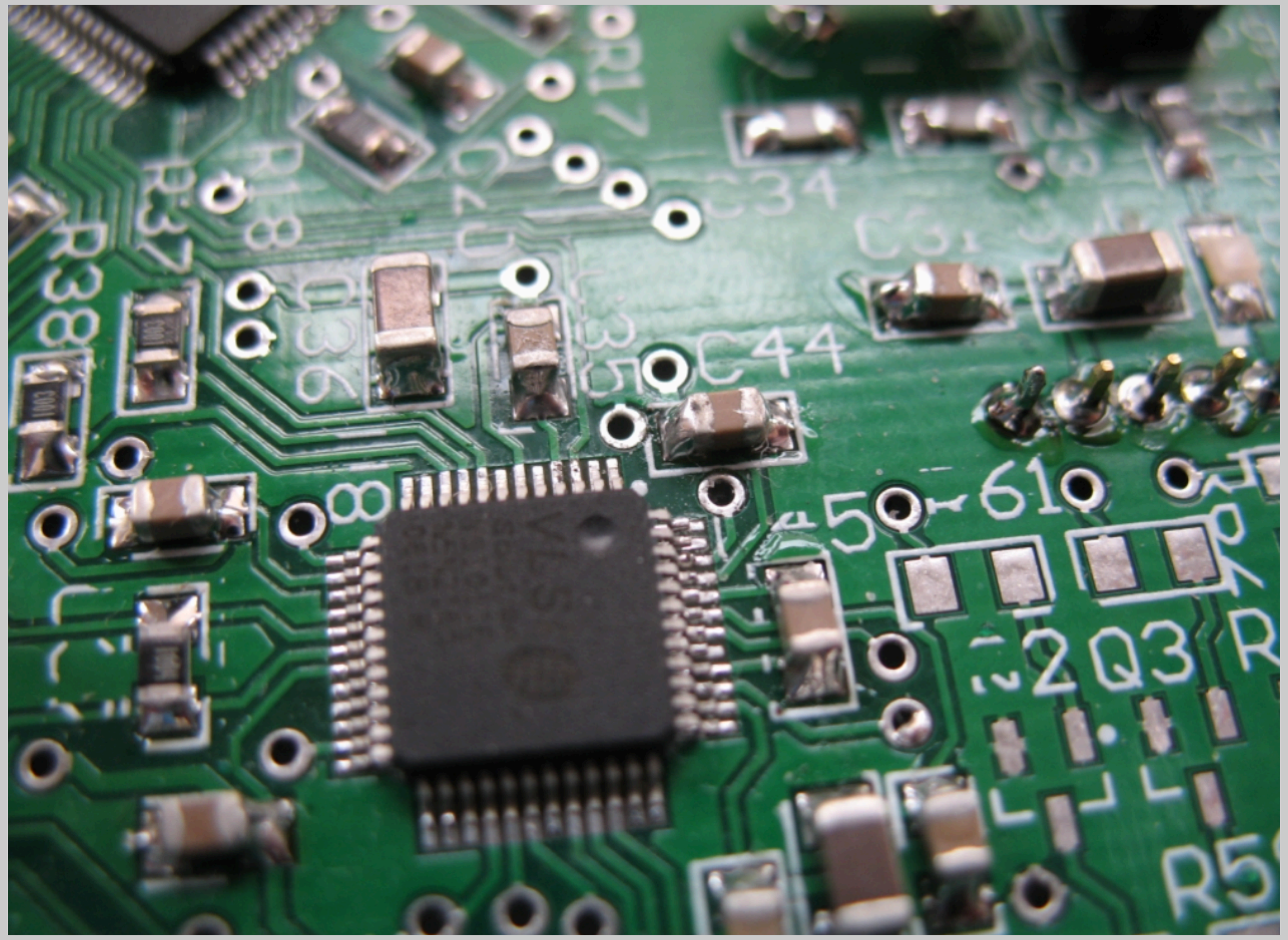
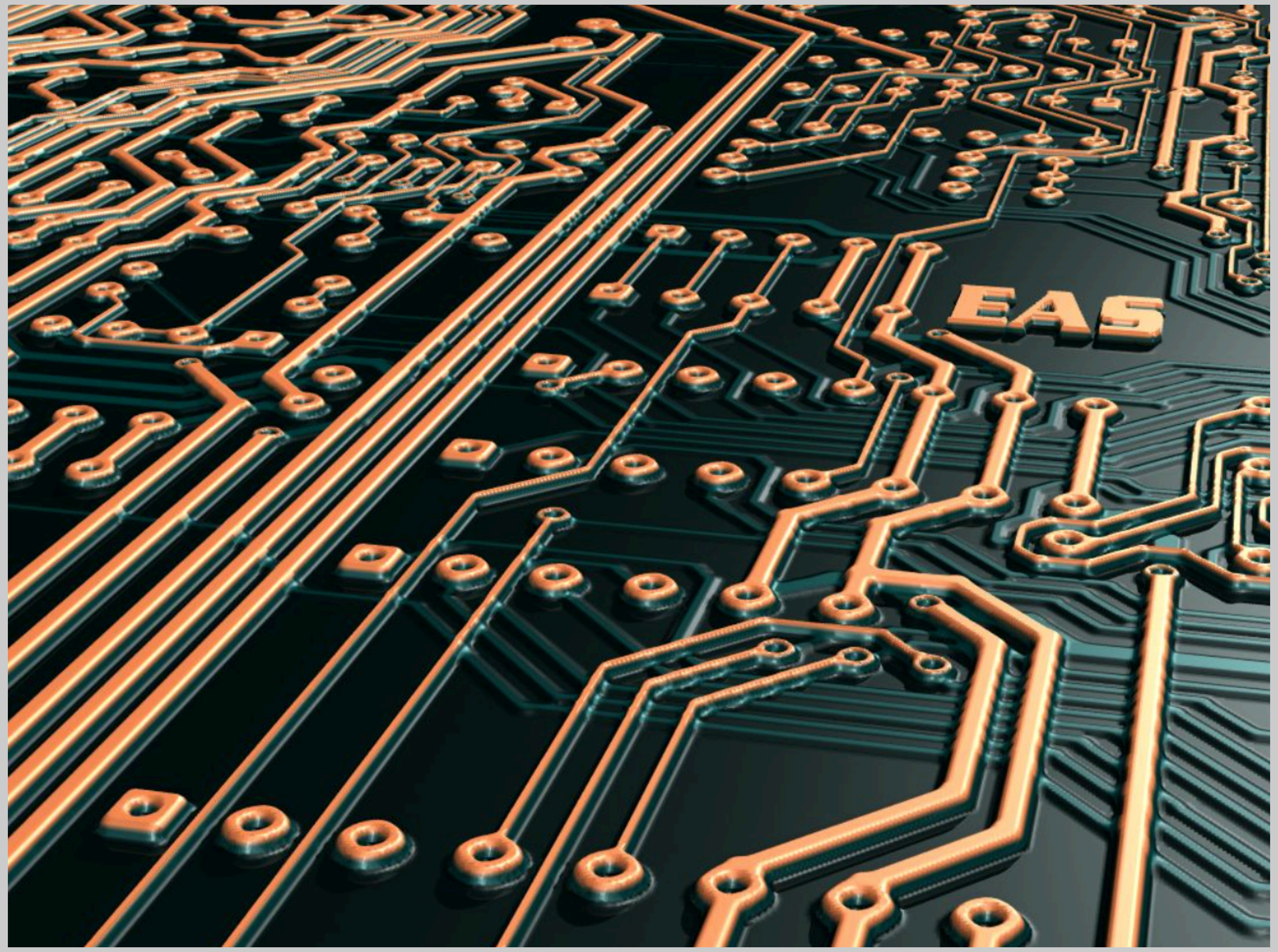
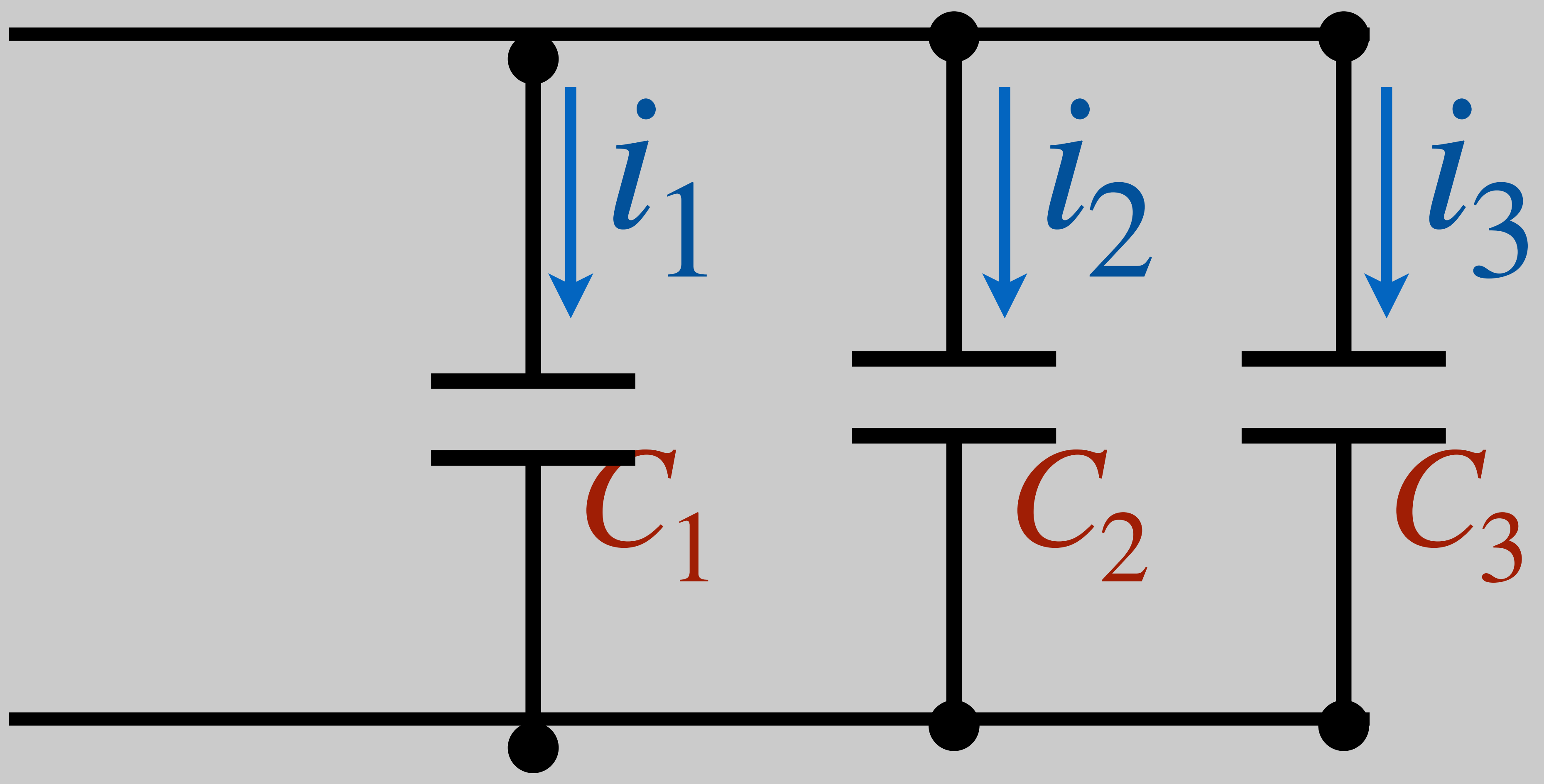


- Unintentional (parasitic)
  - Any close traces with dielectric between becomes a cap!



Parallel Capacitors:



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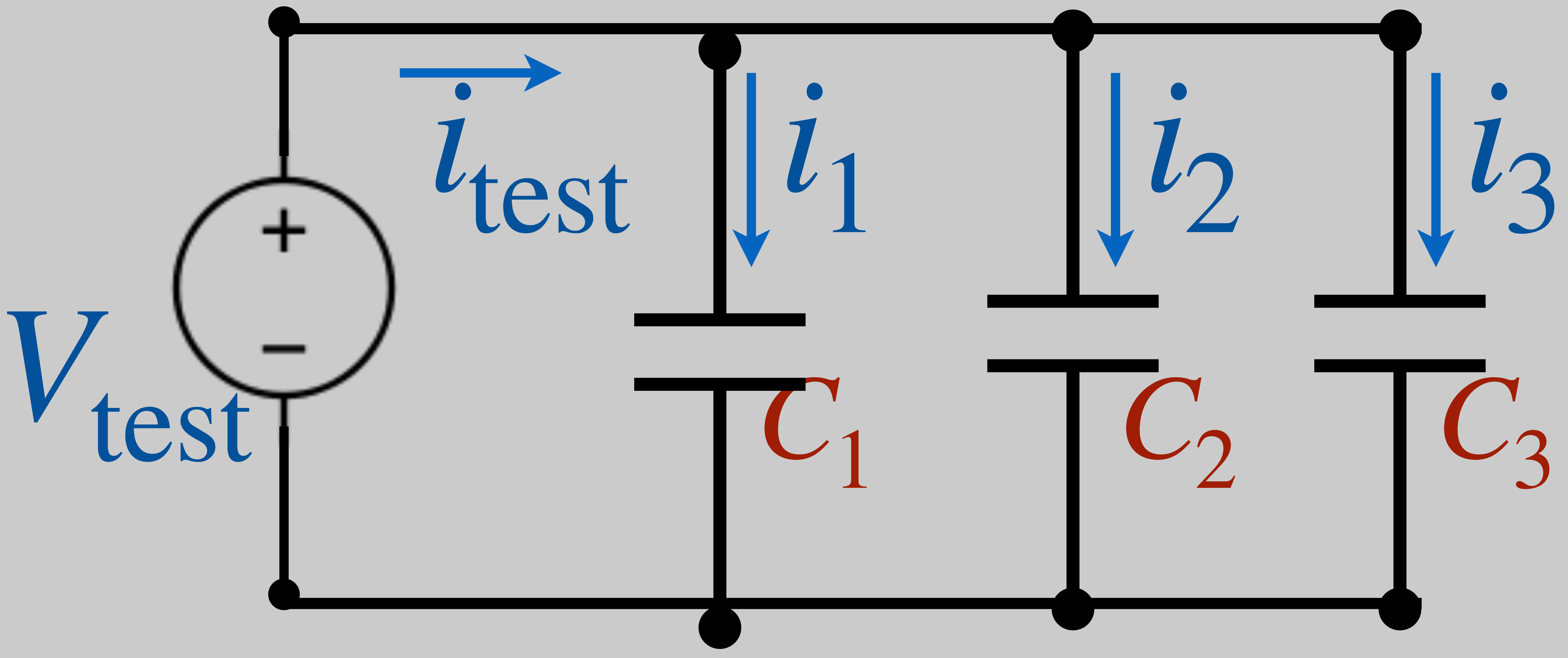
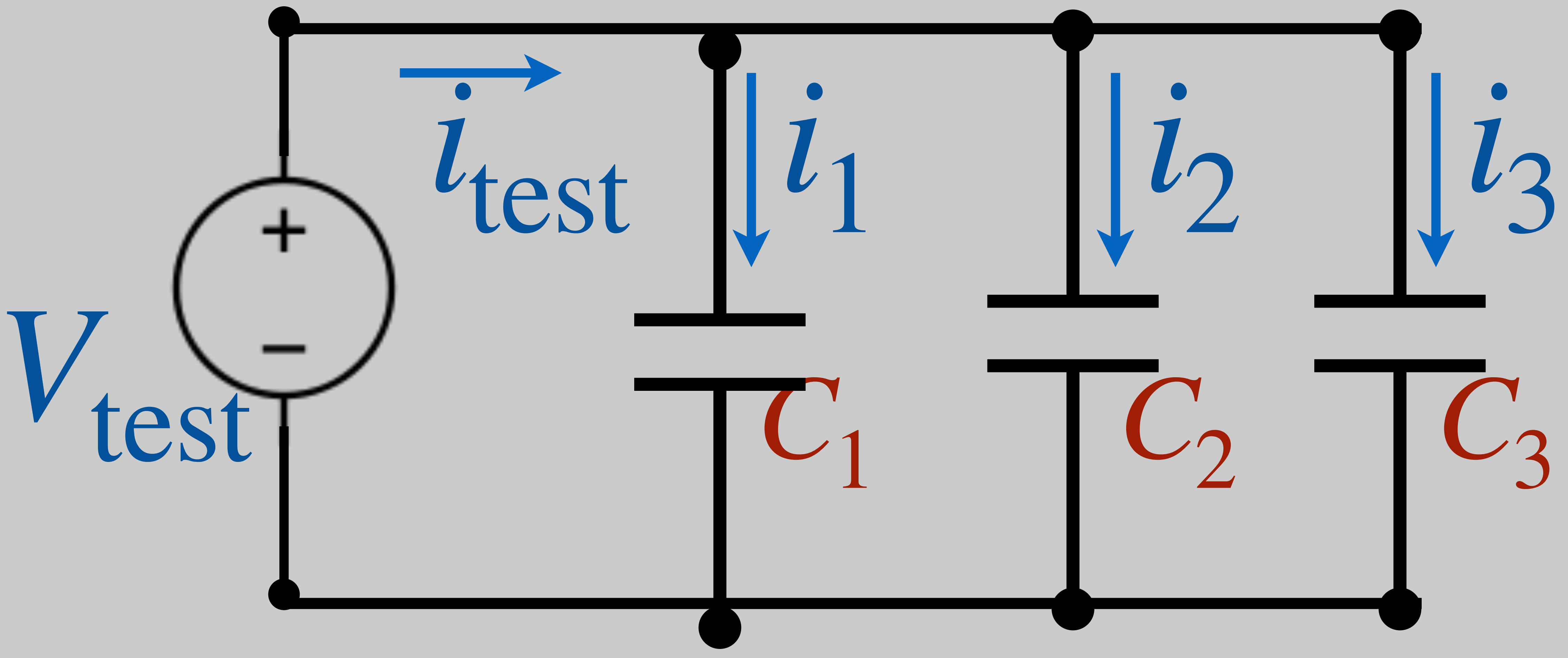


$$i_{test} = i_1 + i_2 + i_3$$

$$C_{eq}\frac{dV_{test}}{dt} = C_1\frac{dV_{test}}{dt} + C_2\frac{dV_{test}}{dt} + C_3\frac{dV_{test}}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3$$

Parallel Capacitors:



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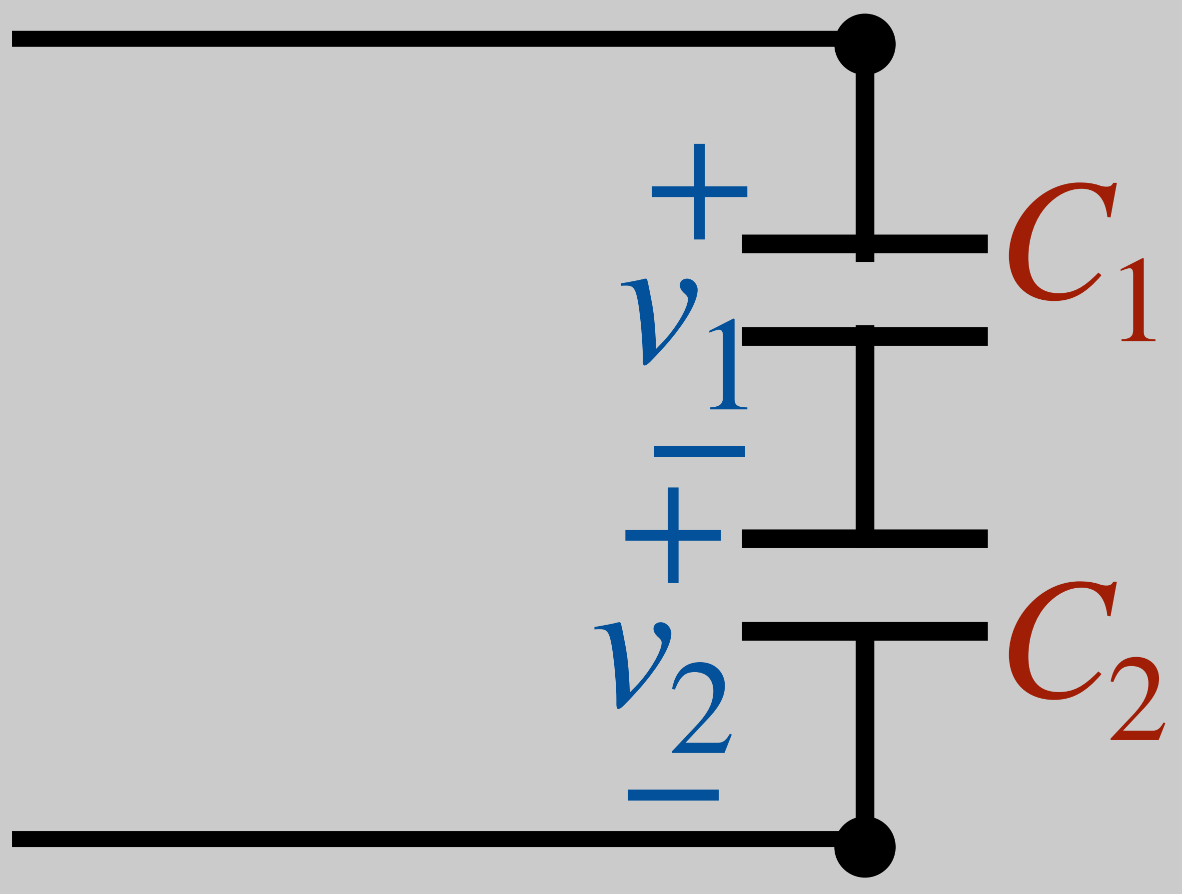
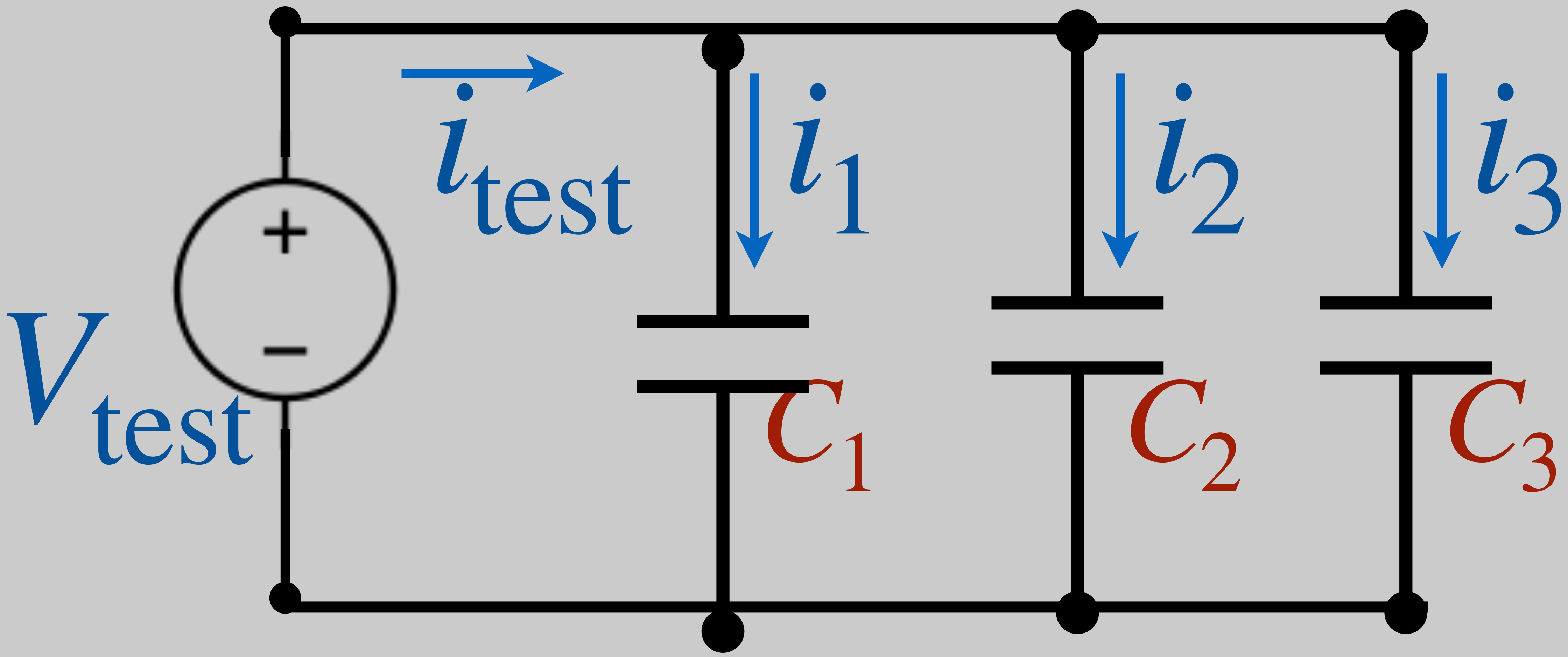
Series Capacitors:



Parallel Capacitors:



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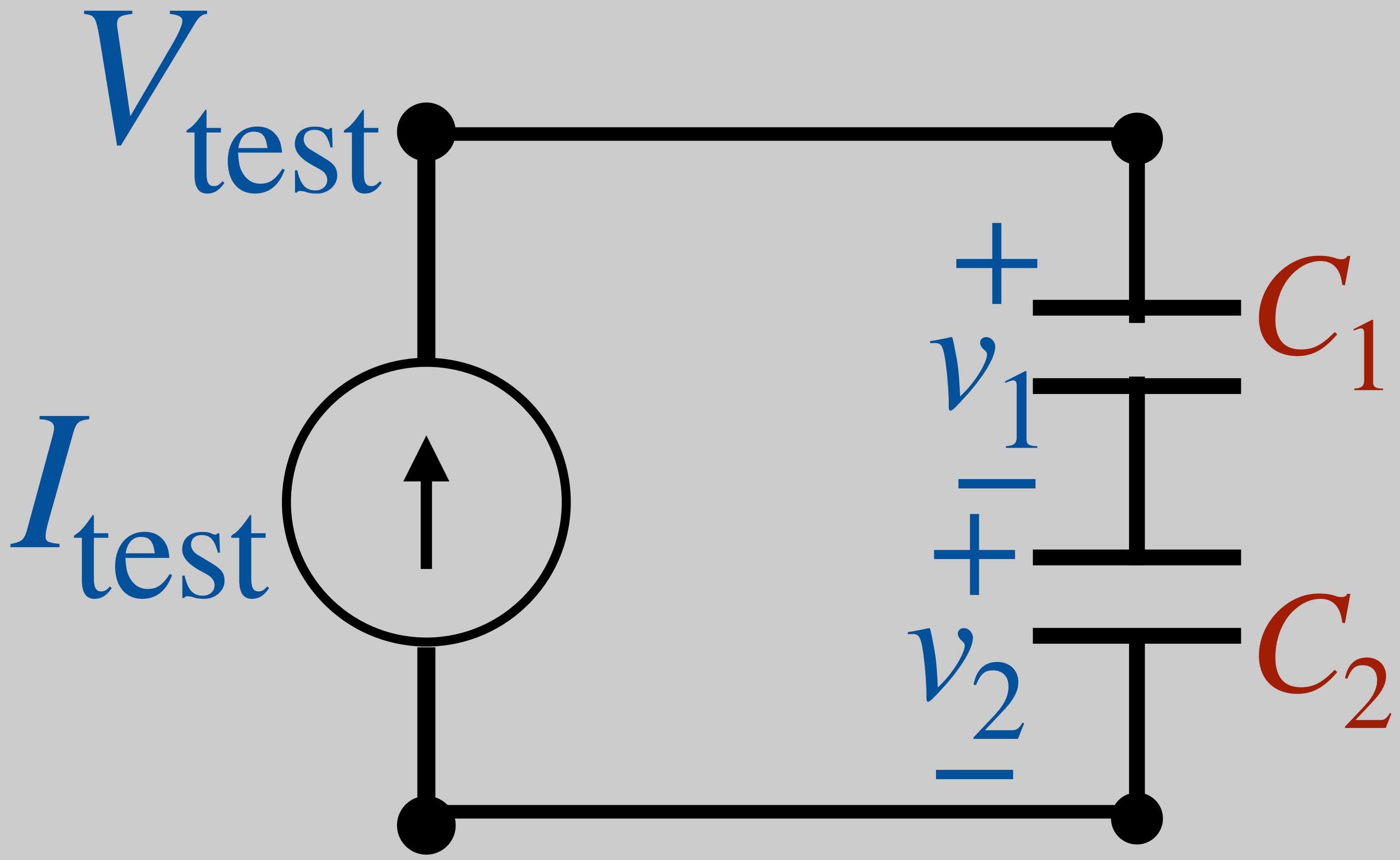
Series Capacitors:



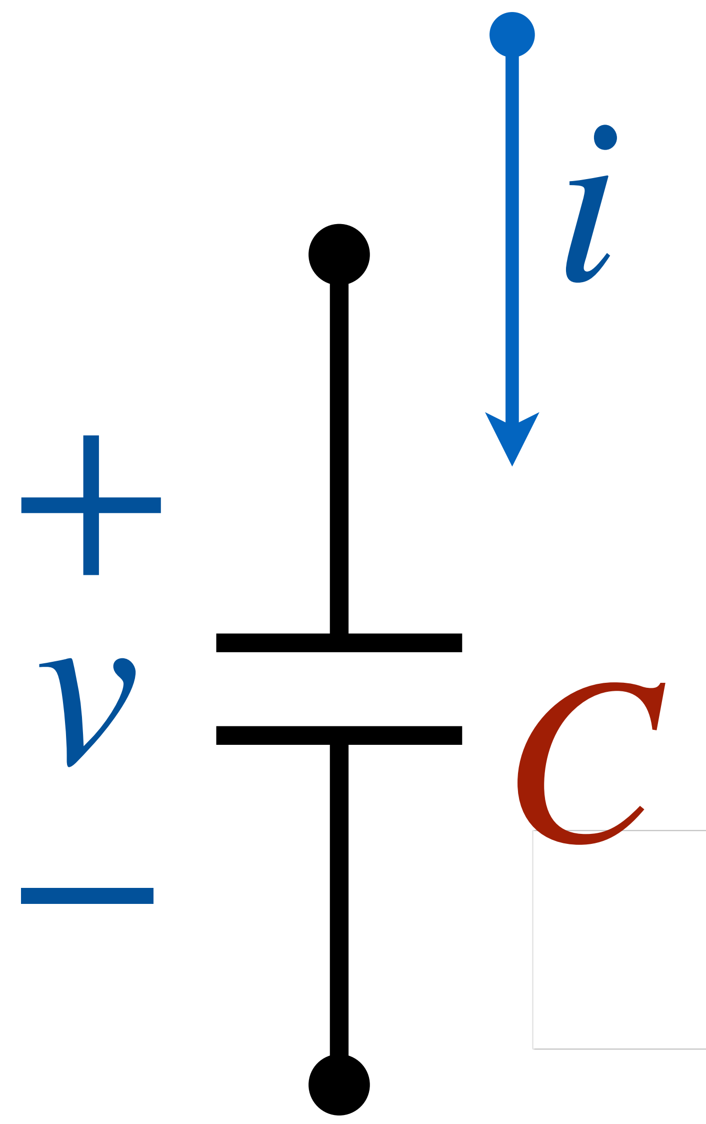
$$i_{test} = C_{eq}\frac{dV_{test}}{dt} = C_{eq}\frac{dV_1}{dt} + C_{eq}\frac{dV_2}{dt}$$

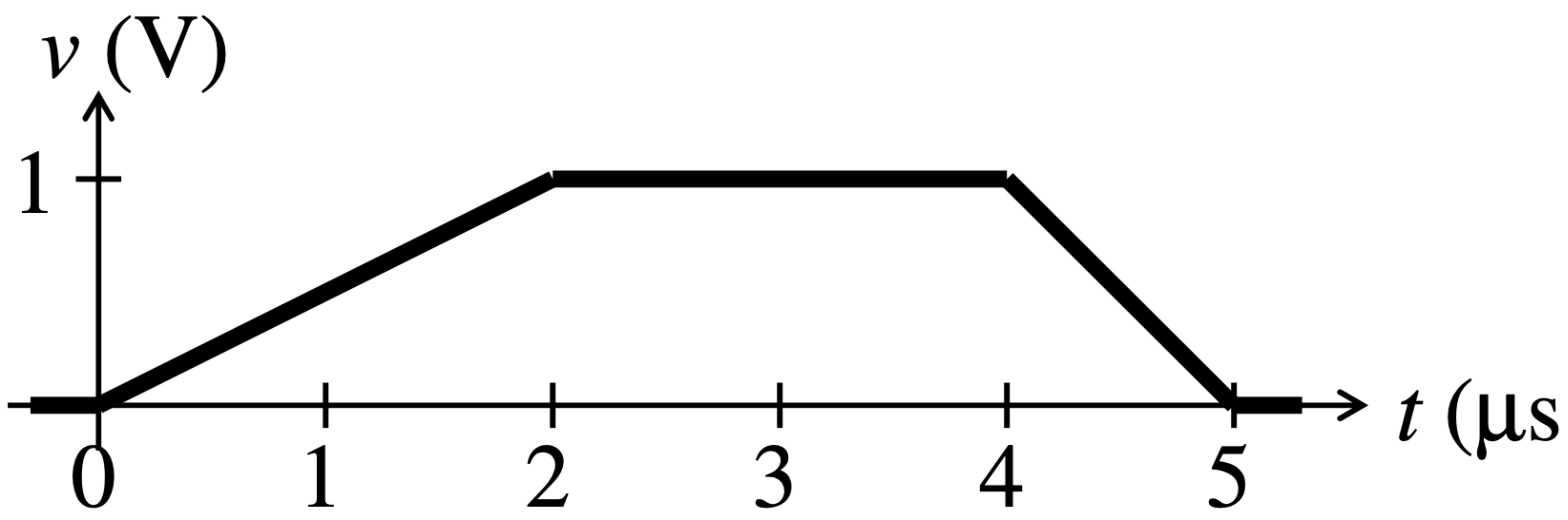
$$i_{test} = C_{eq}\frac{i_{test}}{C_1} + C_{eq}\frac{i_{test}}{C_2}$$

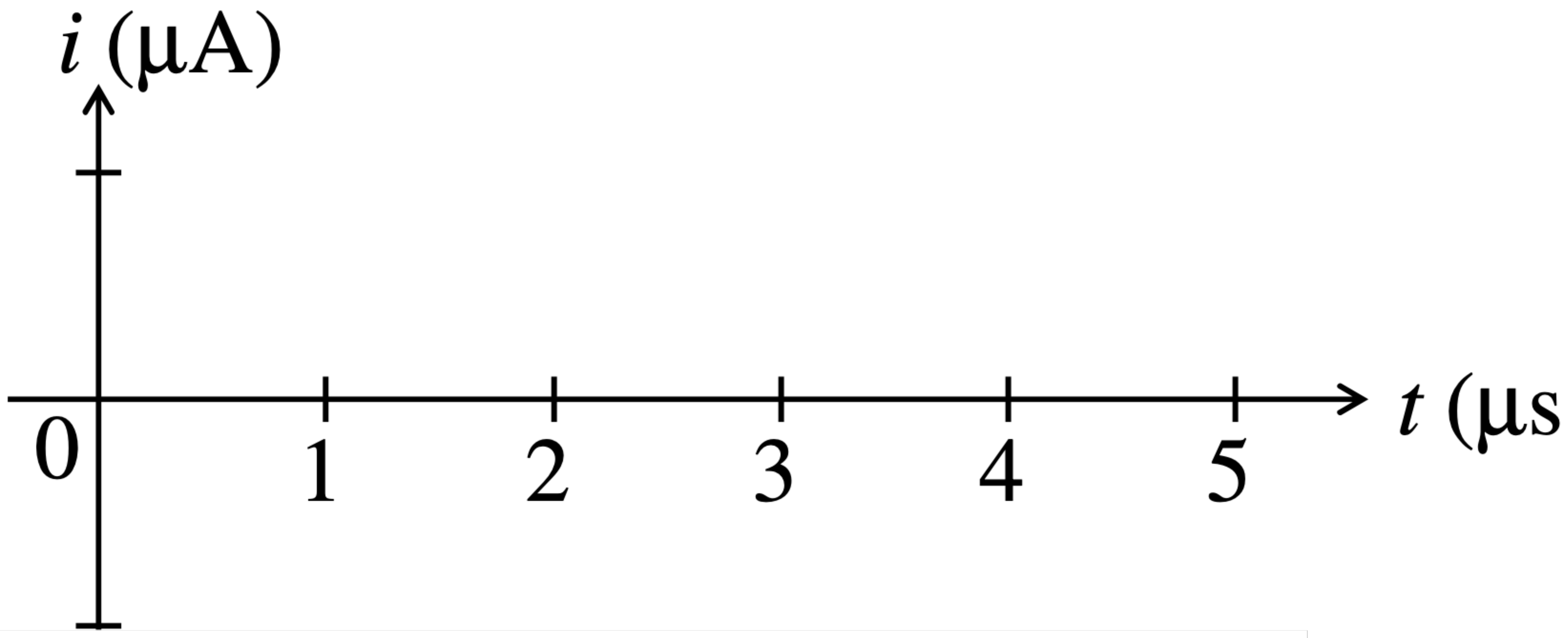
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

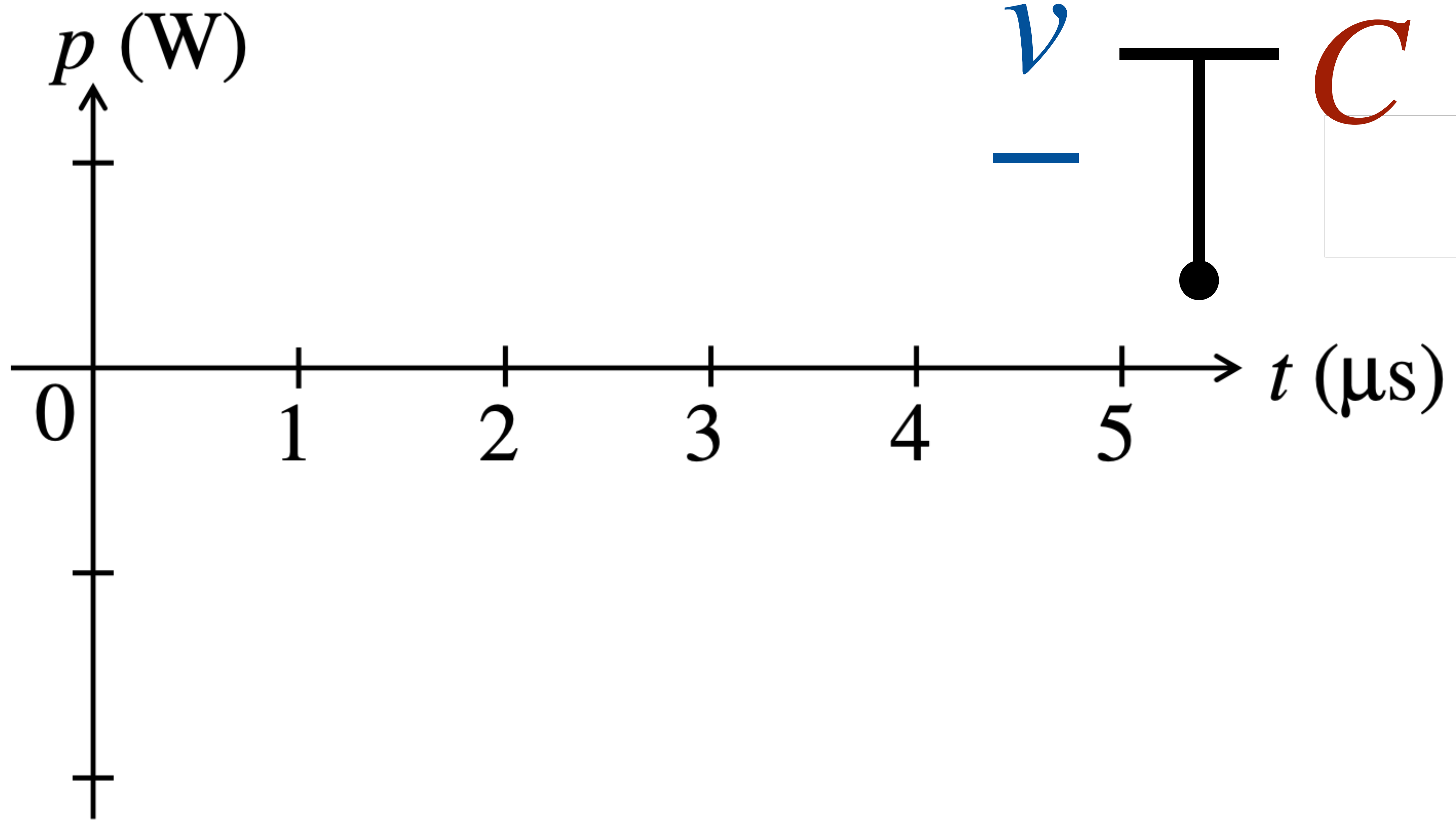
EECS16A Review

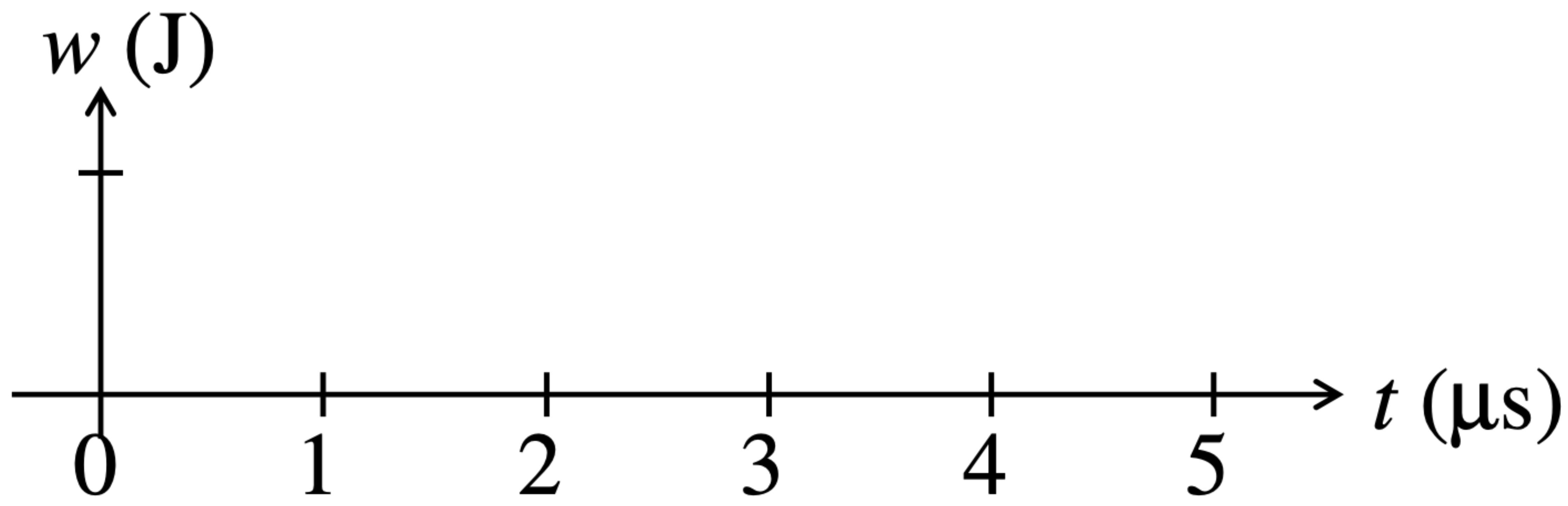
$$i = C\frac{dv}{dt}$$



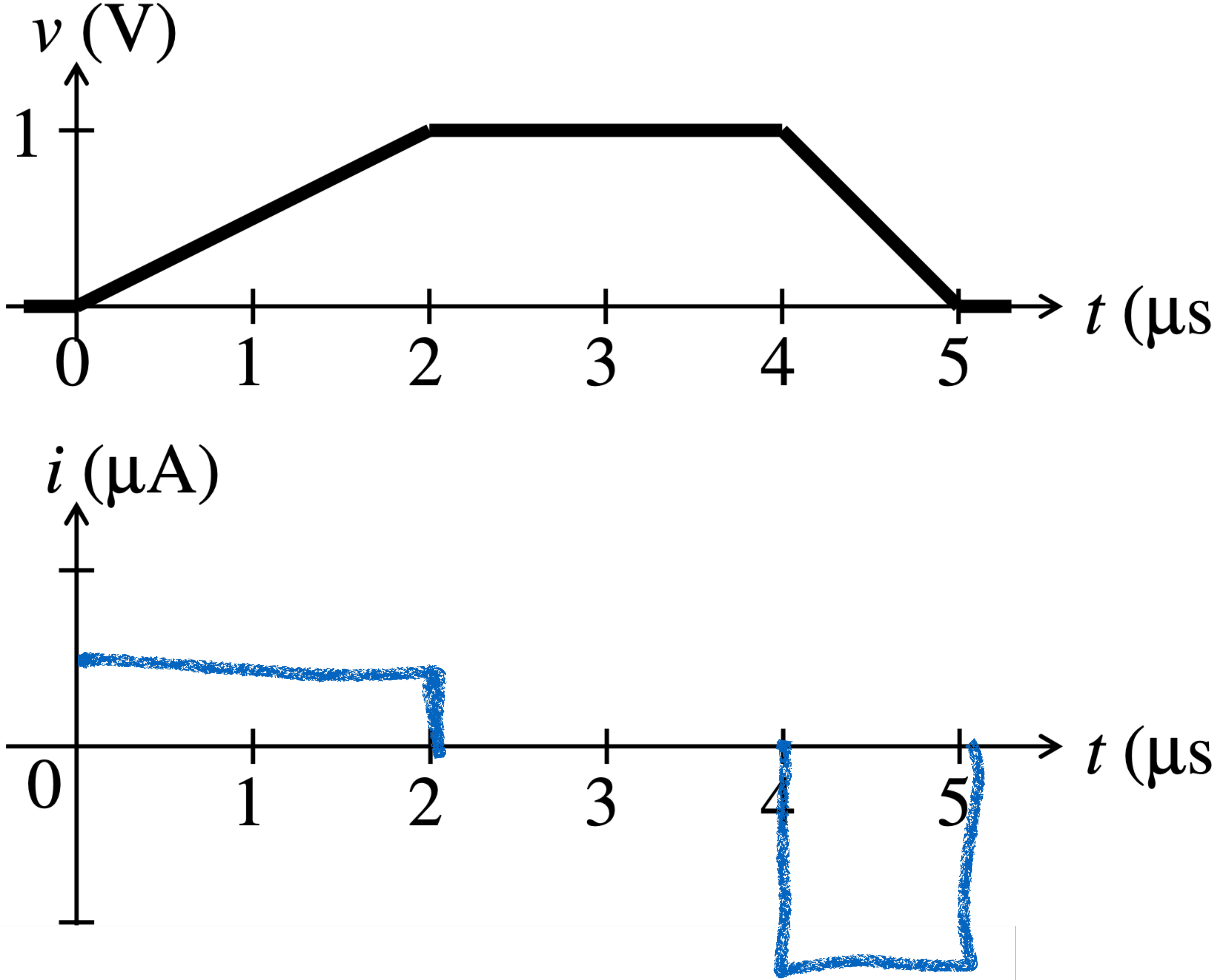


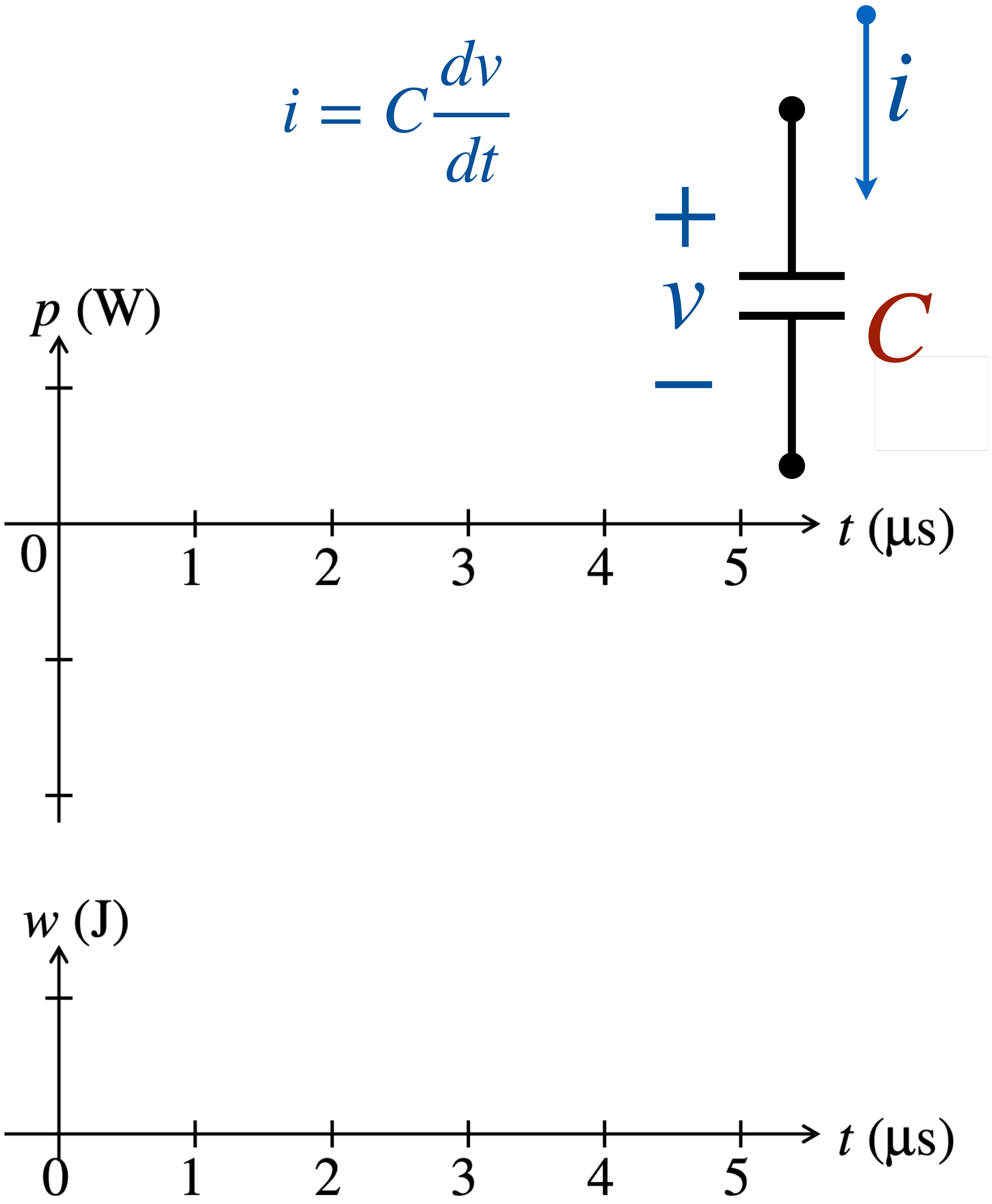




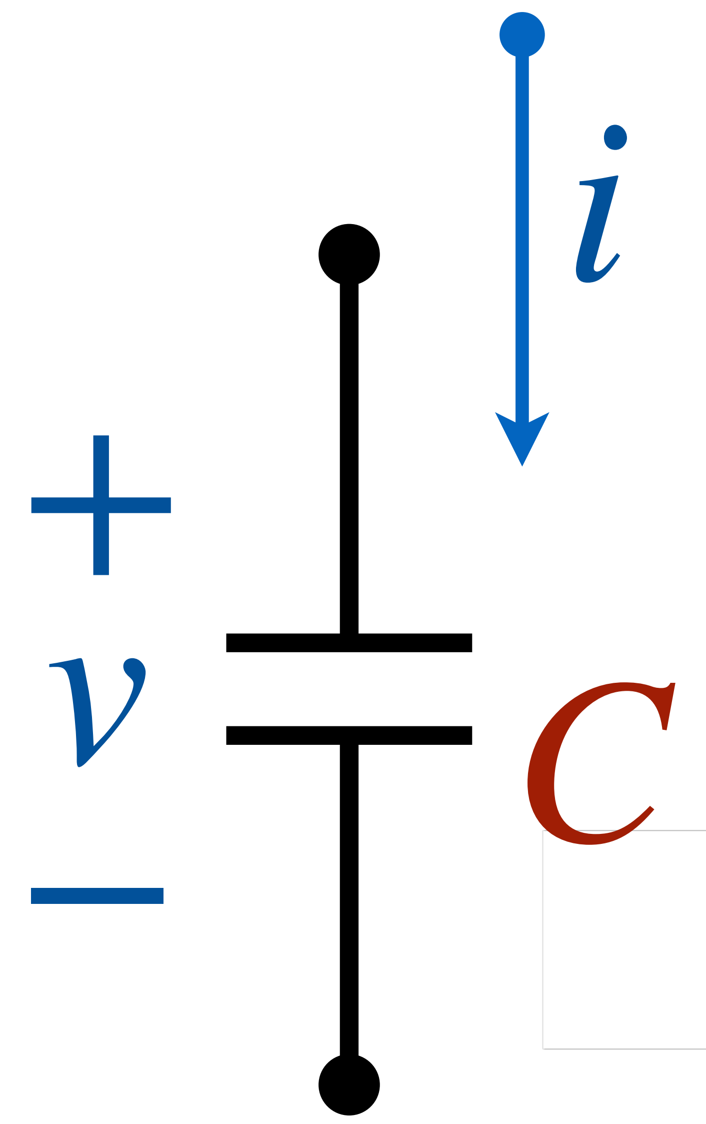


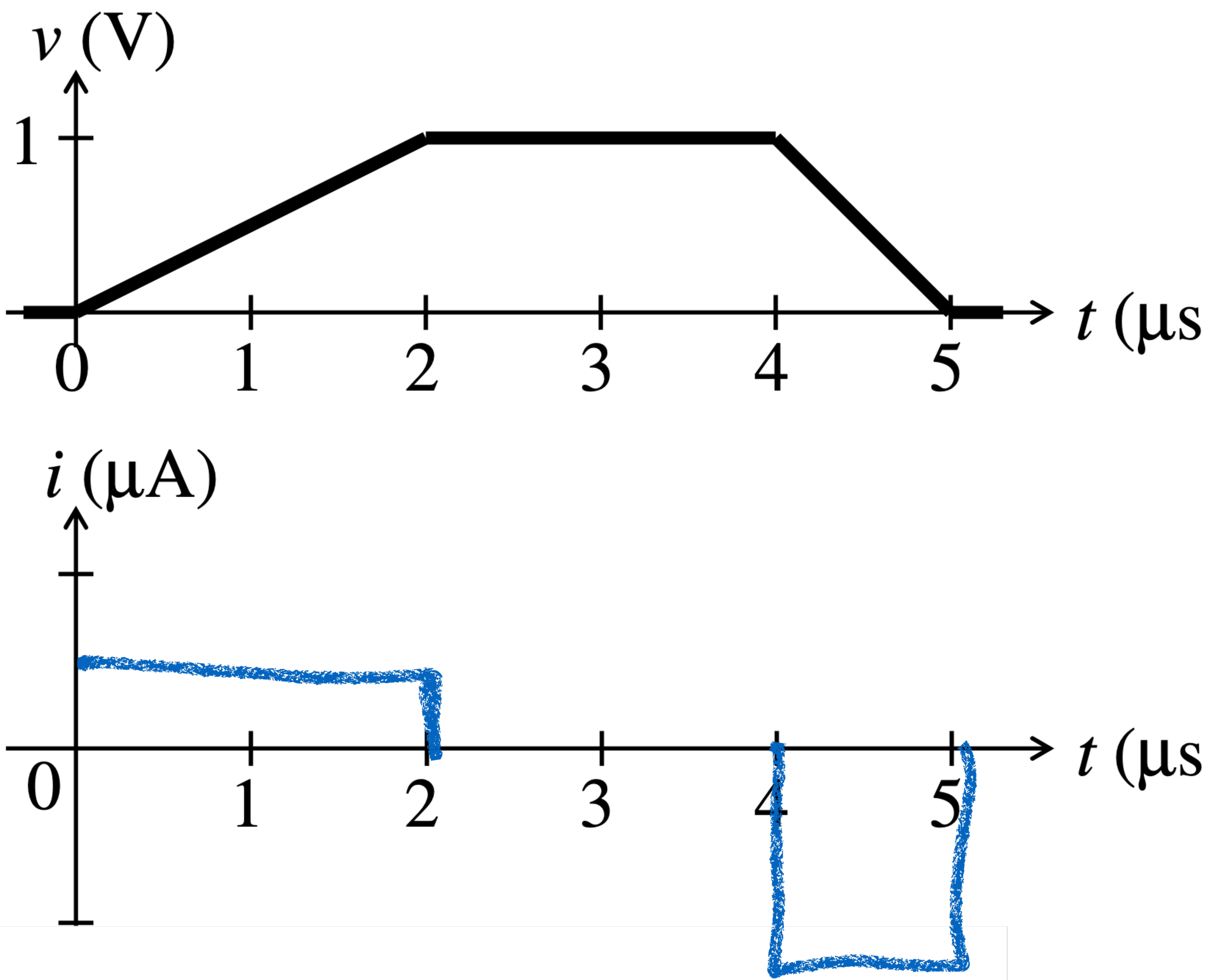
EECS16A Review

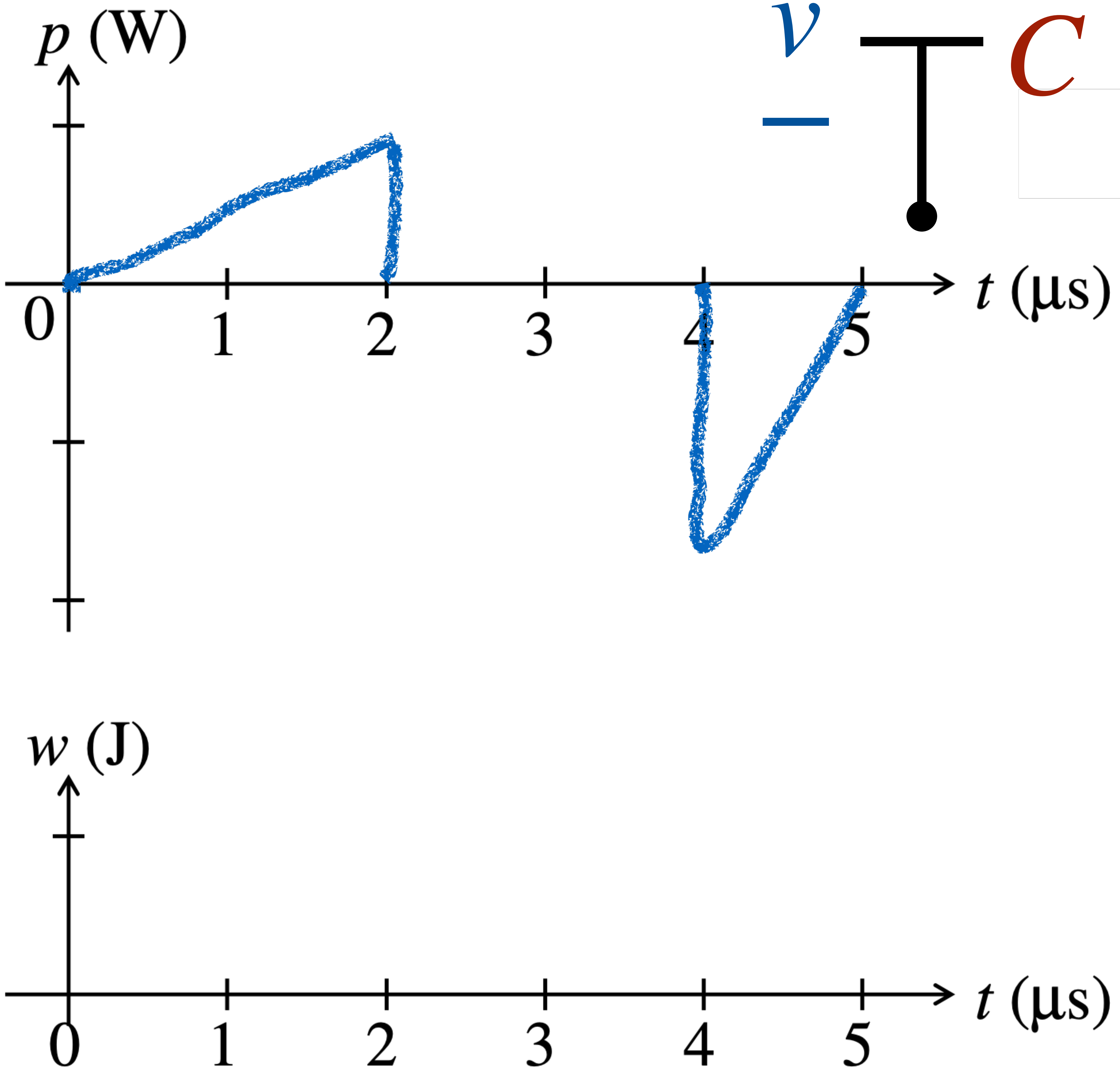




$$i = C\frac{dv}{dt}$$







EECS16A Review

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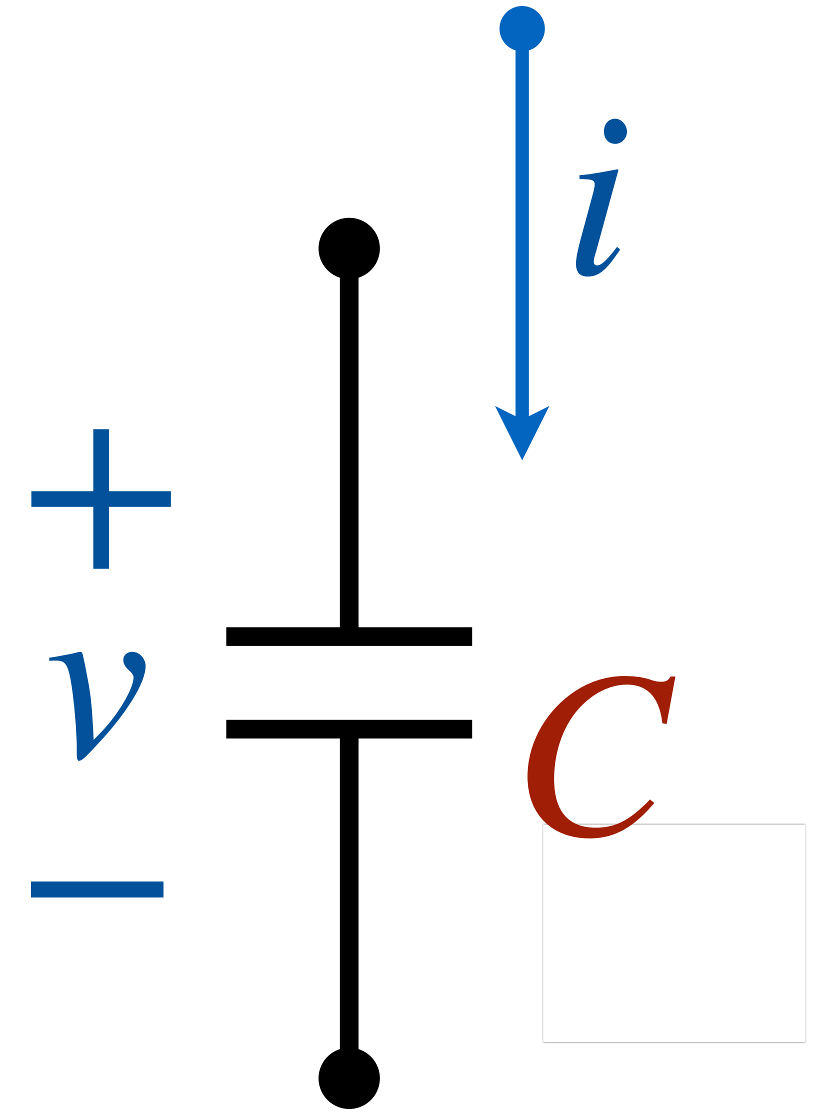
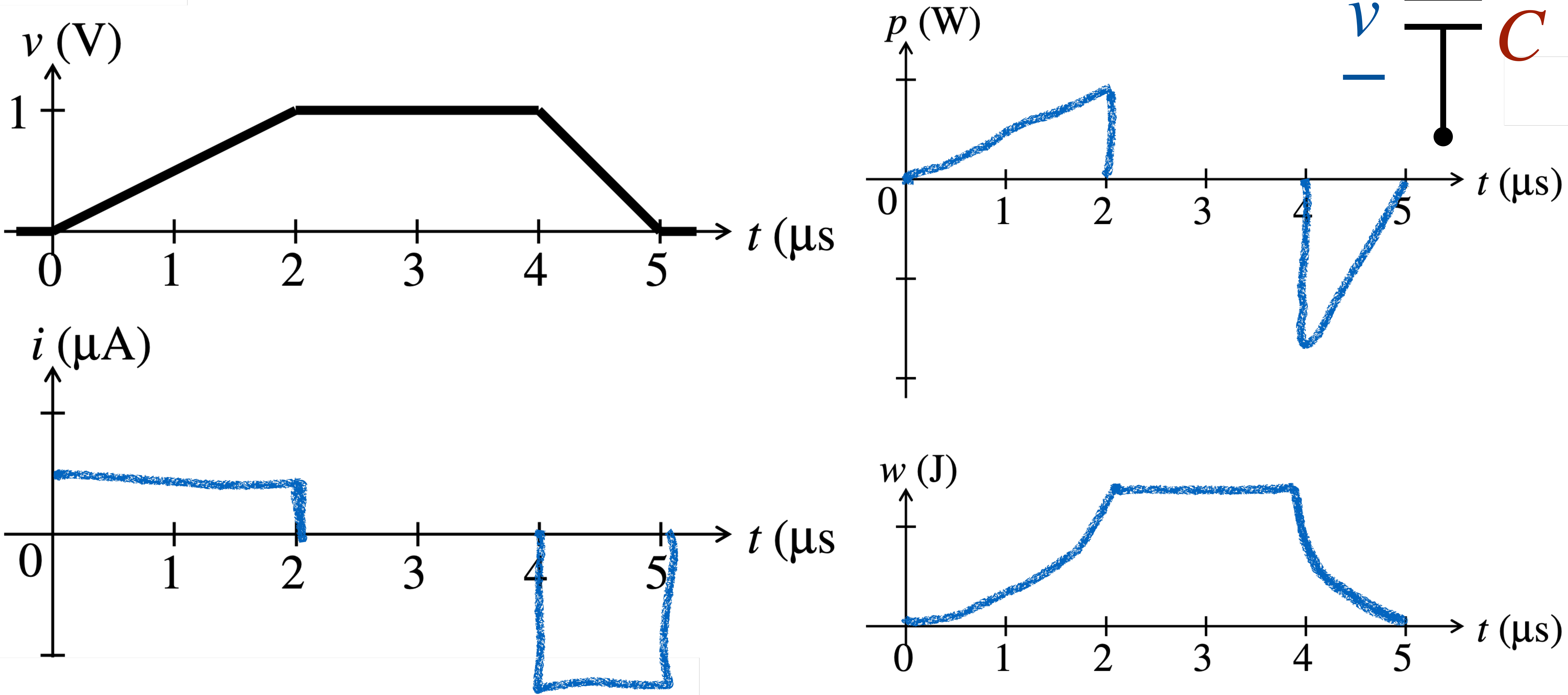
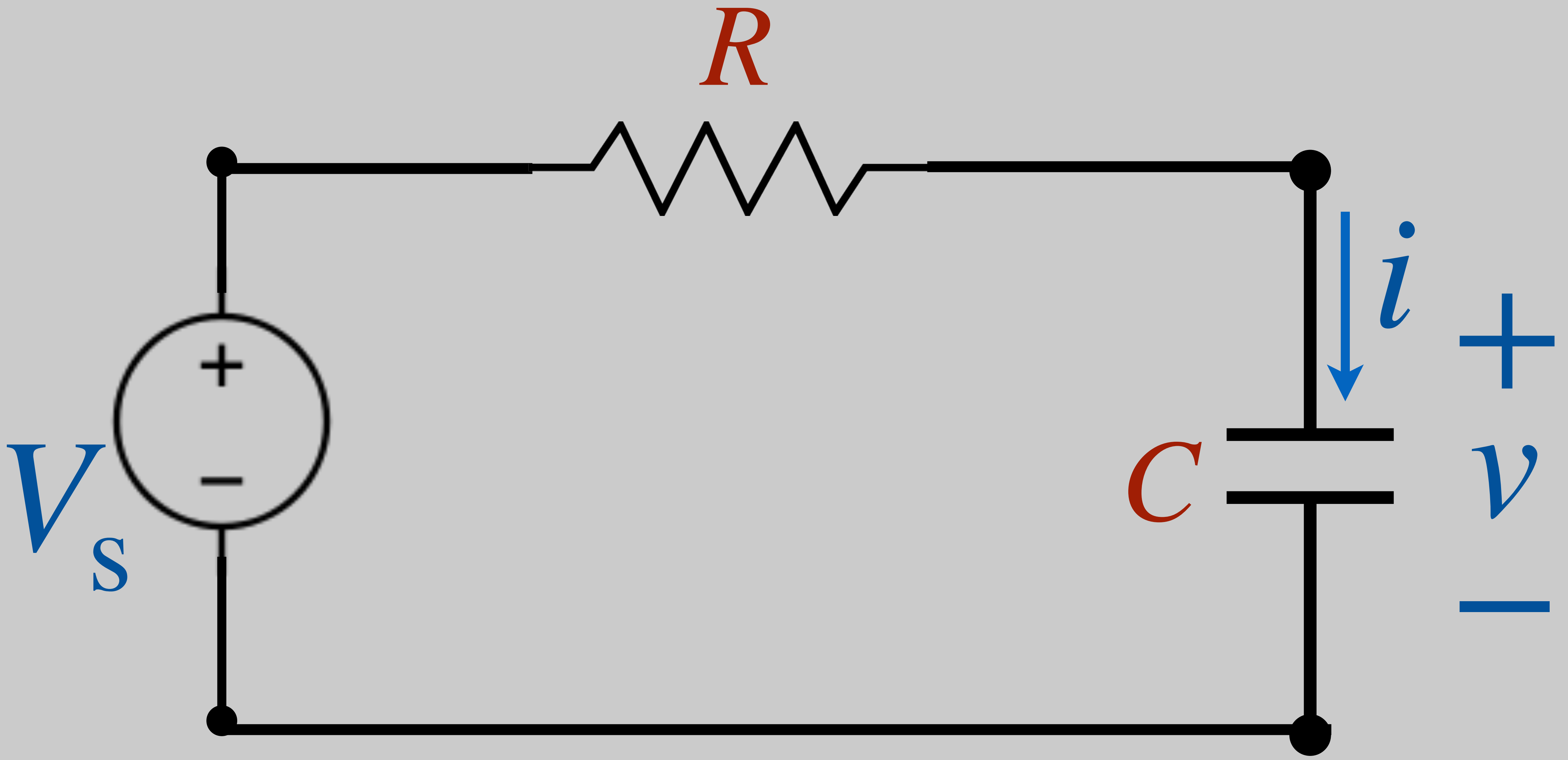
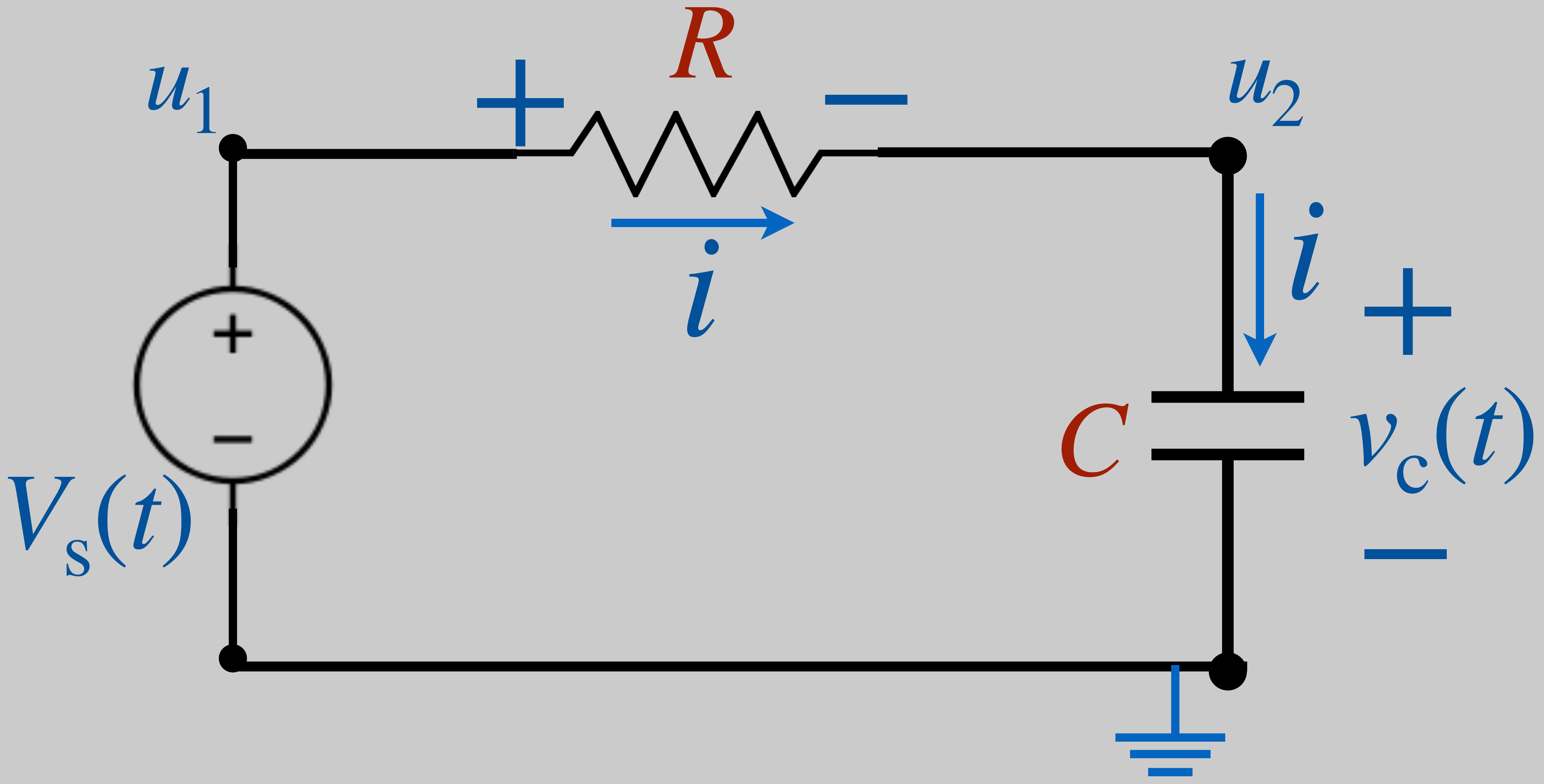




Transient Response



RC Circuits



$$u_1 = V_s$$

$$u_2 = v_c$$

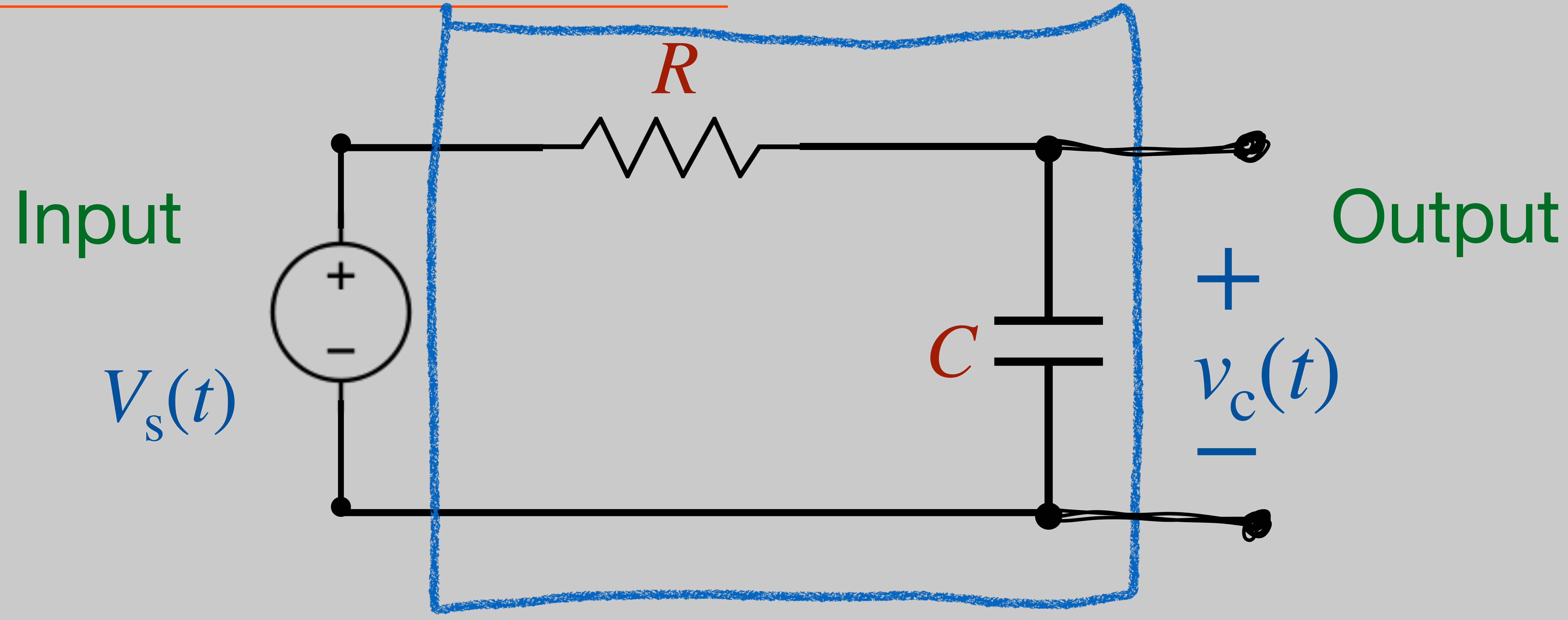
$$V_s - v_c = iR$$

$$i = C\frac{dv_c}{dt}$$

$$V_s - v_c = C\frac{dv_c}{dt}R$$

$$v_c + RC\frac{dv_c}{dt} = V_s \quad \Rightarrow \frac{dv_c}{dt} + \frac{1}{RC}v_c = \frac{1}{RC}V_s$$

RC Circuits



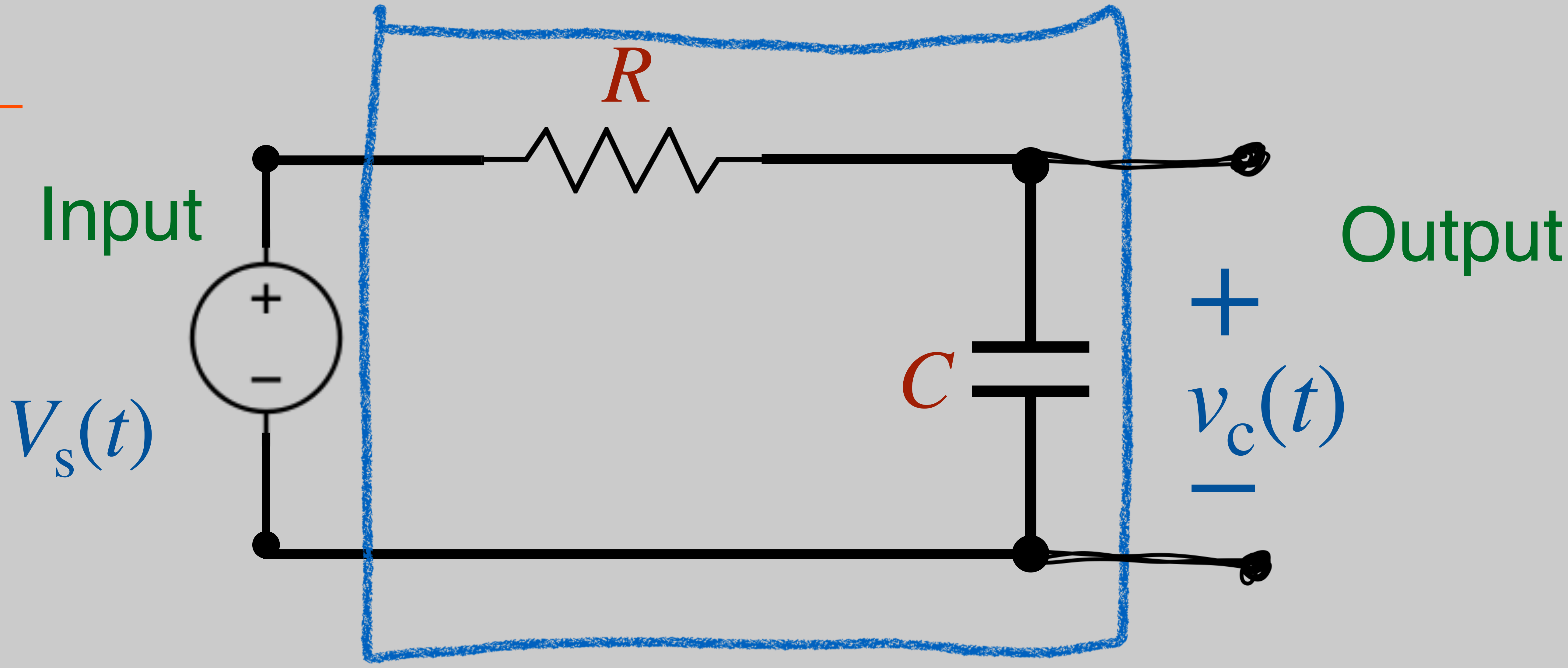
$$RC\frac{dv_c}{dt}+v_c = V_s$$

Constant coefficients
1st order diff. Eq.

$$\frac{dv_c(t)}{dt}+\frac{1}{RC}v_c(t)=\frac{1}{RC}V_s(t)$$

RC Circuits - Steady State

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_s(t)$$



Example 1: Steady state

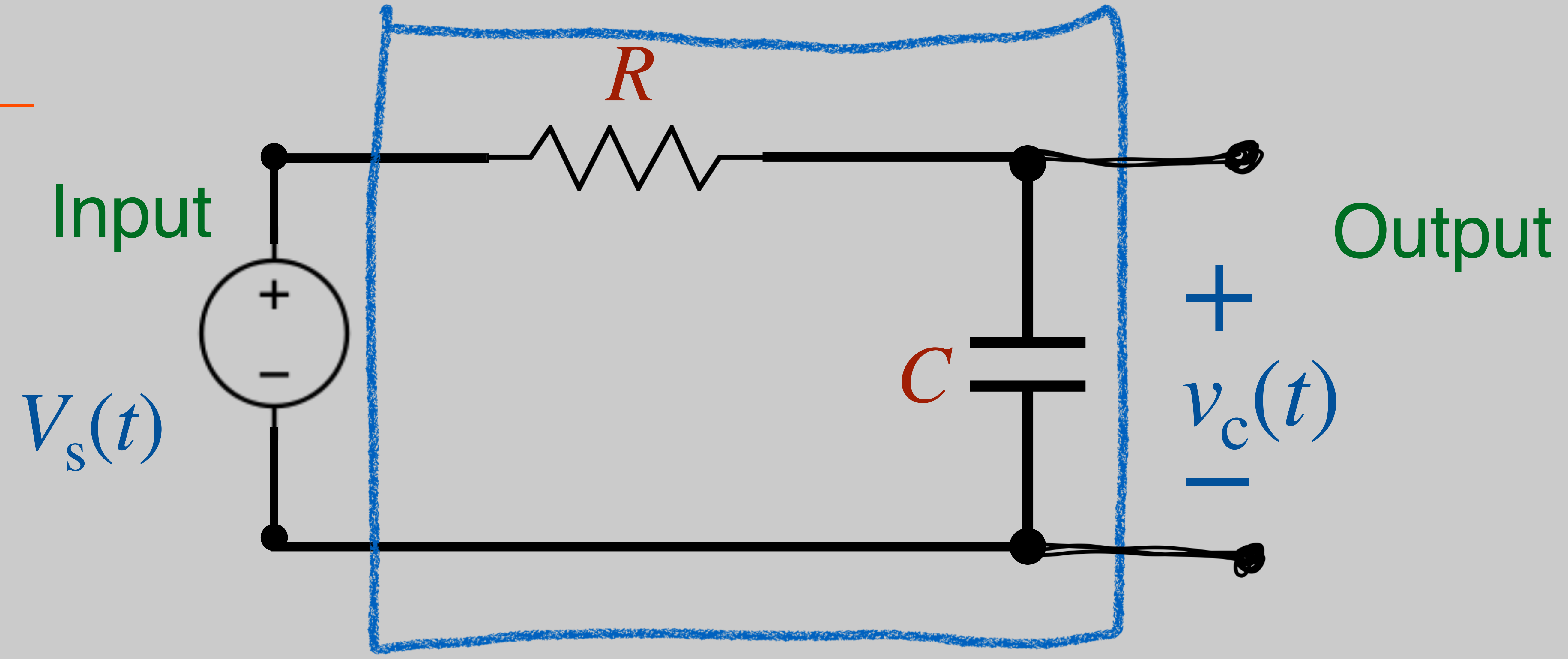
Set $V_s(t) = V_0$

$$\frac{dv_c}{dt} = 0$$

$$0 + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_0 \Rightarrow v_c(t) = V_0$$

Natural Response of RC circuits

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_s(t)$$



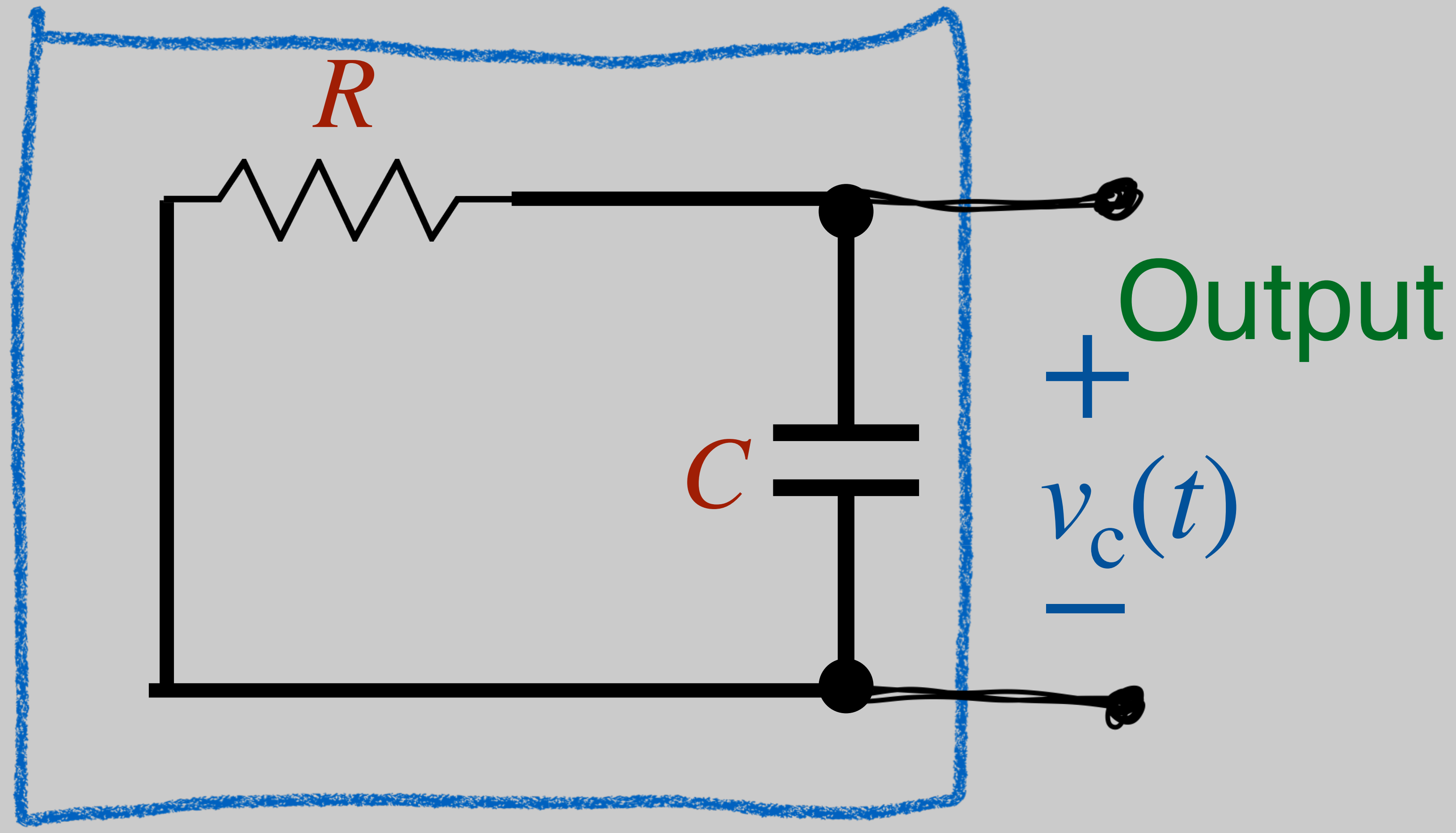
Example 2: Step-down

$V_s(t) = 0 \quad | t > 0$

$v_c(0) = V_0$

$$\Rightarrow \frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = 0$$

Homogeneous differential equation



$$V_s(t) = 0 \quad | t > 0$$

$$v_c(0) = V_0$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = 0$$

Approach:

Guess a solution: $v_c(t) = Ae^{bt}$

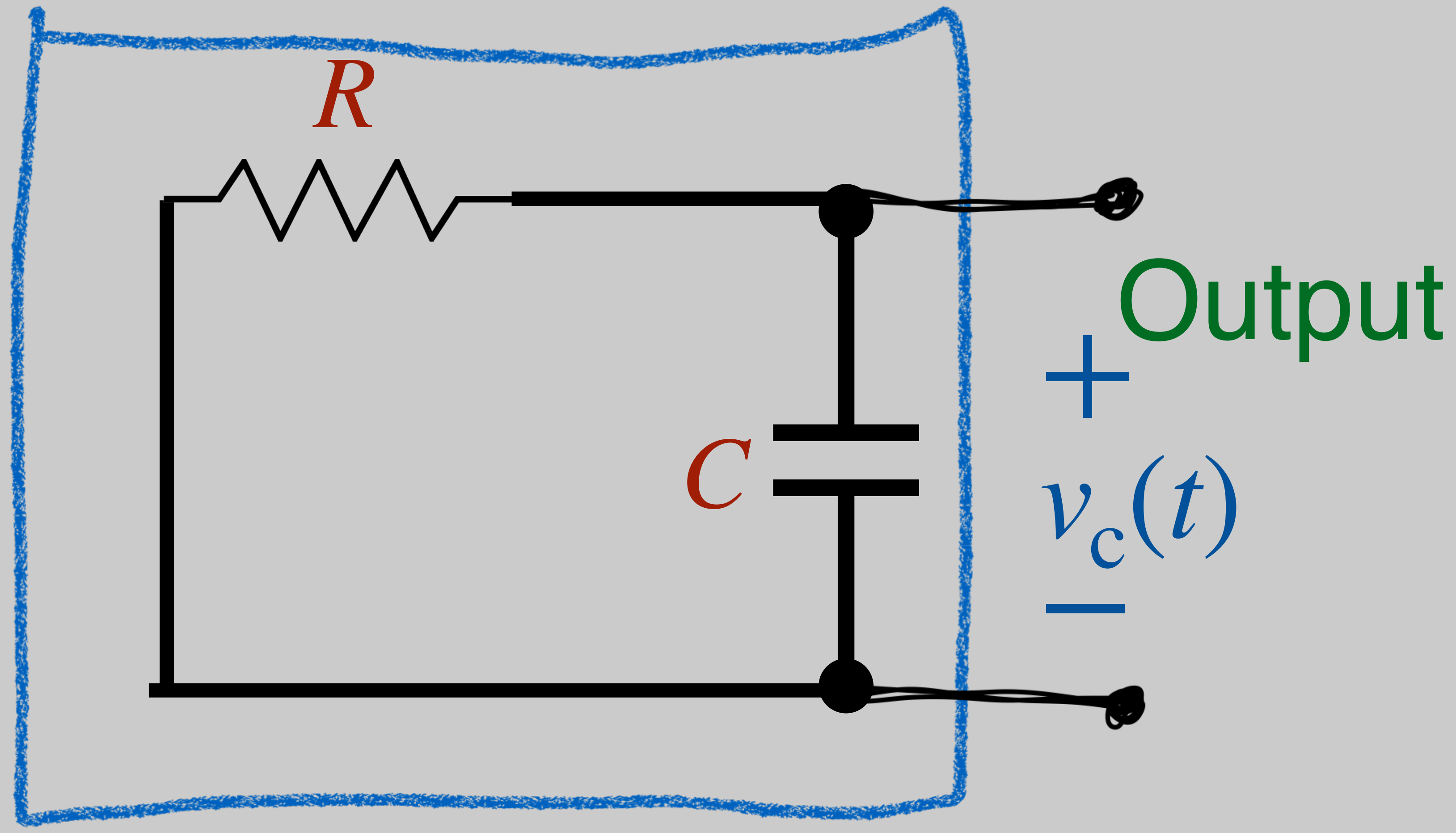
$$A \cdot be^{bt} + \frac{1}{RC} Ae^{bt} = 0 \quad \Rightarrow b = -\frac{1}{RC}$$

Find coeff from initial conditions:

$$v_c(0) = Ae^0 = A = V_0 \quad \Rightarrow A = V_0$$

Natural Response of RC circuits

Homogeneous differential equation



$V_s(t) = 0 \quad | t > 0$

$v_c(0) = V_0$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = 0$$

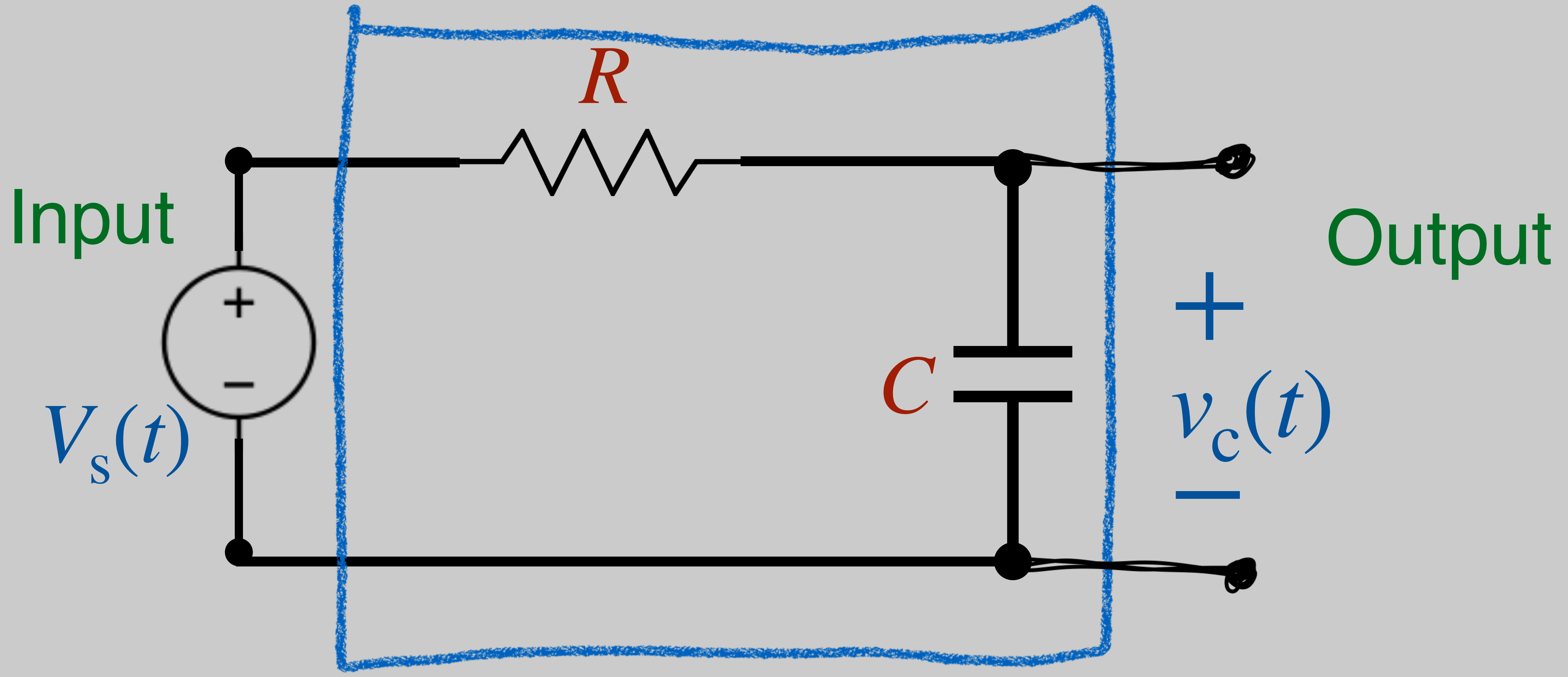
Solution: $v_c(t) = V_0 e^{-\frac{t}{RC}}$

$$RC \rightarrow [\Omega][F] = \frac{[V]}{[A]}\frac{[C]}{[V]} = \frac{[C]}{[A]} = \frac{[C]}{[C]/[s]} = [s]$$

RC is discharge time-constant!

Step Response of RC circuits

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_s(t)$$



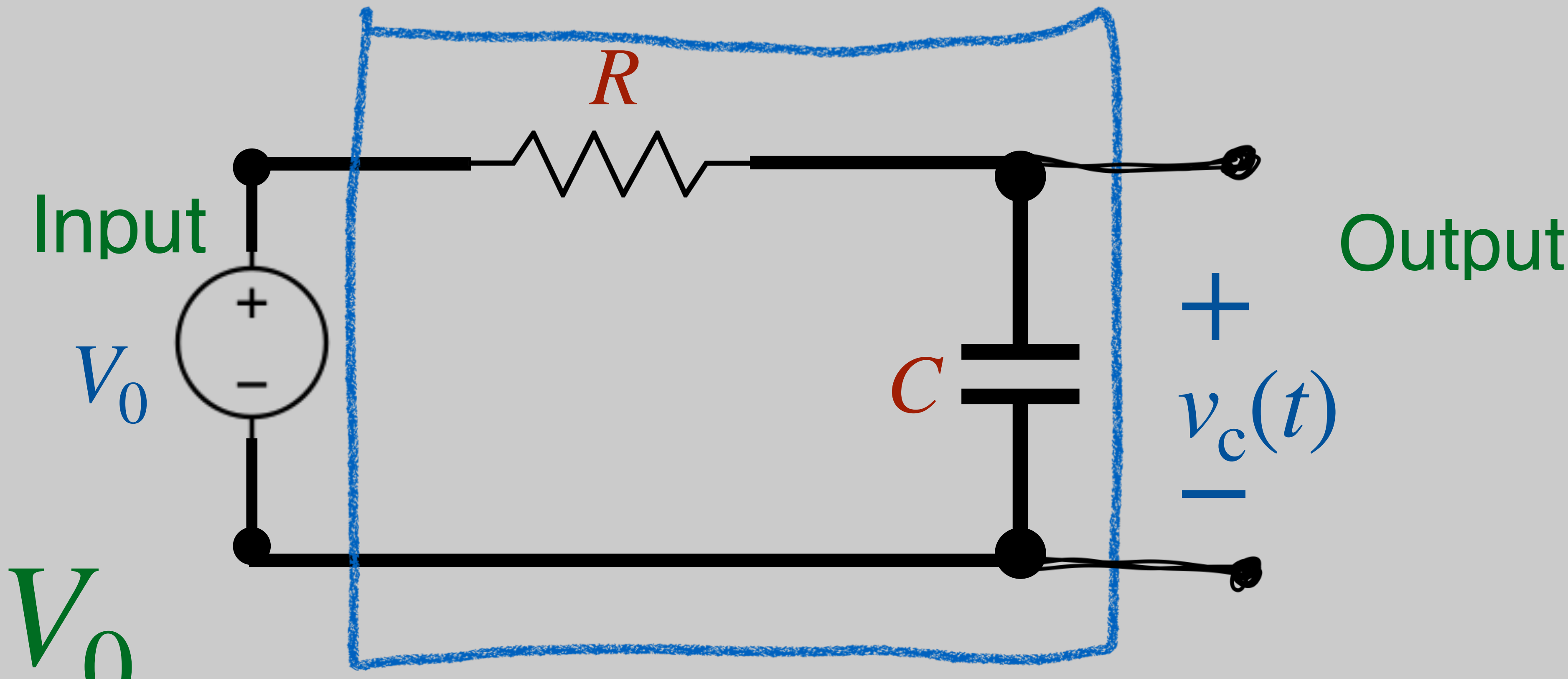
Example 3: Step-Response

$V_s(t) = V_0 \quad | t > t_0$

$v_c(t_0) = 0$

$$\Rightarrow \frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}V_0$$

Non-homogeneous Diff. Eq.



$V_s(t) = V_0 \quad | t > t_0$

$v_c(t_0) = 0$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} V_0$$

constant

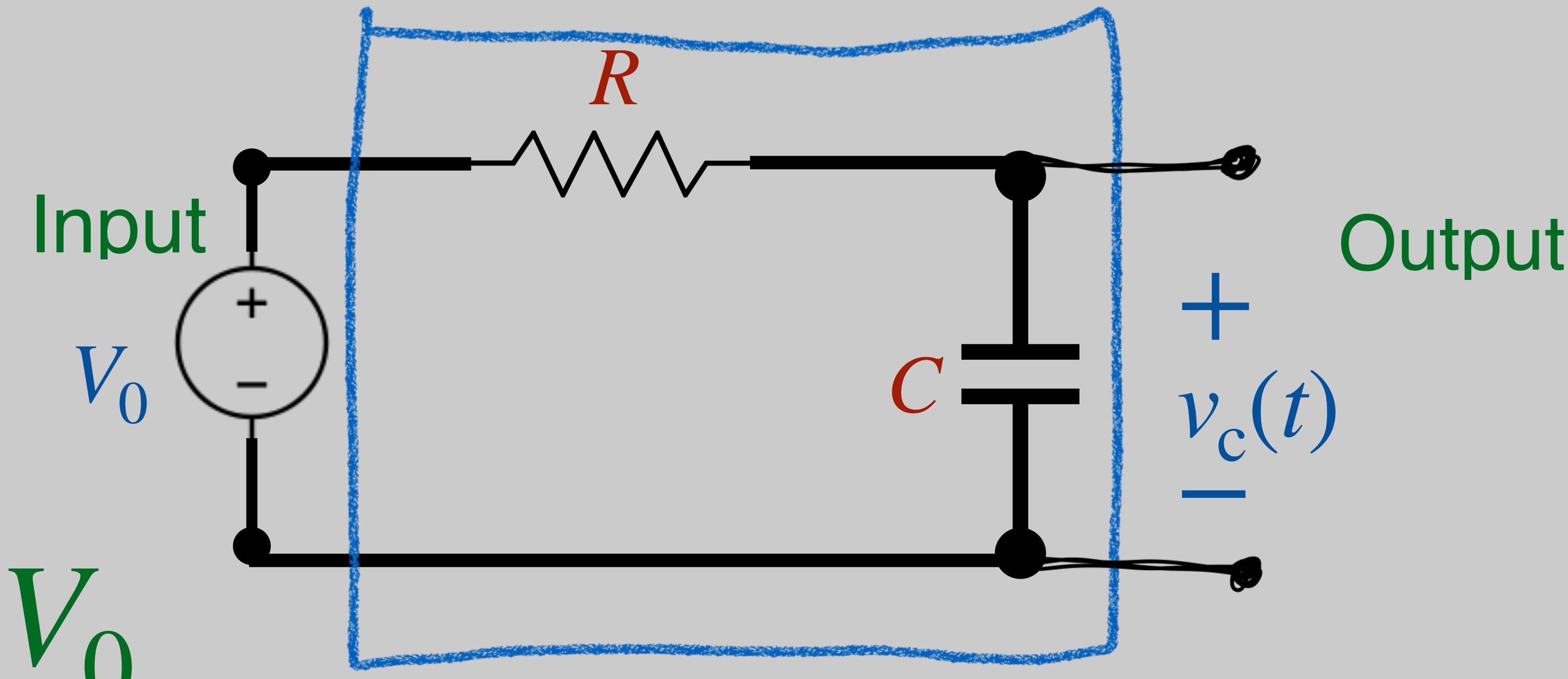
Approach:

From Note 1: $v_c(t) = v_h(t) + v_p(t)$

Homogeneous   Particular

Guess: $v_c = \quad Ke^{-\frac{t}{RC}} \quad + \quad B$

Non-homogeneous Diff. Eq.



$V_s(t) = V_0 \quad | t > t_0$

$v_c(t_0) = 0$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} V_0$$

constant

Approach:

From Note 1: $v_c(t) = v_h(t) + v_p(t)$

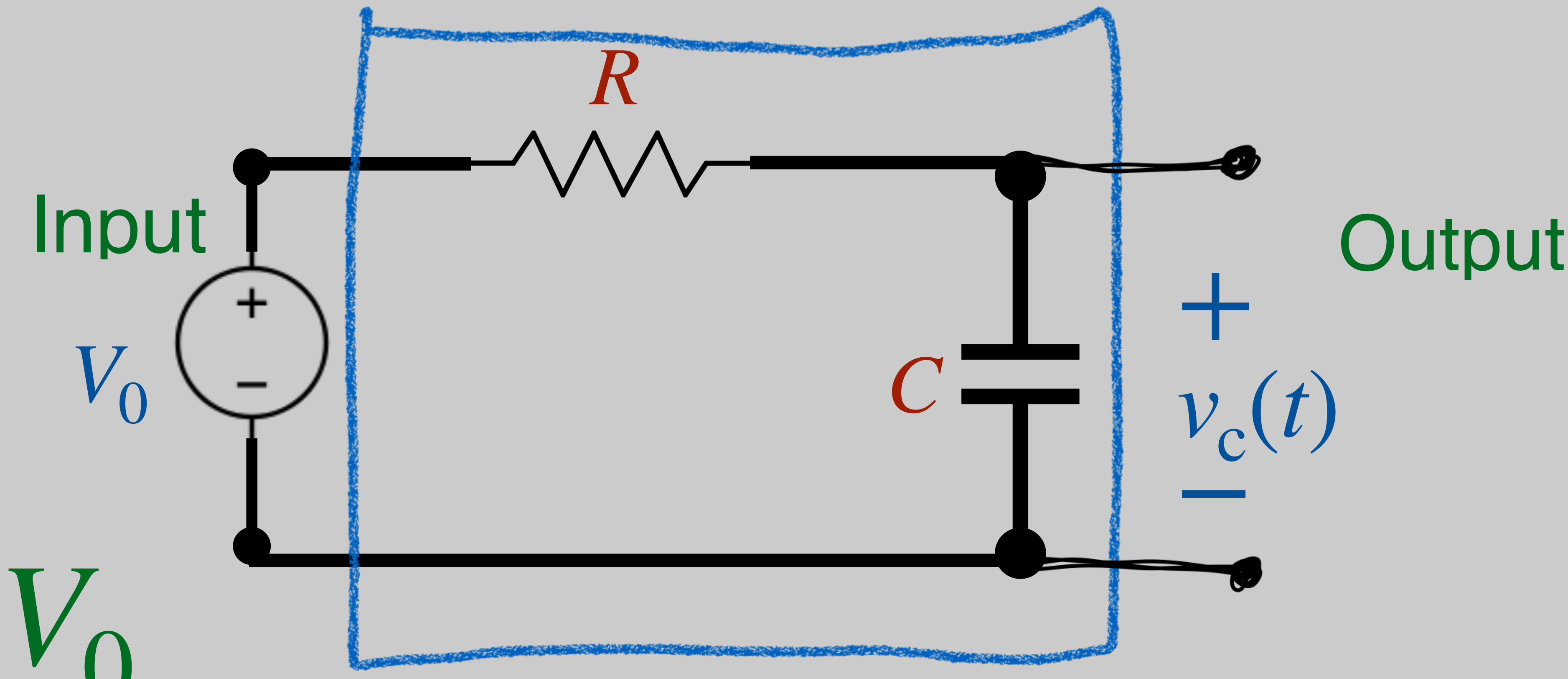
Homogeneous   Particular

Guess: $v_c = Ke^{-\frac{t}{RC}} + B$

Plug in: $\frac{d}{dt}(v_h(t) + v_p(t)) + \frac{1}{RC}(v_h(t) + v_p(t)) = \frac{1}{RC} V_0$

$$-\frac{1}{RC} Ke^{-\frac{t}{RC}} + \frac{1}{RC} Ke^{-\frac{t}{RC}} + \frac{1}{RC} B = \frac{1}{RC} V_0 \quad \Rightarrow B = V_0$$

Non-homogeneous Diff. Eq.



$$V_s(t) = V_0 \quad | t > t_0$$

$$v_c(t_0) = 0$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} V_0$$

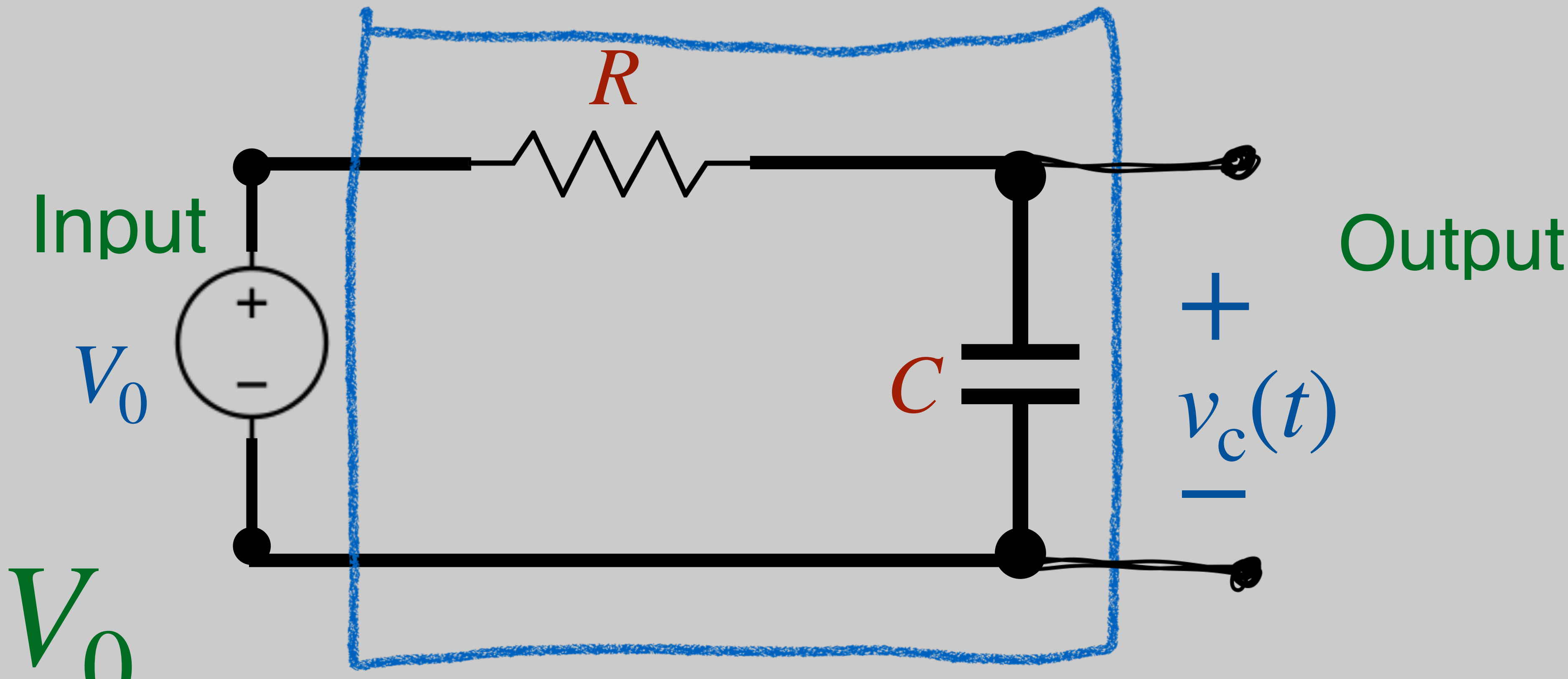
Initial conditions:

$$v_c = Ke^{-\frac{t}{RC}} + V_0$$

$$v_c(t_0) = Ke^{-\frac{t_0}{RC}} + V_0 = 0 \qquad \Rightarrow K = -V_0 \cdot e^{\frac{t_0}{RC}}$$

$$v_c = -V_0 \cdot e^{\frac{t_0}{RC}} \cdot e^{-\frac{t}{RC}} + V_0 = -V_0 \cdot e^{-\frac{t-t_0}{RC}} + V_0$$

$$= V_0 \left(1 - e^{-\frac{t-t_0}{RC}}\right)$$



Non-homogeneous Diff. Eq.

$$V_s(t) = V_0 \quad | t > t_0$$

$$v_c(t_0) = 0$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} V_0$$

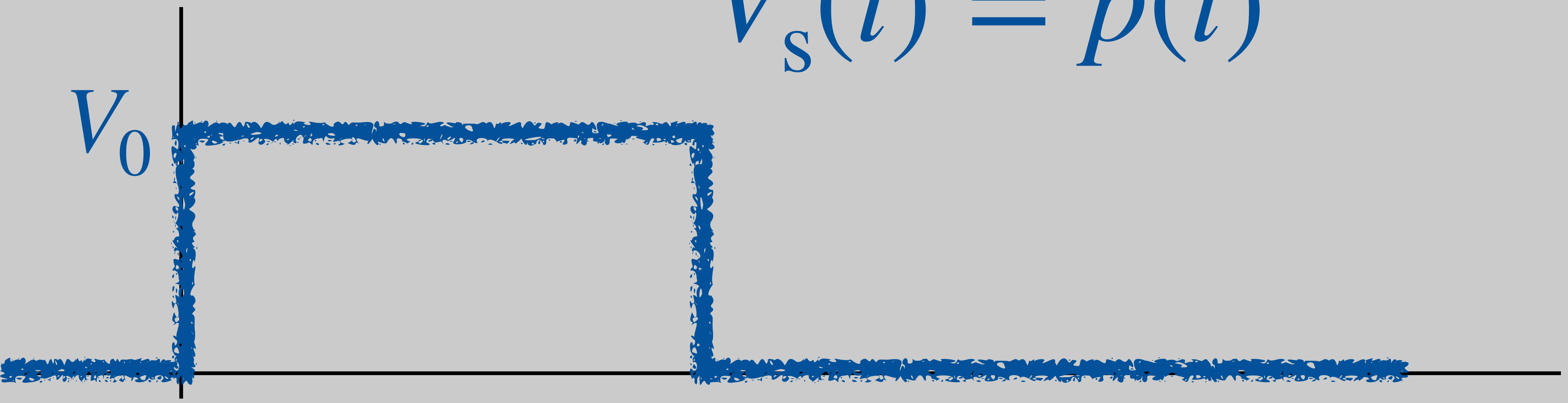
Solution:

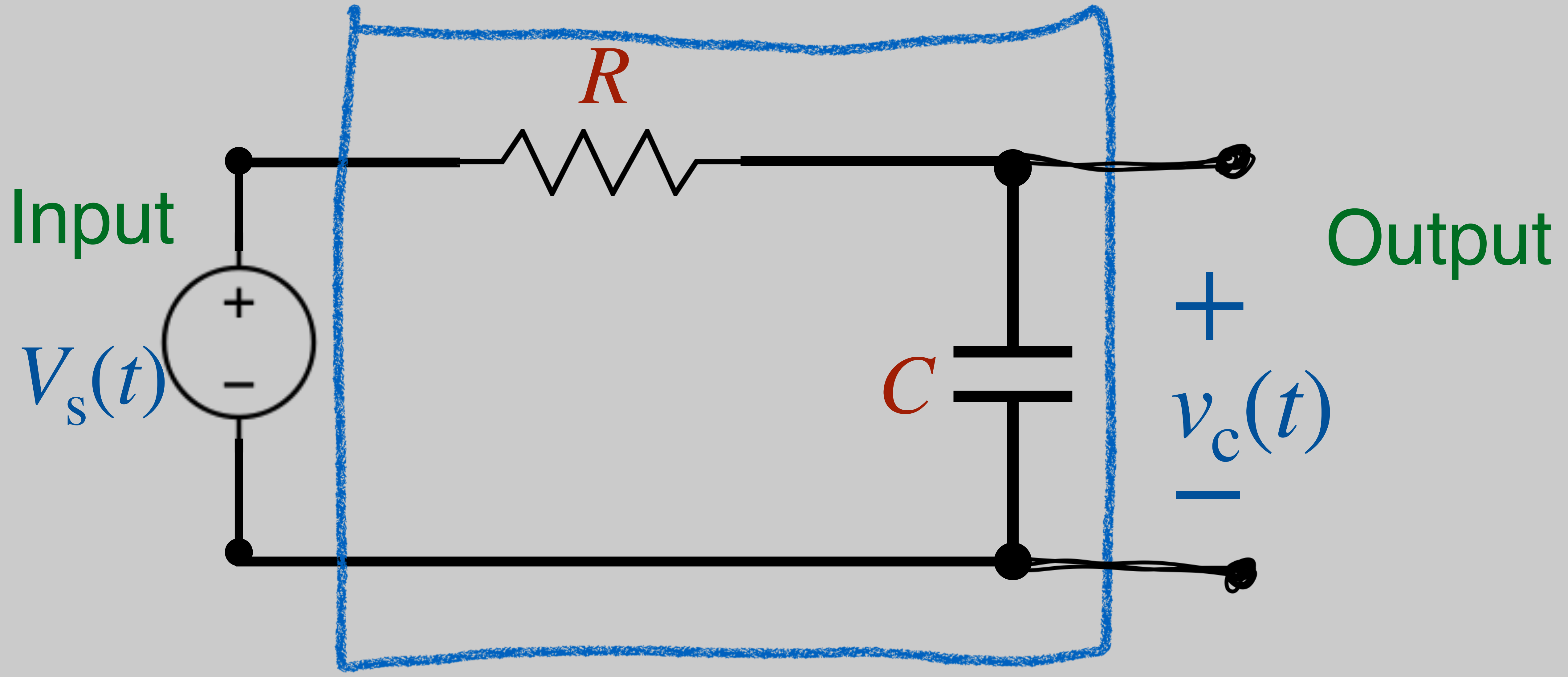
$$v_c = V_0 \left(1 - e^{-\frac{t - t_0}{RC}}\right)$$

Pulse Response of RC circuits

Example 4: Pulse Response:

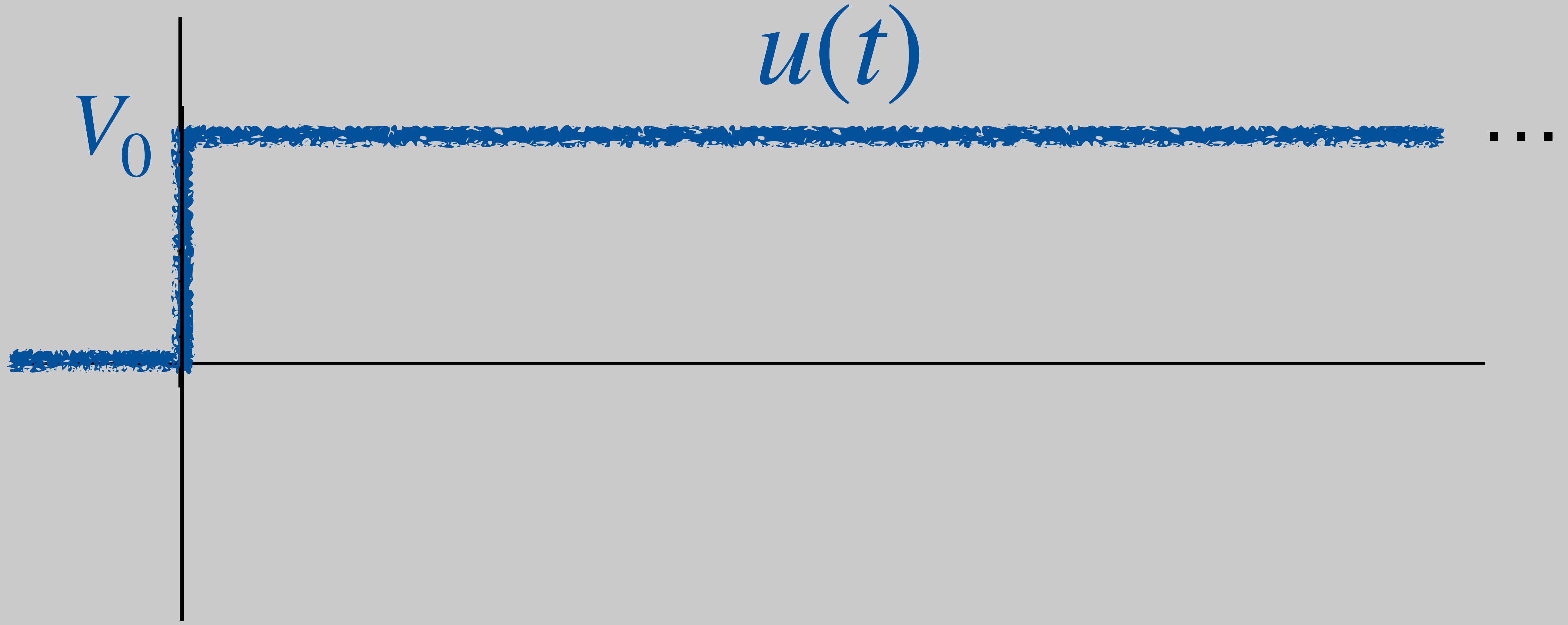
$V_s(t) = p(t)$





Q: approach? A: Super-position!

$u(t)$



Pulse Response of RC circuits

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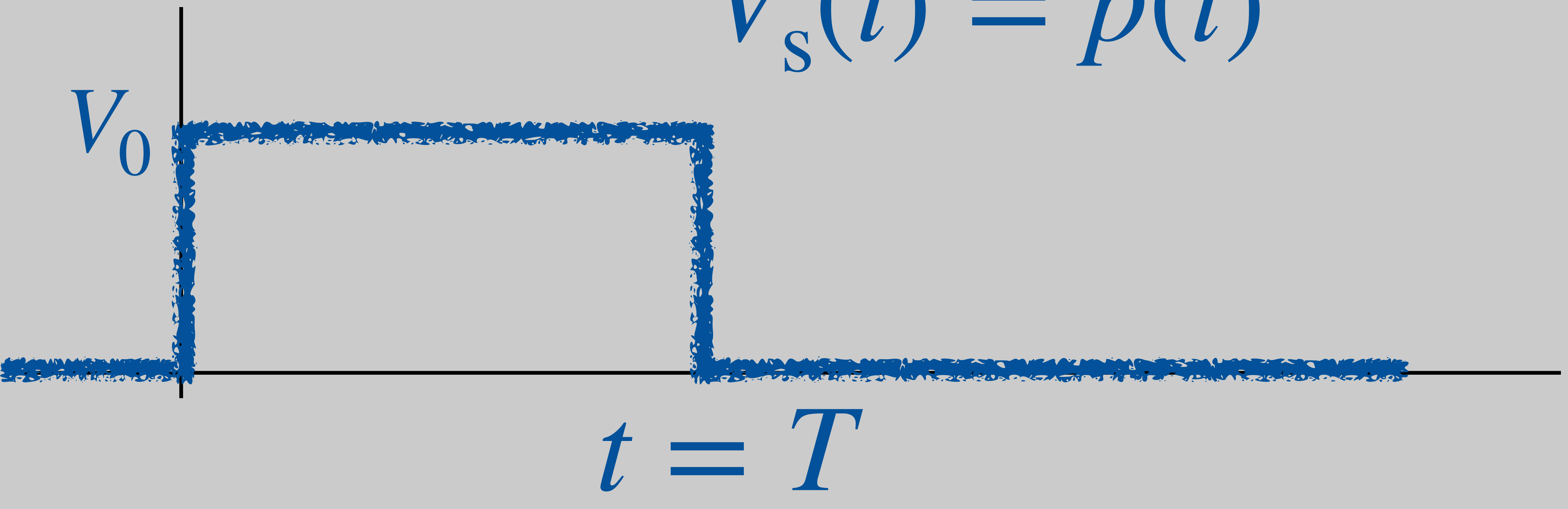

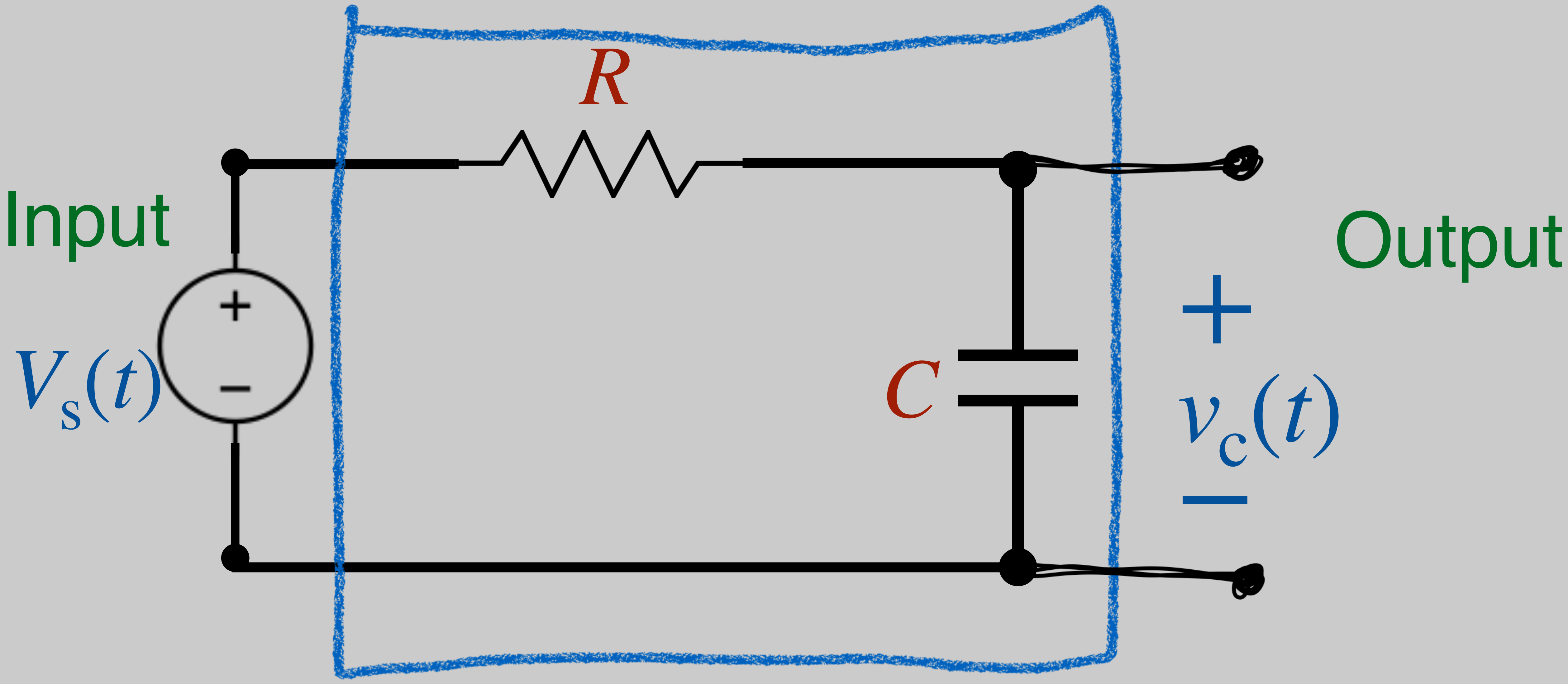

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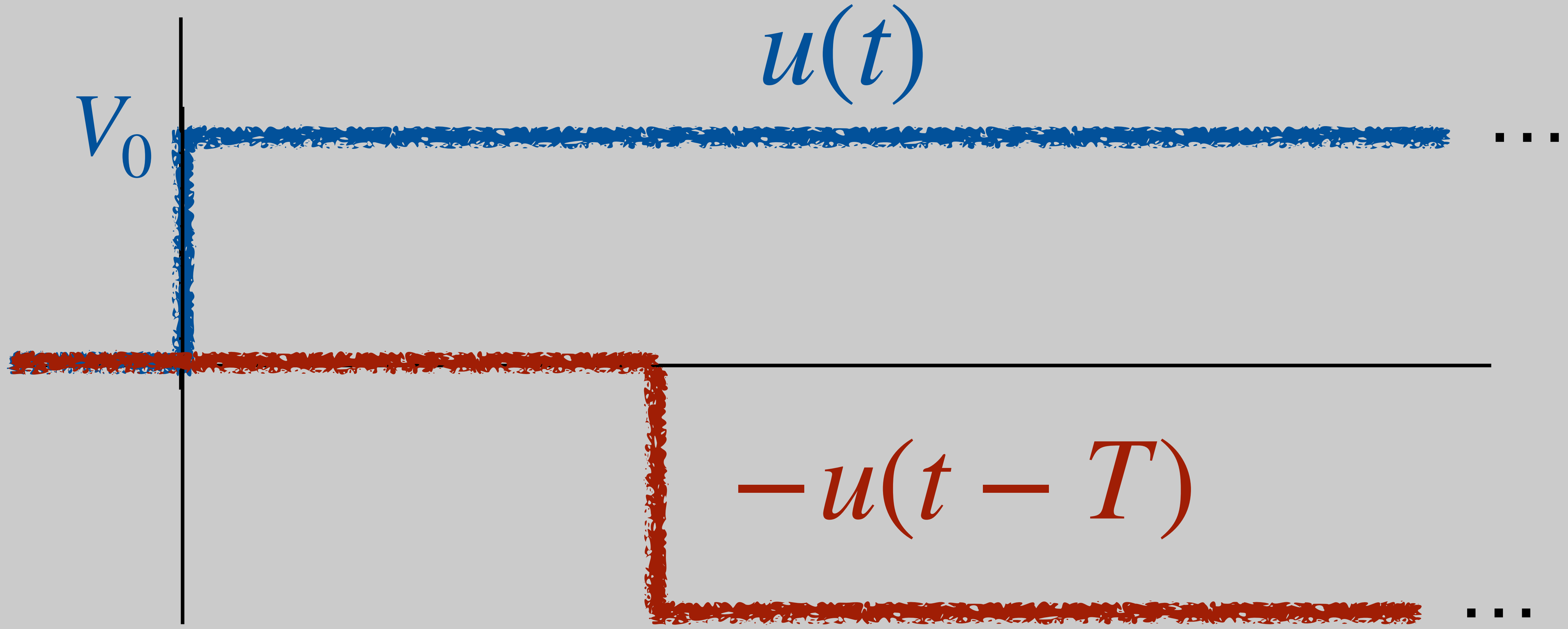
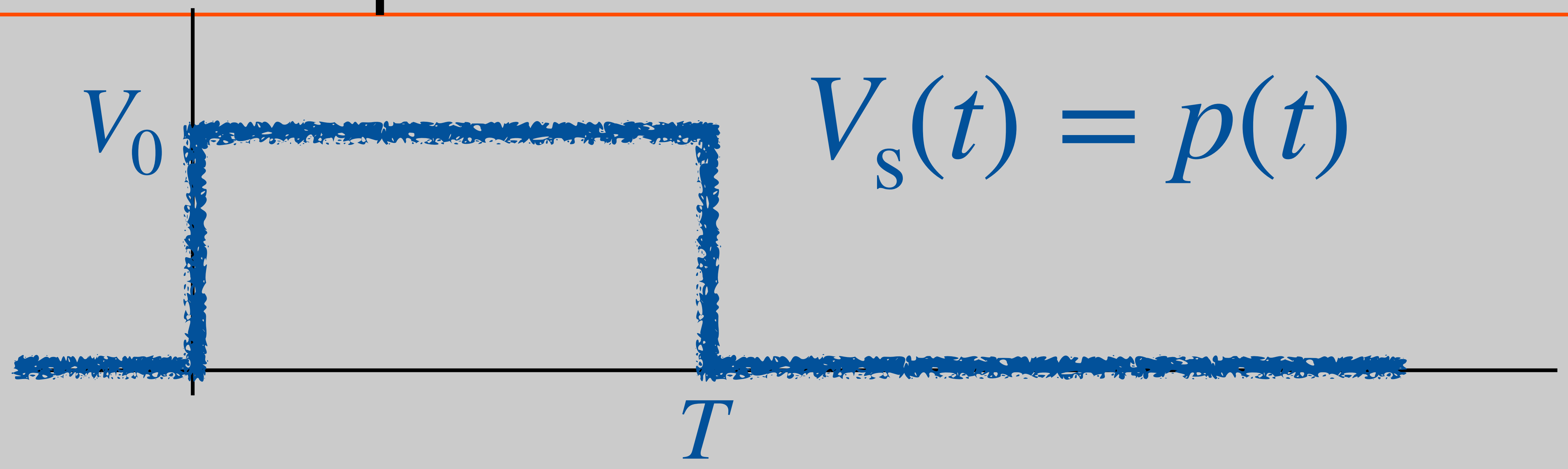


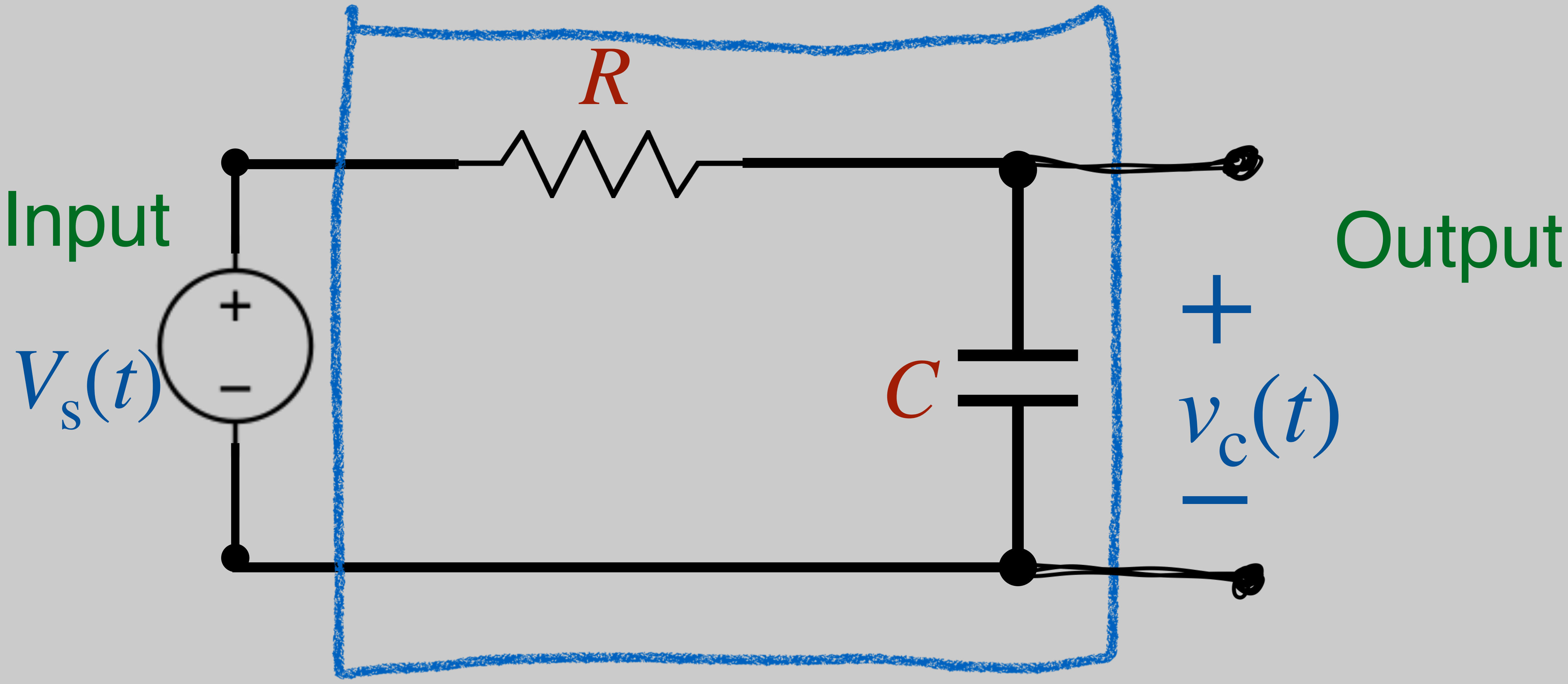
$$p(t) = u(t) - u(t-T)$$

Solve for $u(t)$, solve for $-u(t-T)$ add the results!

Pulse Response of RC circuits



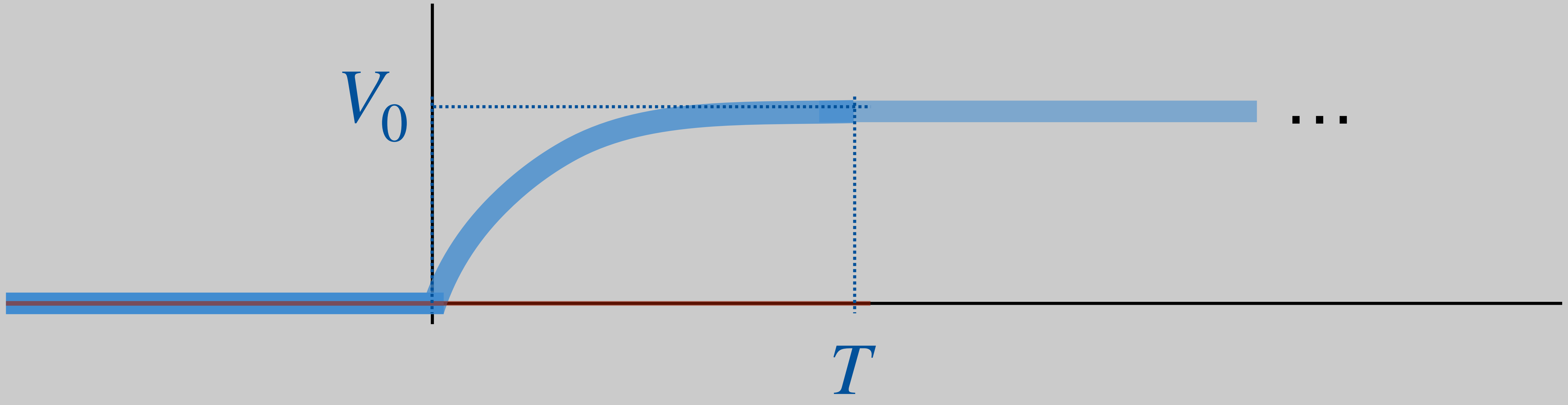
$$V_s(t) = p(t)$$



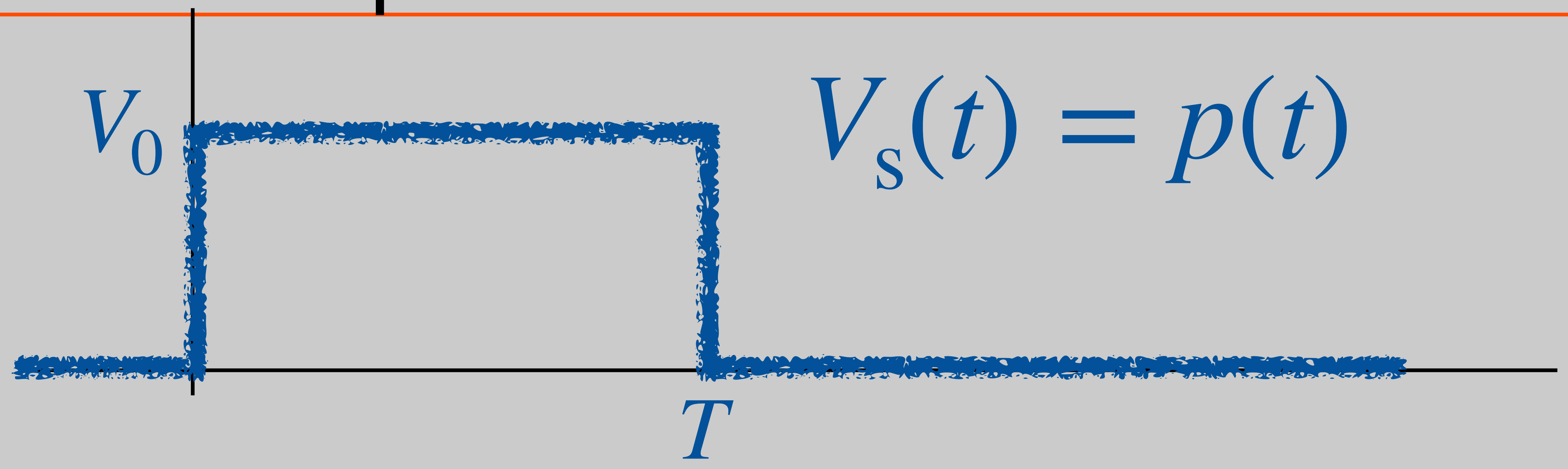
$$p(t) = u(t) - u(t-T)$$

From example 3:

$$V_s(t) = u(t) \Rightarrow v_c = V_0\left(1 - e^{-\frac{t}{RC}}\right) \quad |\, t > 0$$

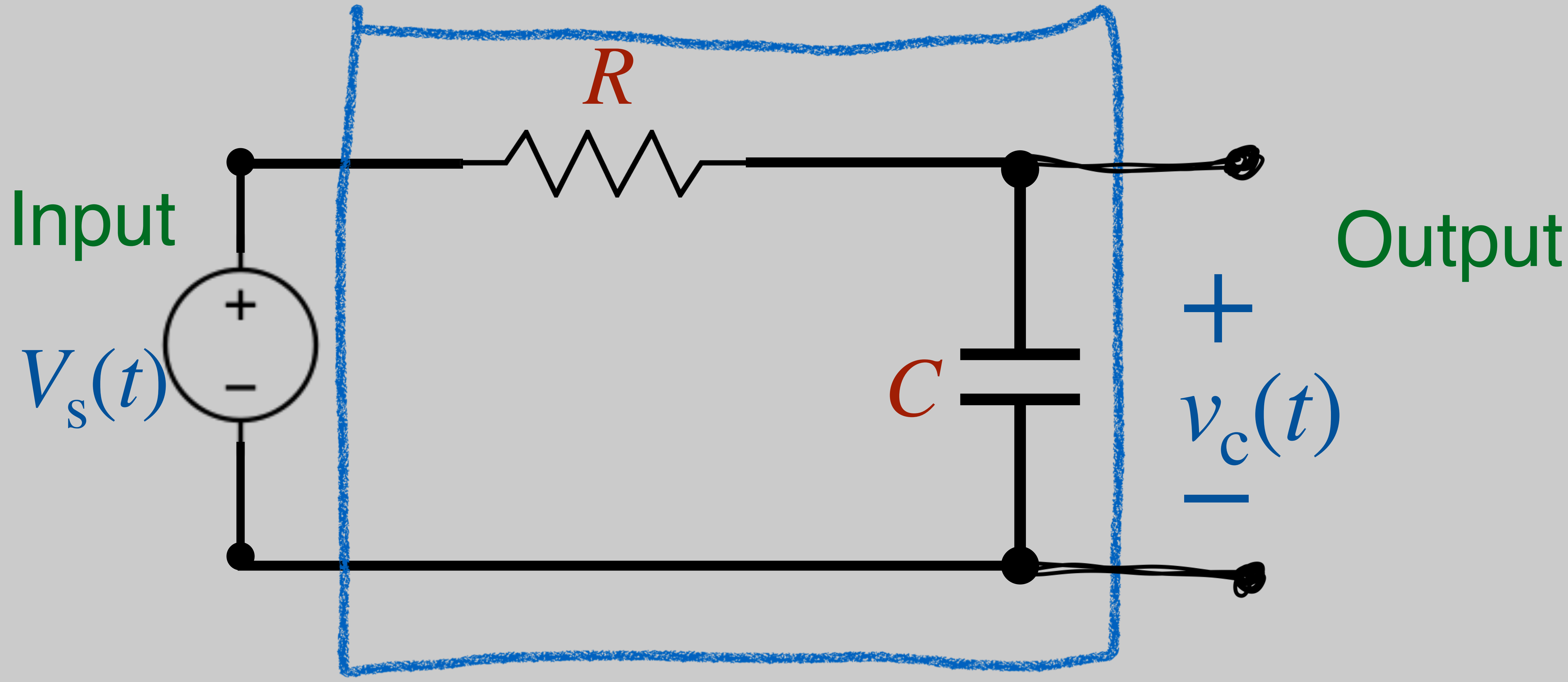


Pulse Response of RC circuits



$$V_s(t) = p(t)$$

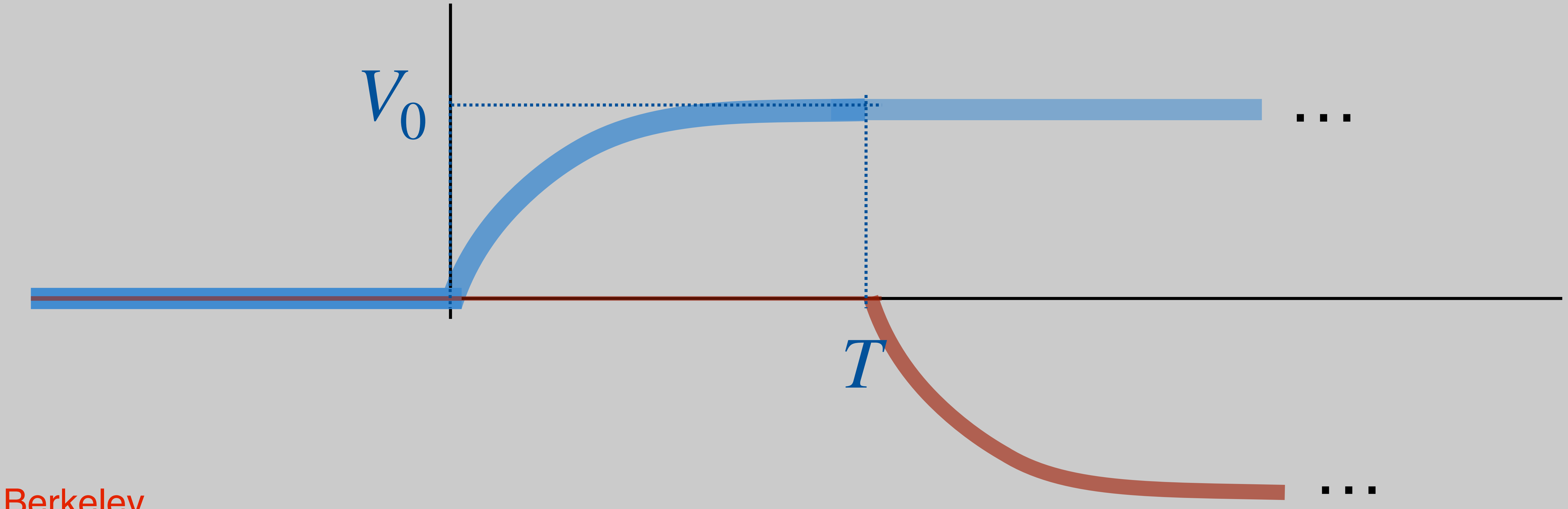
$$p(t) = u(t) - u(t - T)$$



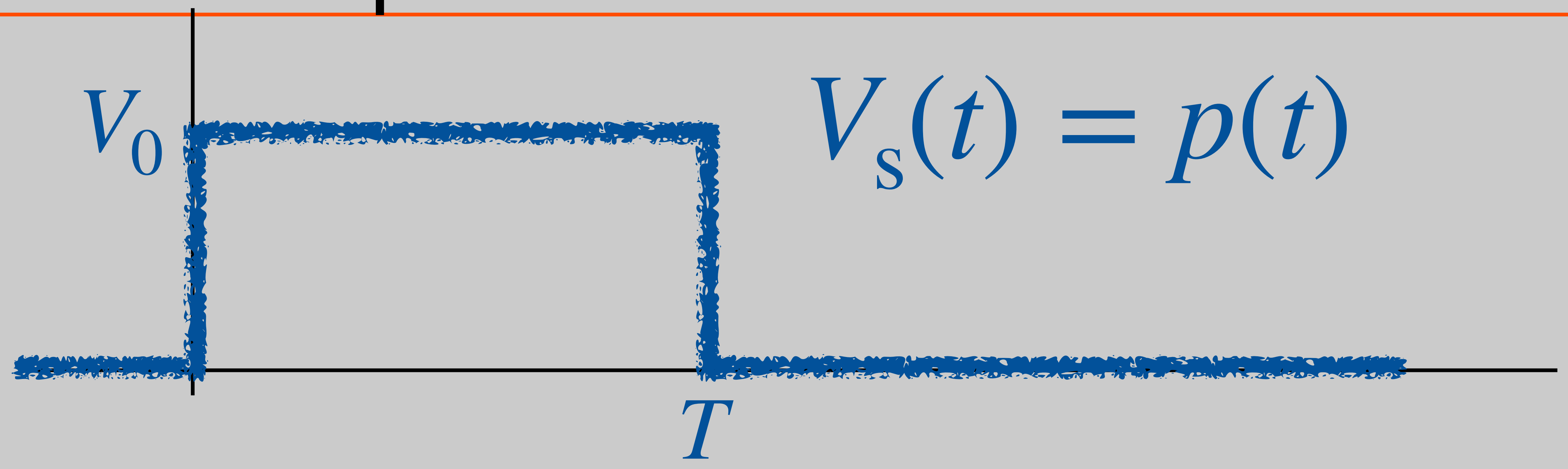
From example 3:

$$V_s(t) = u(t) \Rightarrow v_c = V_0\left(1 - e^{-\frac{t}{RC}}\right) \quad | t > 0$$

$$V_s(t) = u(t - T) \Rightarrow v_c = V_0\left(1 - e^{-\frac{t-T}{RC}}\right) \quad | t > T$$

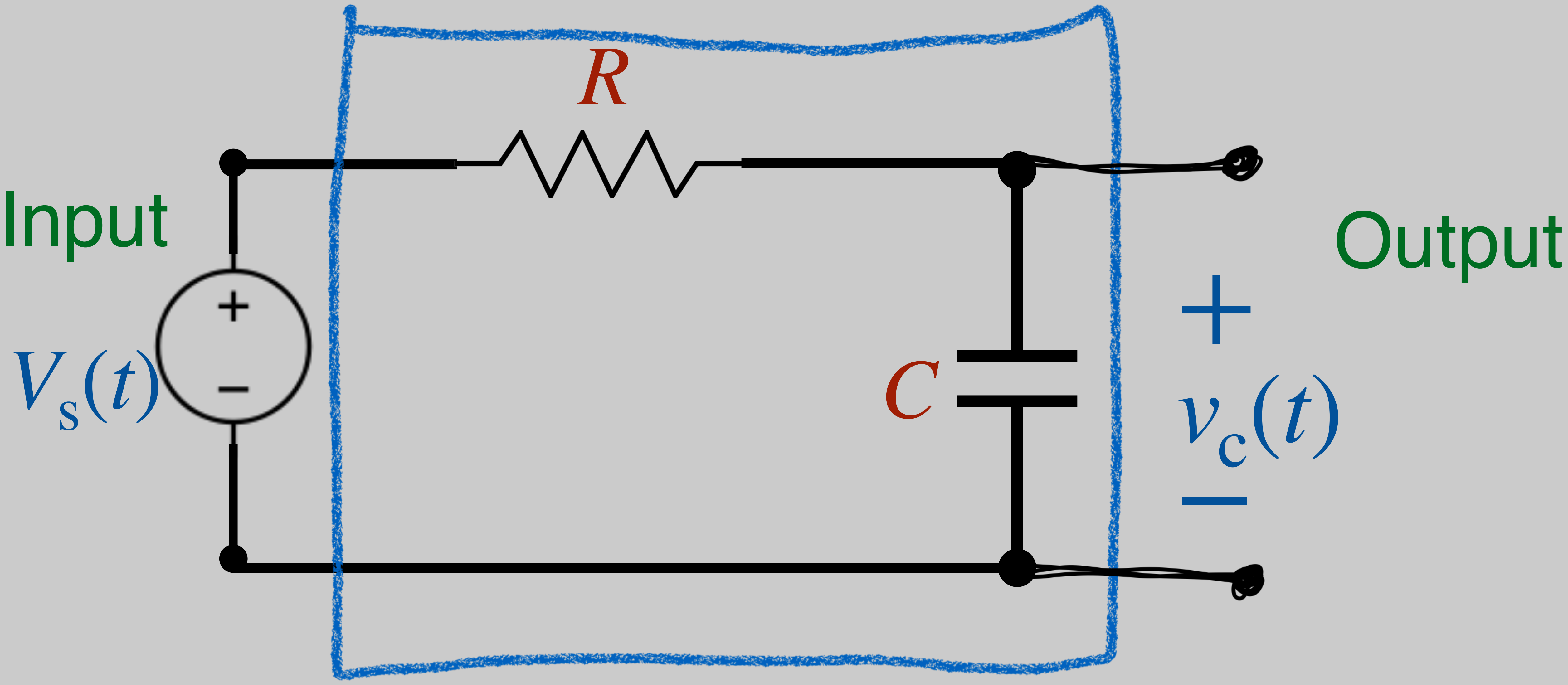


Pulse Response of RC circuits



$$V_s(t) = p(t)$$

$$p(t) = u(t) - u(t-T)$$



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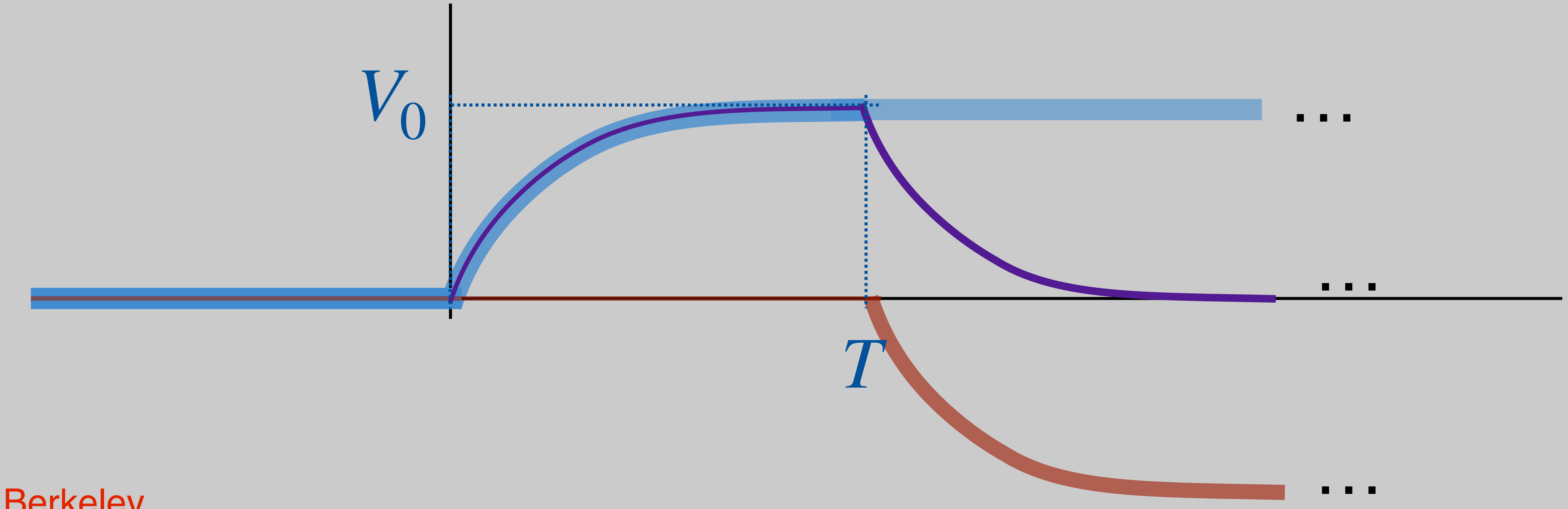
$$V_s(t) = u(t) \Rightarrow v_c = V_0\left(1 - e^{-\frac{t}{RC}}\right) \quad | t > 0$$

$$V_s(t) = u(t-T) \Rightarrow v_c = V_0\left(1 - e^{-\frac{t-T}{RC}}\right) \quad | t > T$$

$$V_s(t) = p(t)$$



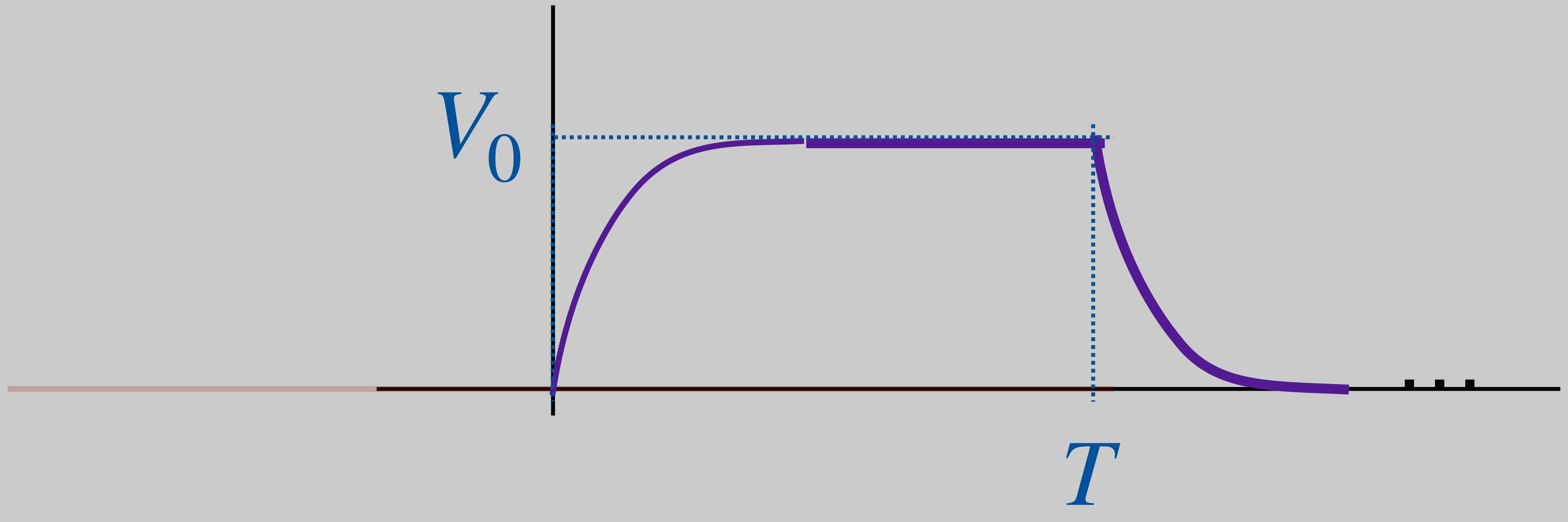
Pulse Response of RC circuits

$$V_s(t) = u(t) \Rightarrow v_c = V_0\left(1 - e^{-\frac{t}{RC}}\right) \quad | t > 0$$

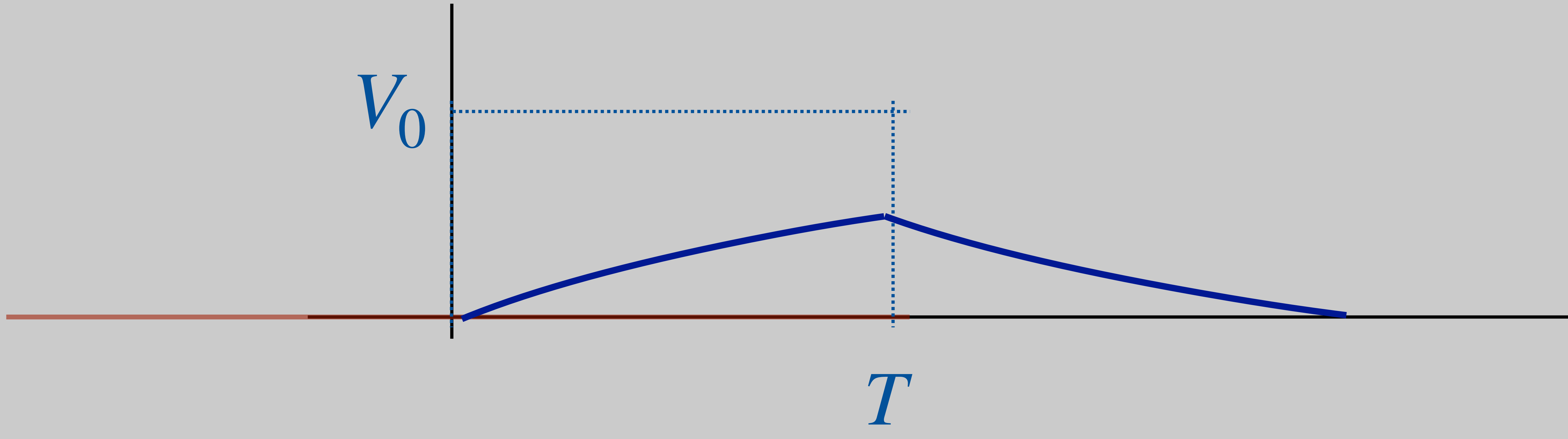
$$V_s(t) = u(t-T) \Rightarrow v_c = V_0\left(1 - e^{-\frac{t-T}{RC}}\right) \quad | t > T$$

$$V_s(t) = p(t)$$

"Fast" Circuit $T \gg RC$

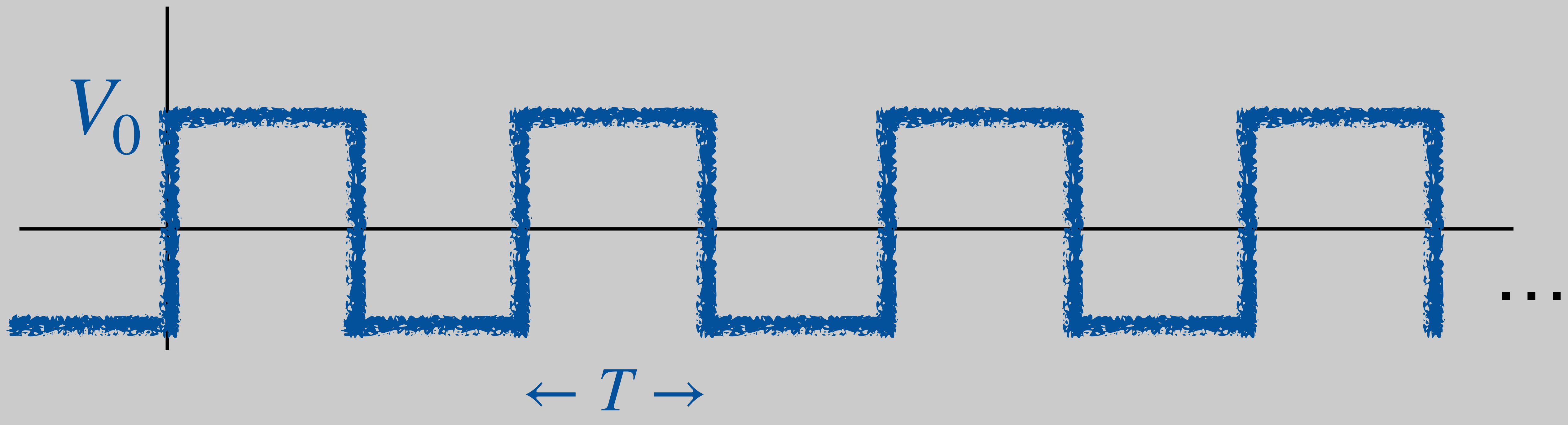


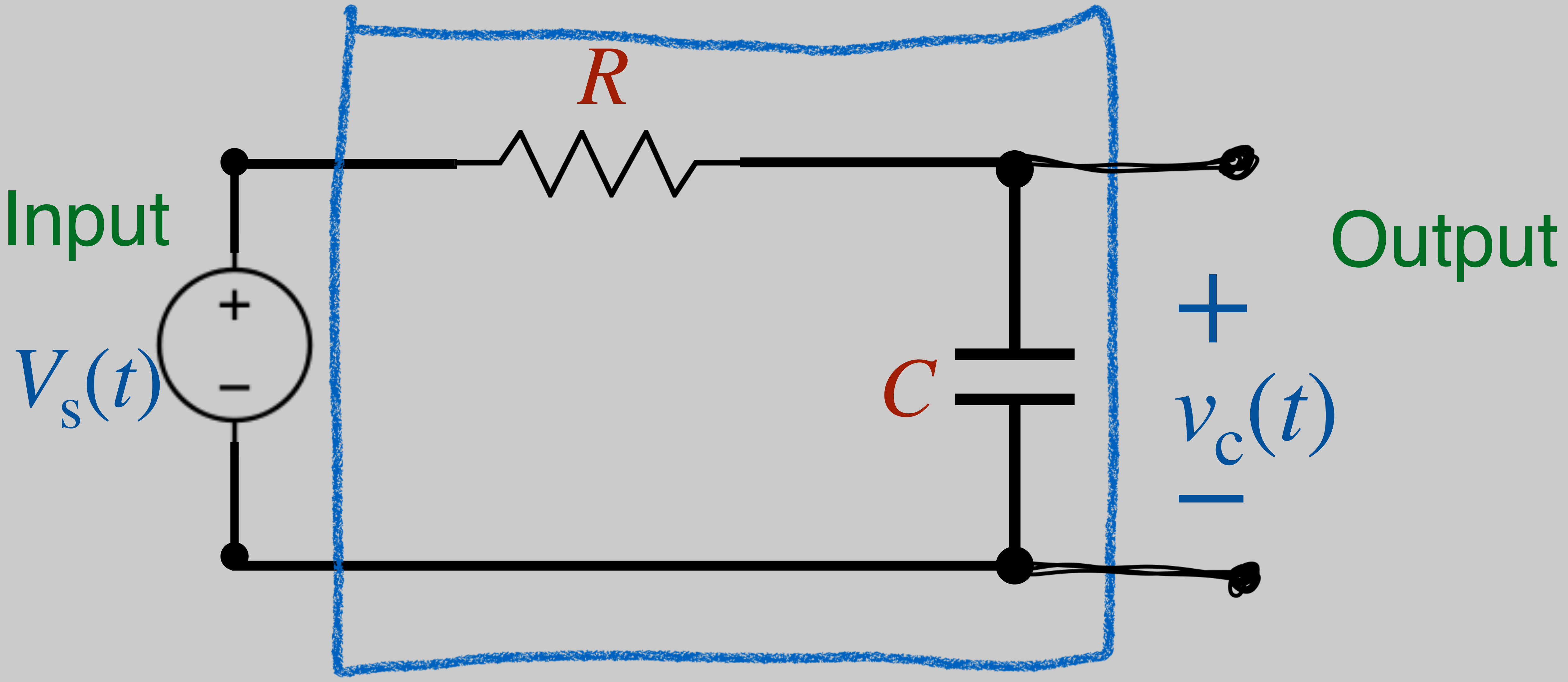
Slow Circuit $T \ll RC$



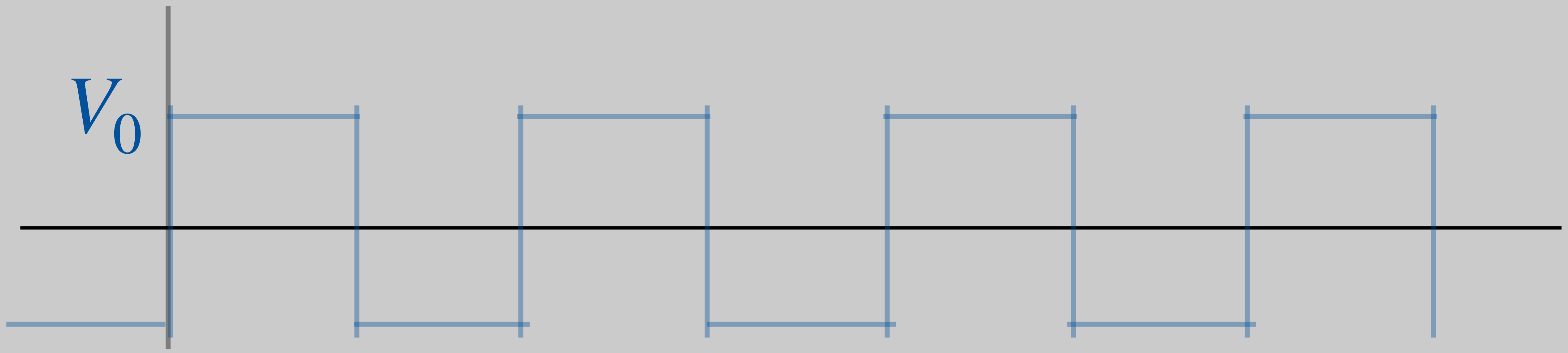
Pulse Response of RC circuits

Pulse train Response:



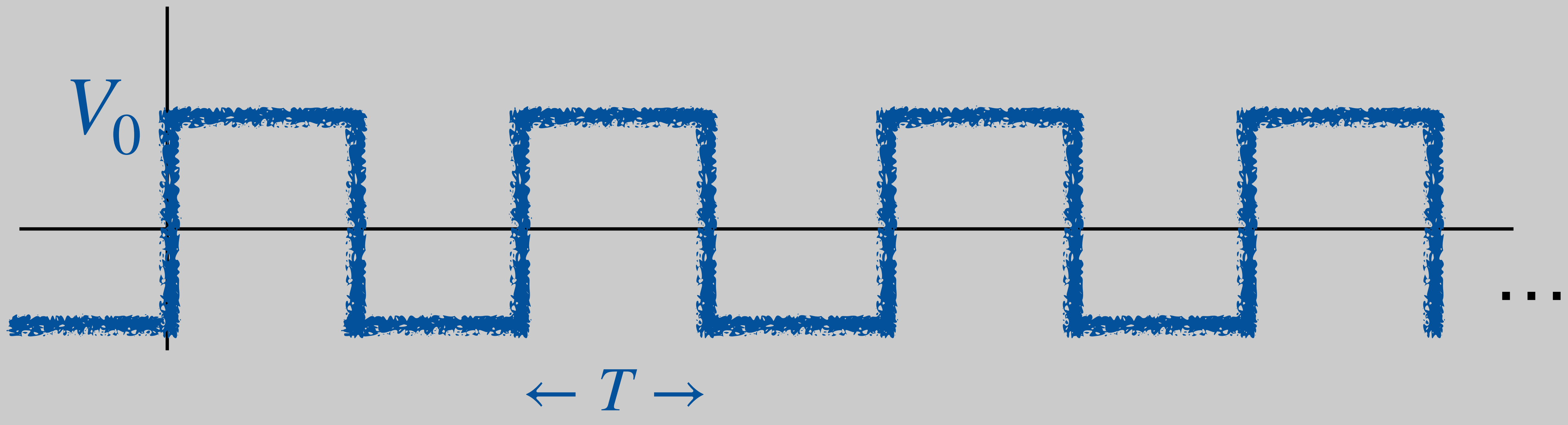


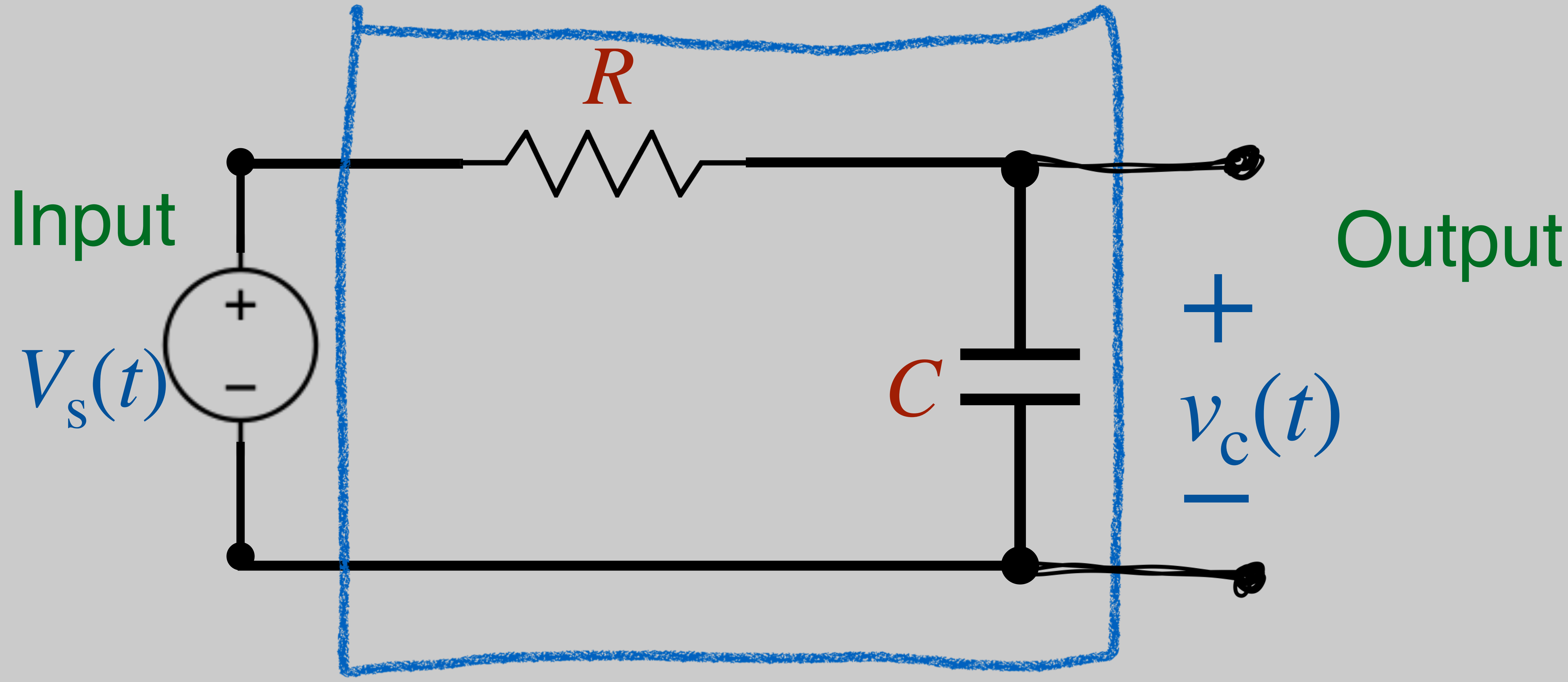
Slow switching - high magnitude



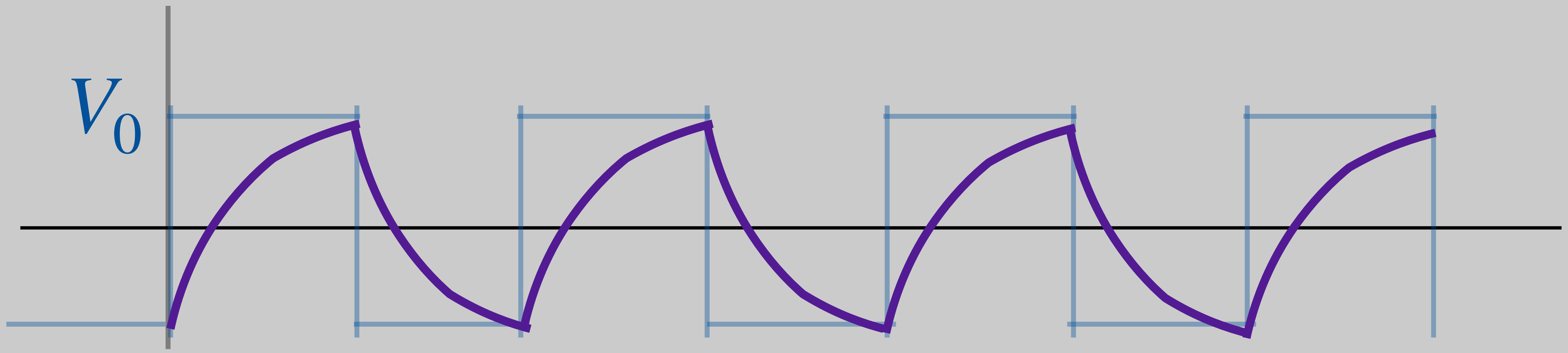
Pulse Response of RC circuits

Pulse train Response:

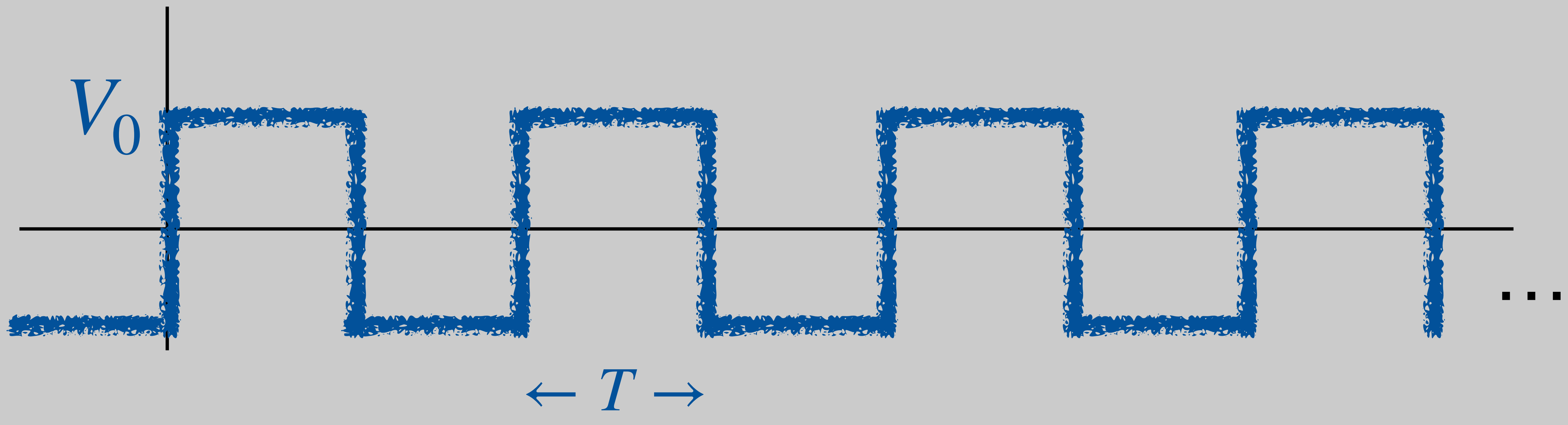


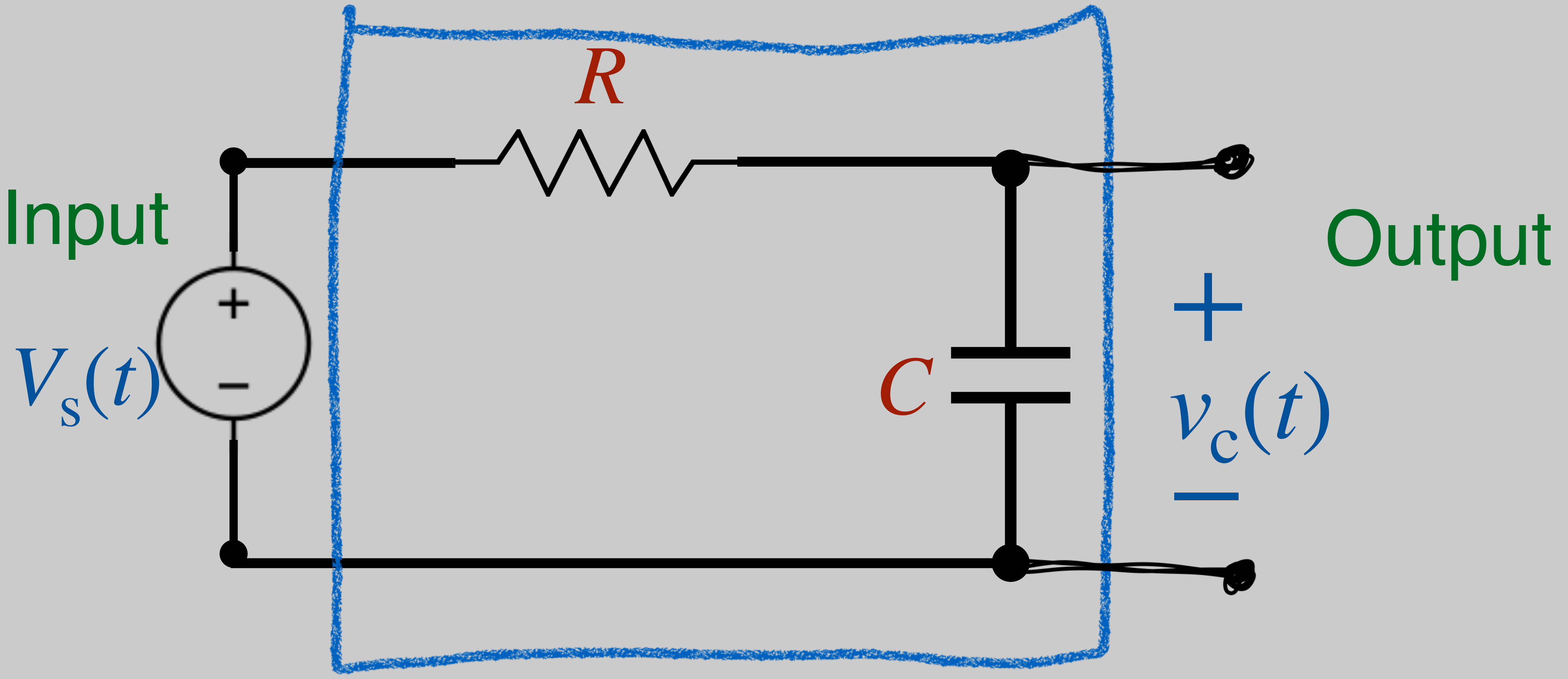


Slow switching - high magnitude

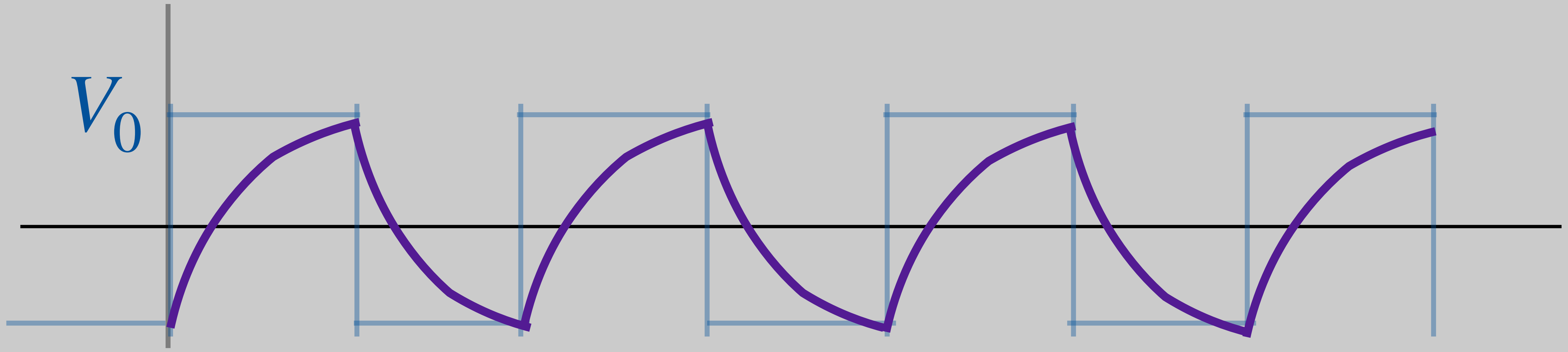


Pulse train Response:

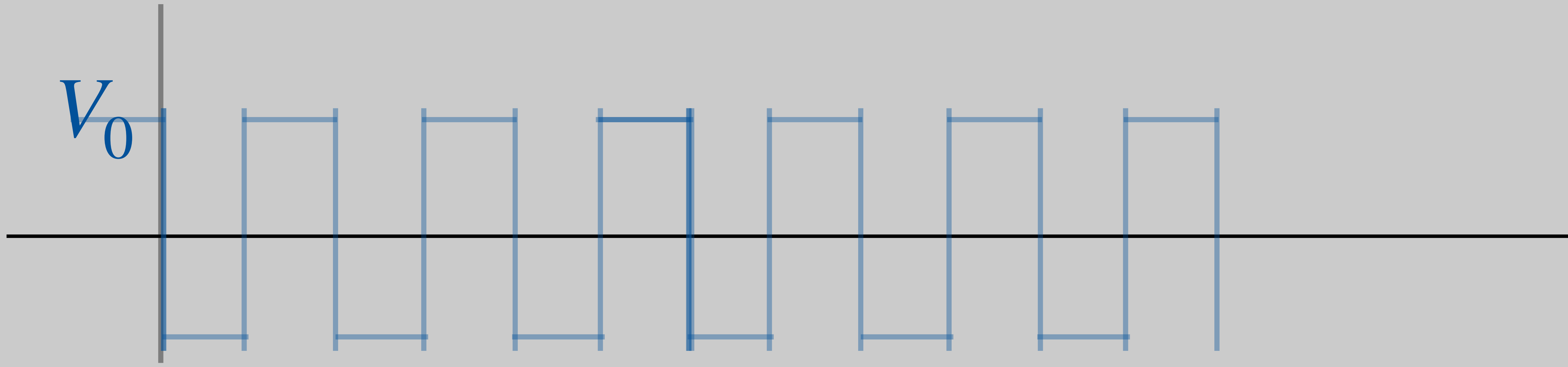




Slow switching - high magnitude



Fast switching - low magnitude



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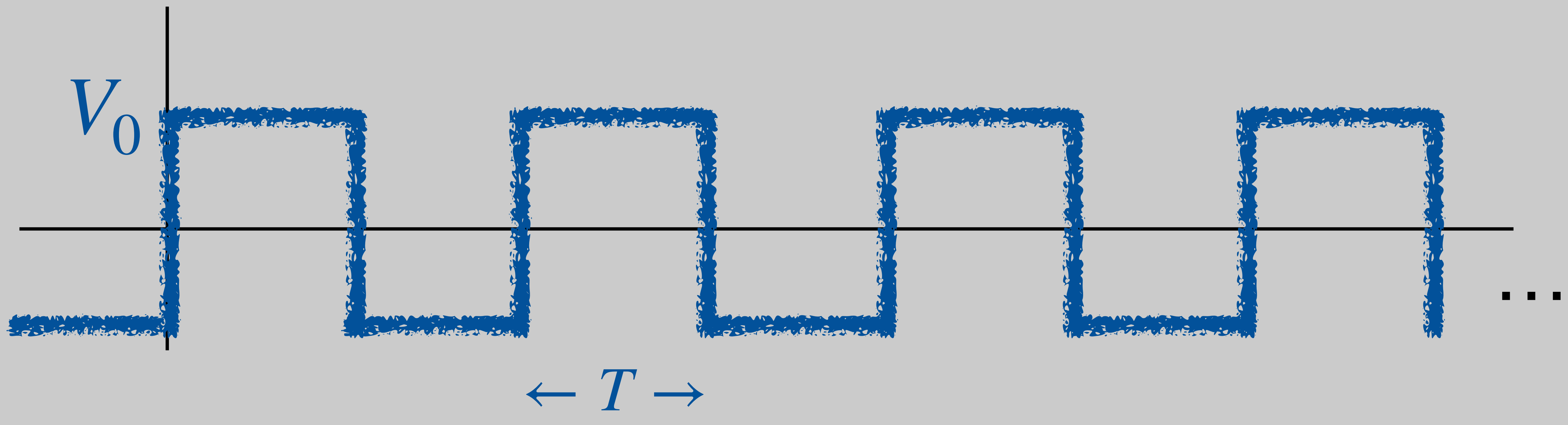
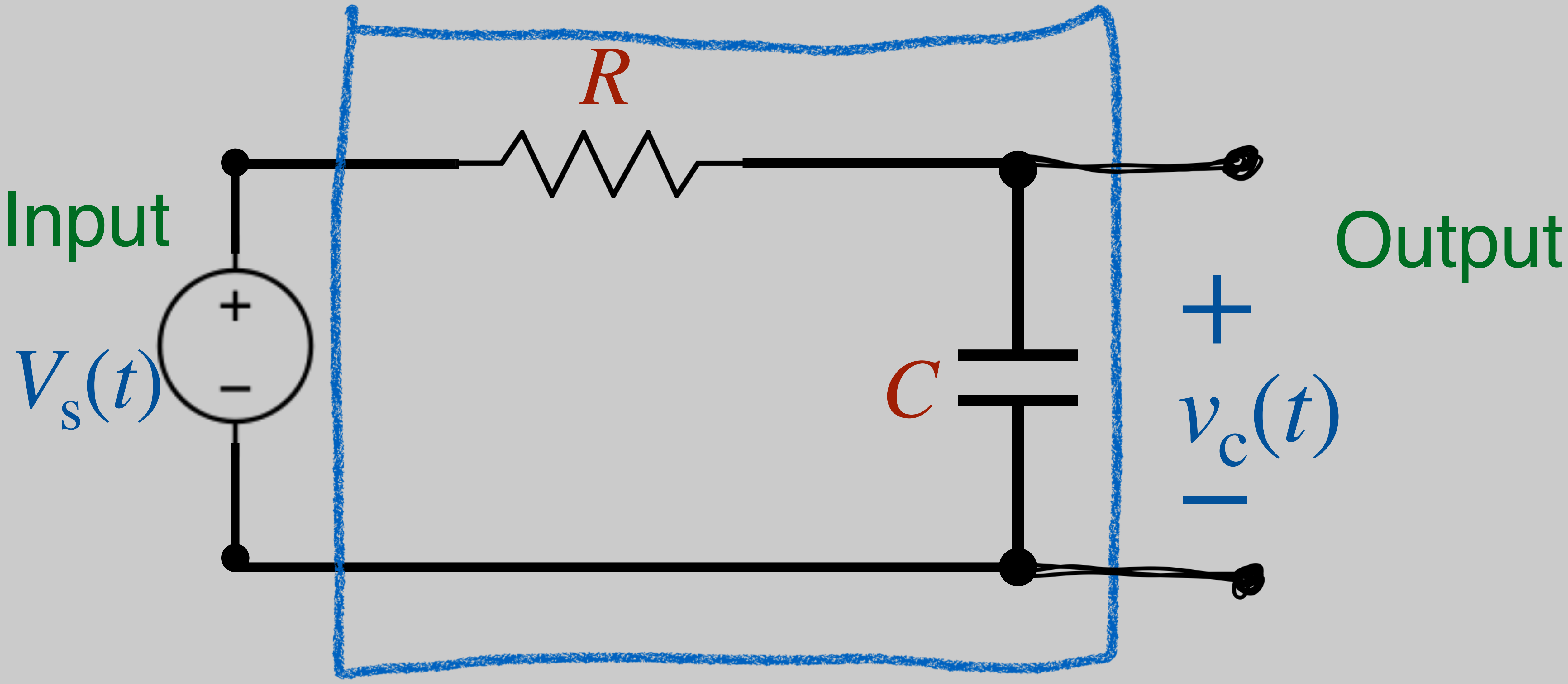
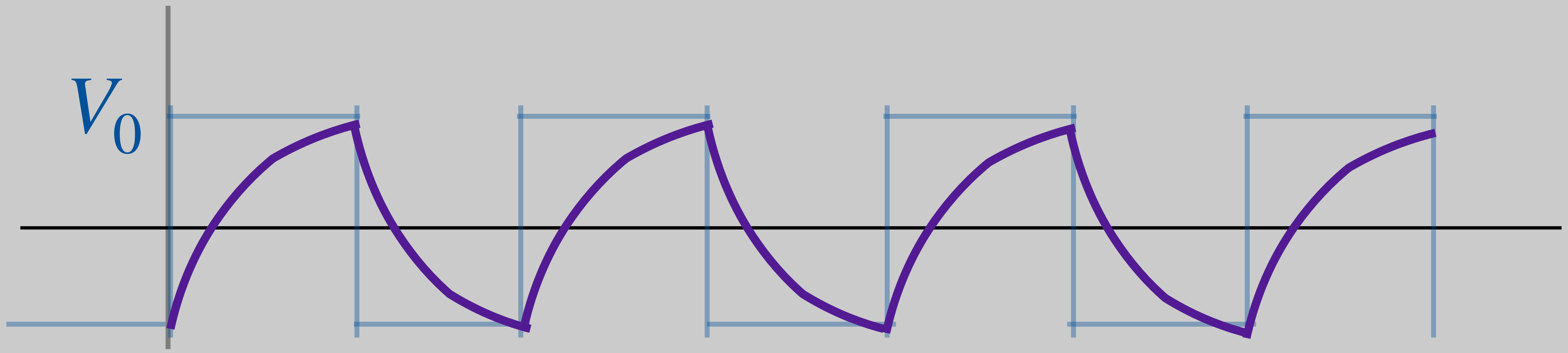
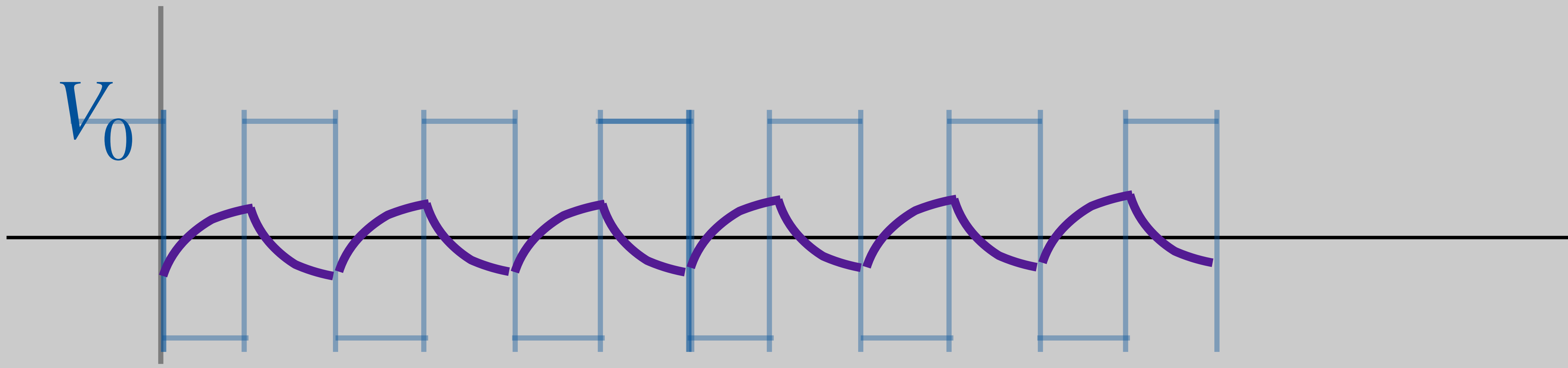




Slow switching - high magnitude



Fast switching - low magnitude



Low-Pass Filter!