
EECS 16B Designing Information Devices and Systems II
 Spring 2021 UC Berkeley

Homework 6

This homework is due on Friday, February 26, 2021, at 11:00PM. Self-grades and HW Resubmission are due on Tuesday, March 2, 2021, at 11:00PM.

1. Exam Policy and Practice

Your midterm is coming up on **March 15th, 2021**. To prepare for midterm proctoring, and to receive credit on this question, we ask you to read and complete the **Proctoring Document**. Make sure to **complete all action items in the document in order to receive credit for this question**.

2. Reading Lecture Notes

Staying up to date with lectures is an important part of the learning process in this course. Here are links to the notes that you need to read for this week: [Note4](#), [Note5](#)

- (a) Consider an RC circuit with a sinusoidal voltage input $V(t) = A \cos(\omega t)$. We are interested in finding the voltage on the capacitor in steady state (after a long time has passed). Can we solve this using our standard differential equation techniques? Can we solve this with phasors? Which one is more concise and why?

Solution:

Standard Differential Equations: We can solve this using our standard differential equation techniques.

Recall that if the D.E. takes the form $\frac{d}{dt}x(t) = \lambda x(t) + u(t)$, we can solve using $x(t) = x_0 e^{\lambda t} + \int_0^t e^{\lambda(t-\theta)} u(\theta) d\theta$. This works, but requires us to perform an integral.

Phasor Analysis: Using phasor analysis, we can solve this by using circuit analysis techniques. Phasor analysis will be more concise because fundamentally, phasor analysis is built for sinusoidal inputs (since they are complex exponentials) at steady state. Phasor analysis only requires one to solve a simple RC circuit using standard circuit analysis techniques. **Phasor analysis is much more concise than solving the D.E..**

- (b) There are two ways to make a low pass filter (discussed in the notes). What are they? Are there any major differences?

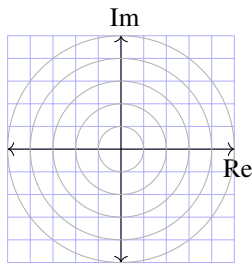
Solution: The first method is to construct a series R-C circuit (measuring the voltage across C). The second method is to construct a series L-R circuit (measuring the voltage across R). These are extremely similar. In fact, the transfer function for both cases has the same form: $H(\omega) = \frac{1}{1+j\frac{\omega}{\omega_c}}$. The only difference is that $\omega_c = \frac{1}{RC}$ for R-C circuits and $\omega_c = \frac{R}{L}$ for L-R circuits.

3. Phasors

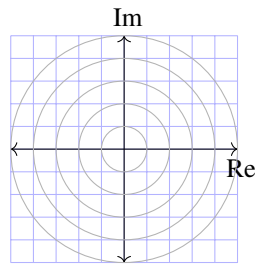
- (a) Consider a resistor ($R = 1.5\Omega$), a capacitor ($C = 1F$), and an inductor ($L = 1H$) connected in series. **Give expressions for the impedances of Z_R, Z_C, Z_L for each of these elements as a function of the angular frequency ω .**

Solution: The impedances are as follows: $Z_R = R = 1.5$, $Z_C = \frac{1}{j\omega C} = \frac{1}{j\omega} = -\frac{j}{\omega}$ and $Z_L = j\omega L = j\omega$.

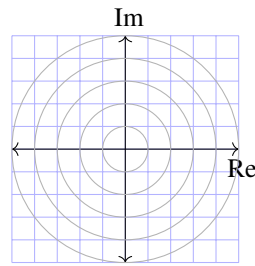
(b) Draw the individual impedances as “vectors” on the same complex plane for the case $\omega = \frac{1}{2}$ rad/sec. Also draw the combined impedance Z_{total} of their series combination. Then give the magnitude and phase of Z_{total} .



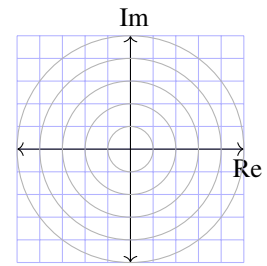
a $Z_R(@\omega = 0.5)$



b $Z_C(@\omega = 0.5)$

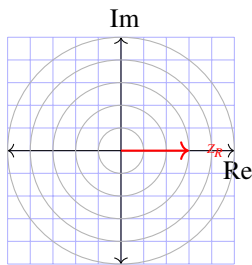


c $Z_L(@\omega = 0.5)$

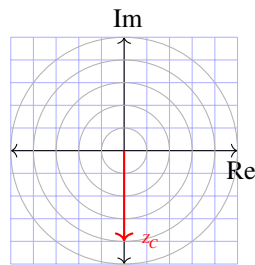


d $Z_{total}(@\omega = 0.5)$

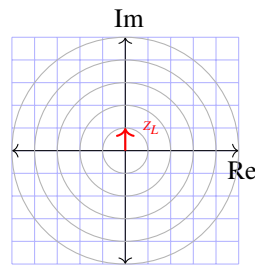
Solution: Substituting for $\omega = \frac{1}{2}$ in the above answers, we get, $Z_R = 1.5$, $Z_C = -2j$ and $Z_L = j(0.5)$. Since the elements are in series, $Z_{total} = Z_L + Z_C + Z_R = 1.5 - j(1.5)$. This has magnitude $1.5\sqrt{2}$ and phase $-\frac{\pi}{4}$. Following are the plots:



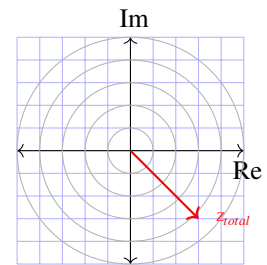
a $Z_R(@\omega = 0.5)$



b $Z_C(@\omega = 0.5)$

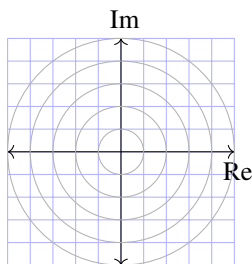


c $Z_L(@\omega = 0.5)$

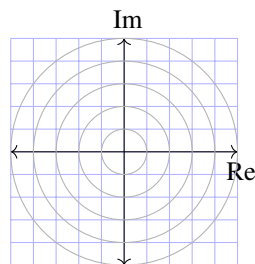


d $Z_{total}(@\omega = 0.5)$

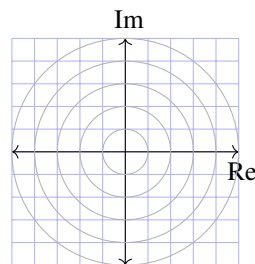
(c) Draw the individual impedances as “vectors” on the same complex plane for the case $\omega = 1$ rad/sec. Also draw the combined impedance Z_{total} of their series combination. Then give the magnitude and phase of Z_{total} .



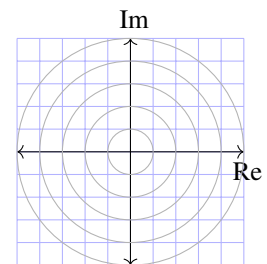
a $Z_R(@\omega = 1)$



b $Z_C(@\omega = 1)$



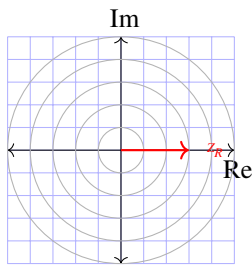
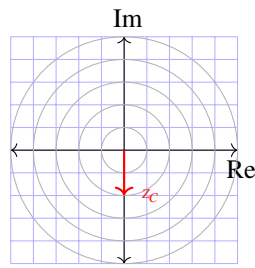
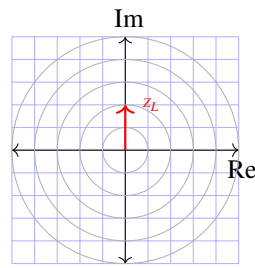
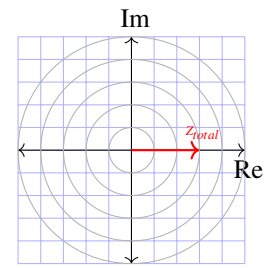
c $Z_L(@\omega = 1)$



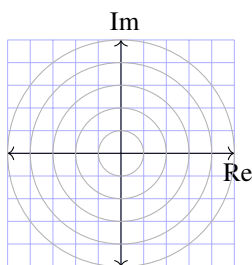
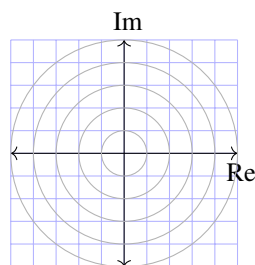
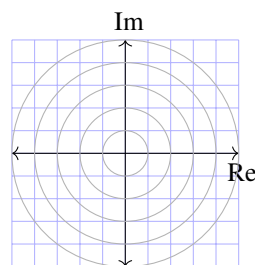
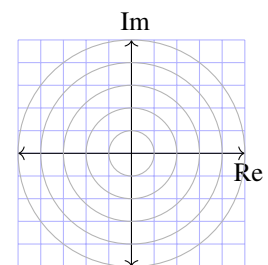
d $Z_{total}(@\omega = 1)$

Solution:

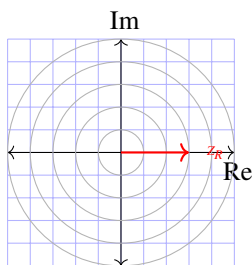
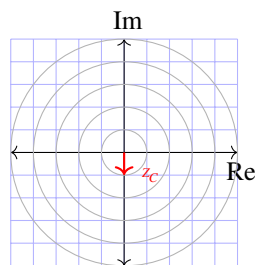
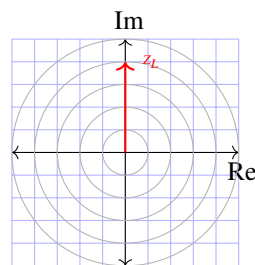
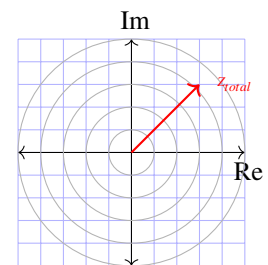
Following the same method as last time, with $\omega = 1$, $Z_R = 1.5$, $Z_C = -j$, $Z_L = j$ and $Z_{total} = 1.5$. This has magnitude 1.5 and phase 0.

a $Z_R(@\omega = 1)$ b $Z_C(@\omega = 1)$ c $Z_L(@\omega = 1)$ d $Z_{total}(@\omega = 1)$

- (d) Draw the individual impedances as “vectors” on the same complex plane for the case $\omega = 2$ rad/sec. Also draw the combined impedance Z_{total} of their series combination. Then give the magnitude and phase of Z_{total} .

a $Z_R(@\omega = 2)$ b $Z_C(@\omega = 2)$ c $Z_L(@\omega = 2)$ d $Z_{total}(@\omega = 2)$

Solution: Again, following the same method as last time, with $\omega = 2$, $Z_R = 1.5$, $Z_C = -j(0.5)$, $Z_L = 2j$ and $Z_{total} = 1.5 + j(1.5)$. This has magnitude $1.5\sqrt{2}$ and phase $+\frac{\pi}{4}$.

a $Z_R(@\omega = 2)$ b $Z_C(@\omega = 2)$ c $Z_L(@\omega = 2)$ d $Z_{total}(@\omega = 2)$

- (e) For the previous series combination of RLC elements, what is the “natural frequency” ω_n (i.e. the frequency ω_n where the series impedance is purely real). **Fact:** We call this the natural frequency since it is the frequency that the circuit will oscillate at if it was underdamped (if R was small enough).

Solution: From our above answers, clearly the natural frequency, $\omega_n = 1$ rad/s. This is where the imaginary parts of the impedance cancel each other.

4. Color Organ Filter Design

In the fourth lab, we will design low-pass, band-pass, and high-pass filters for a color organ. There are red, green, and blue LEDs. Each color will correspond to a specified frequency range of the input audio signal. The intensity of the light emitted will correspond to the amplitude of the audio signal.

- (a) First, you remember that you saw in lecture that you can build simple filters using a resistor and a capacitor. **Design a simple first-order passive low-pass filter with the following frequency range using 1 μF capacitors.** (“Passive” means that the filter does not require any power supply to operate on the input signal. Passive components include resistors, capacitors, inductors, diodes, etc., while an example of an active component would be an op-amp).

- Low-pass filter: cut-off frequency $f_c = 2400\text{Hz}$, $\omega_c = 2\pi \cdot 2400 \frac{\text{rad}}{\text{sec}}$

Recall that the cutoff-frequency of such a filter is just where the magnitude of the filter is $\frac{1}{\sqrt{2}}$ of its peak value.

Show your work to find the resistor value that creates this low-pass filter. Draw the schematic-level representation of your design. Please mark V_{in} , V_{out} , and the ground node(s) in your schematic. Round your results to two significant figures.

Solution:

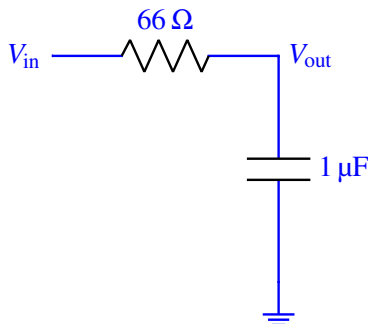
Low-pass filter

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC} = 2400\text{Hz}$$

$$R = \frac{1}{2\pi \cdot 1\mu\text{F} \cdot 2400\text{Hz}} = 66\Omega$$

Therefore, we need a 66 Ω resistor.



- (b) Now design a simple first-order passive high-pass filters with the following frequency range using 1 μF capacitors.

- High-pass filter: cut-off frequency $f_c = 100\text{Hz}$, $\omega_c = 2\pi \cdot 100 \frac{\text{rad}}{\text{sec}}$

Show your work to find the resistor value that creates this high-pass filter. Draw the schematic-level representation of your design. Please mark V_{in} , V_{out} , and the ground node(s) in your schematic. Round your results to two significant figures.

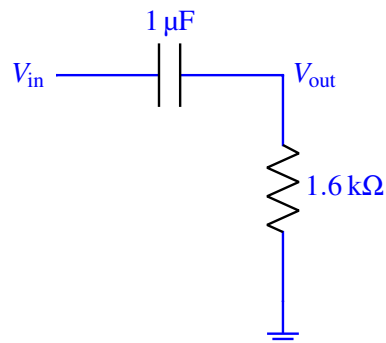
Solution:

High-pass filter

$$f_c = \frac{1}{2\pi RC} = 100\text{Hz}$$

$$R = \frac{1}{2\pi \cdot 1\mu\text{F} \cdot 100\text{Hz}} = 1.6\text{k}\Omega$$

Therefore, we need a 1.6 k Ω resistor. Note that we want a 24 times lower frequency, which means a 24 times higher time constant, which means a 24 times higher resistor.



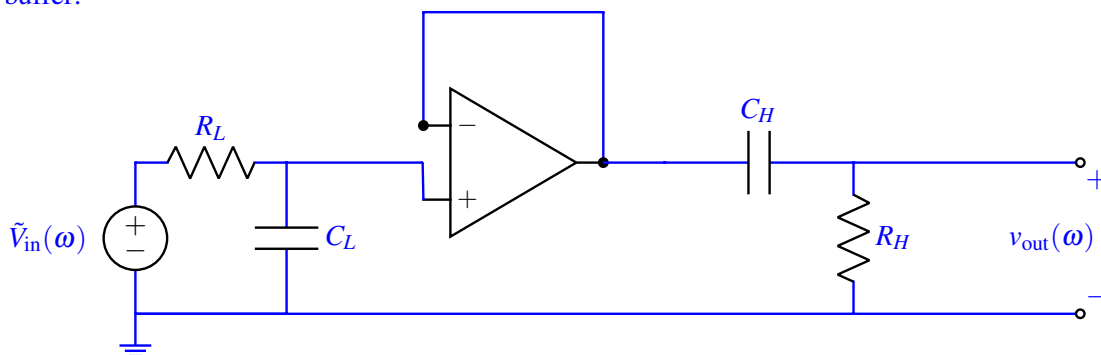
- (c) You can try to build a bandpass filter by cascading the first-order low-pass and high-pass filters you designed in parts (a) and (b). To do this, you might be tempted to connect the V_{out} node of your low-pass filter directly to the V_{in} node of your high-pass filter. However, if you did this, just as you saw in 16A for voltage dividers, the purported high-pass filter would “load” the low-pass filter and you might get some potentially complicated mess instead of what you wanted.

Show how you can use an ideal op-amp configured as a unity gain buffer to eliminate this loading effect to cascade the low-pass and high-pass filters, and write the resulting transfer function of the combined circuit. Draw the magnitude and phase transfer functions of the combined circuit. What kind of filter is this? You can optionally use the included Jupyter notebook `tf_gen.ipynb`. (HINT: Read Section 2.1 in *Note6*.)

(NOTE: In Python, use `1j` when your transfer function has a `j`.)

Solution:

Consider the circuit given below, which is the low pass and the high pass, connected with a unity gain buffer:



We know that when we cascade circuits, the combined transfer function is the multiplication of the individual elements. For the Low Pass Filter $H_L(\omega)$, Unity Gain Buffer $H_{\text{unity}}(\omega)$, and High Pass Filter $H_H(\omega)$.

$$H(\omega) = H_L(\omega) \cdot H_{\text{unity}}(\omega) \cdot H_H(\omega)$$

And we know that:

$$H_L(\omega) = \frac{1}{1 + j\omega R_L C_L}, \quad H_{\text{unity}}(\omega) = 1, \quad H_H(\omega) = \frac{j\omega R_H C_H}{1 + j\omega R_H C_H}$$

Combining the transfer functions, we get:

$$H(\omega) = \frac{1}{(1 + j\omega R_L C_L)} \cdot \frac{j\omega R_H C_H}{(1 + j\omega R_H C_H)}$$

The magnitude and phase transfer functions are shown below. We can see that this is a band pass filter.

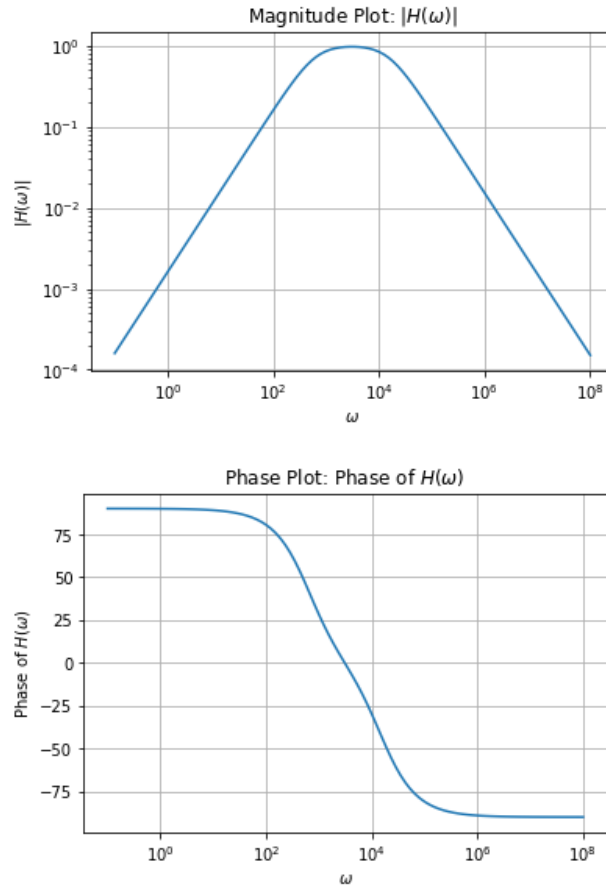


Figure 7: Magnitude and Phase transfer functions

- (d) Write down an expression for the time-domain output waveform $V_{out}(t)$ of this filter if the input voltage is $V_{in}(t) = 1 \sin(1000t)$ V. Round your answer to 2 significant digits.

Solution: We can find the transfer function at this point:

$$|H(\omega = 10^3)| = 0.85$$

$$\angle H(\omega = 10^3) = 0.49 \text{ rad} = 28.23^\circ$$

Therefore the output will be:

$$V_{out}(t) = 0.85 \sin(1000t + 0.49)$$

5. Phasors and Eigenvalues

Suppose that we have the two-dimensional system of differential equations expressed in matrix/vector form:

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + \vec{b}u(t) \quad (1)$$

where for this problem, the matrix A and the vector \vec{b} are both real.

- (a) **Give a necessary condition on the eigenvalues λ_k of A such that any impact of an initial condition will eventually completely die out.** (i.e. the system will reach steady-state.)

You don't have to prove this. The idea here is to make sure that you understand what kind of thing is required. (*HINT: Read Section 2 in Note4.*)

Solution: The condition is that all eigenvalues must have real parts that are less than zero. In equations

$$\forall k, \operatorname{Re}(\lambda_k) < 0$$

This condition derives from the fact that the differential equations in the eigenspace take the form of (*Recall how the diagonalization we have done in the past takes us to an coordinate system where the matrix representing the differential equation has only diagonal entries being the eigenvalues, corresponding to differential equations of the form $\frac{d}{dt}(t) = \lambda z(t)$.*)

$$e^{\lambda_k t}$$

So, if all the eigenvalues have strictly negative real parts, then the exponential terms will die out.

If any of the eigenvalues have strictly positive real parts, then the exponential terms corresponding to them will blow up as growing exponentials.

The case of $\lambda = 0$ or having a zero real part in general (purely imaginary eigenvalues) is a little more ambiguous in feeling. This suggests that some constant offset (for the case of $\lambda = 0$) or some steady oscillation at a natural frequency of the system can persist throughout all time. But persisting isn't dying out and so we really want the eigenvalues to have strictly negative real parts for us to be able to ignore the initial conditions.

The argument above implicitly assumes that we can find enough linearly independent eigenvectors to get a basis. But what if we can't? We will explicitly address that case later in the course, but so far, we have seen in the cases that we have explored that what seems to happen is that even in the new basis, we seem to get a copy of an existing eigenvalue showing up again. This gives us some confidence that the condition that we are expressing is probably the right one, but we aren't fully sure yet since we have no proof.

- (b) Now assume that $u(t)$ has a phasor representation \tilde{U} . In other words, $u(t) = \tilde{U}e^{+j\omega t} + \overline{\tilde{U}}e^{-j\omega t}$.

Assume that the vector solution $\vec{x}(t)$ to the system of differential equations (1) can also be written in phasor form as

$$\vec{x}(t) = \tilde{X}e^{+j\omega t} + \overline{\tilde{X}}e^{-j\omega t}. \quad (2)$$

Derive an expression for \tilde{X} involving $A, \vec{b}, j\omega, \tilde{U}$, and the identity matrix I .

(*HINT: Plug (2) into (1) and simplify, using the rules of differentiation and grouping terms by which exponential $e^{\pm j\omega t}$ they multiply.*)

Solution: As the hint suggests, plugging back (2) into (1) we get the following:

$$\frac{d}{dt}(\tilde{X}e^{j\omega t} + \overline{\tilde{X}}e^{-j\omega t}) = A(\tilde{X}e^{j\omega t} + \overline{\tilde{X}}e^{-j\omega t}) + \vec{b}(\tilde{U}e^{j\omega t} + \overline{\tilde{U}}e^{-j\omega t}) \quad (3)$$

$$(j\omega\tilde{X}e^{j\omega t} - j\omega\overline{\tilde{X}}e^{-j\omega t}) = (A\tilde{X} + \vec{b}\tilde{U})e^{j\omega t} + (A\overline{\tilde{X}} + \vec{b}\overline{\tilde{U}})e^{-j\omega t} \quad (4)$$

$$(5)$$

Note that \tilde{X} and \tilde{U} do not depend on time since they are phasors. Next, we can group the coefficients with the same exponential terms,

$$j\omega\tilde{X} = A\tilde{X} + \vec{b}\tilde{U} \quad (6)$$

$$-j\omega\overline{\tilde{X}} = A\overline{\tilde{X}} + \vec{b}\overline{\tilde{U}} \quad (7)$$

$$\Rightarrow \overline{(j\omega)\tilde{X}} = \overline{(A\tilde{X} + \vec{b}\tilde{U})} \quad (8)$$

$$\Rightarrow (j\omega)\overline{\tilde{X}} = (A\overline{\tilde{X}} + \vec{b}\overline{\tilde{U}}) \quad (9)$$

We see that equations (6) and (9) match, which is good. Note that, here we are assuming A and \vec{b} are real. Next, we can solve (6) to get \tilde{X} :

$$j\omega\tilde{X} = A\tilde{X} + \vec{b}\tilde{U} \quad (10)$$

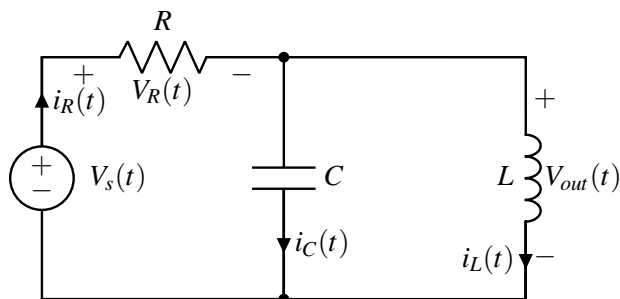
$$\Rightarrow (j\omega I - A)\tilde{X} = \vec{b}\tilde{U} \quad (11)$$

$$\Rightarrow \tilde{X} = (j\omega I - A)^{-1}\vec{b}\tilde{U} \quad (12)$$

6. Phasor-Domain Circuit Analysis

The analysis techniques you learned previously in 16A for resistive circuits are equally applicable for analyzing circuits driven by sinusoidal inputs in the phasor domain. In this problem, we will walk you through the steps with a concrete example.

Consider the following circuit where the input voltage is sinusoidal. The end goal of our analysis is to find an equation for $V_{out}(t)$.



The components in this circuit are given by:

$$V_s(t) = 10\sqrt{2}\cos\left(100t - \frac{\pi}{4}\right)$$

$$R = 5\Omega$$

$$L = 50\text{mH}$$

$$C = 2\text{mF}$$

- (a) Give the amplitude V_0 , oscillation frequency ω , and phase ϕ of the input voltage V_s .

Solution: A sinusoid takes the form $v(t) = V_0\cos(\omega t + \phi)$. Given $V_s(t)$, we find:

$$V_0 = 10\sqrt{2}\text{V}$$

$$\omega = 100\text{rad/sec}$$

$$\phi = -\frac{\pi}{4}\text{rad}$$

- (b) Transform the circuit into the phasor domain. What are the impedances of the resistor, capacitor, and inductor? What is the phasor \tilde{V}_s of the input voltage $V_s(t)$?

Solution:

$$Z_L = j\omega L$$

$$Z_C = \frac{1}{j\omega C}$$

$$Z_R = R$$

$$\tilde{V}_s = \frac{|V_s|}{2}e^{j\angle V_s} = 5\sqrt{2}e^{-j\frac{\pi}{4}}$$

(c) Use the circuit equations to **solve for** \tilde{V}_{out} , the phasor representing the output voltage.

Solution:

We have

$$i_R = \frac{\tilde{V}_S - \tilde{V}_{out}}{R}$$

$$i_L = \frac{\tilde{V}_{out}}{j\omega L}$$

$$i_C = \tilde{V}_{out} \cdot j\omega C$$

Rewriting the current relation in terms of voltage phasors gives:

$$\frac{\tilde{V}_S - \tilde{V}_{out}}{R} = \frac{\tilde{V}_{out}}{j\omega L} + \tilde{V}_{out} \cdot j\omega C$$

$$\frac{\tilde{V}_S}{R} = \tilde{V}_{out} \left(\frac{1}{j\omega L} + j\omega C + \frac{1}{R} \right)$$

$$\frac{\tilde{V}_S}{R} = \tilde{V}_{out} \left(\frac{R + (j\omega)^2 RLC + j\omega L}{j\omega RL} \right)$$

Solving for \tilde{V}_{out} :

$$\tilde{V}_{out} = \tilde{V}_S \left(\frac{j\omega L}{R - \omega^2 RLC + j\omega L} \right)$$

Plugging in for the values of R, L, C and ω :

$$\tilde{V}_{out} = \tilde{V}_S \left(\frac{j \cdot 100 \cdot 50 \times 10^{-3}}{5 - 100^2 \cdot 5 \cdot 50 \times 10^{-3} \cdot 2 \times 10^{-3} + j100 \cdot 50 \times 10^{-3}} \right)$$

$$= \tilde{V}_S \left(\frac{j5}{5 - 5 + j5} \right)$$

$$= \tilde{V}_S \left(\frac{j5}{j5} \right)$$

$$\tilde{V}_{out} = \tilde{V}_S$$

We found that $\tilde{V}_{out} = \tilde{V}_S$ because this circuit is in resonance; i.e., the capacitor and inductor have the exact values that cause current and voltage to endlessly oscillate between them at this frequency. If we chose a different value for ω with these same component values, the circuit would not be in resonance and \tilde{V}_{out} and \tilde{V}_S would no longer be equal.

Alternative Solution:

The equivalent impedance of the parallel combination of inductor and capacitor is given by

$$\begin{aligned}
 Z_{eq} &= \frac{Z_L Z_C}{Z_L + Z_C} \\
 &= \frac{L/C}{j\omega L + \frac{1}{j\omega C}} \\
 &= \frac{L/C}{j\left(\omega L - \frac{1}{\omega C}\right)} \\
 &= \frac{(50 \times 10^{-3})/(2 \times 10^{-3})}{j\left(100 \cdot 50 \times 10^{-3} - \frac{1}{100 \cdot 2 \times 10^{-3}}\right)} \\
 &= \frac{25}{j(5-5)} = \frac{25}{j0} = \infty
 \end{aligned}$$

Hence the parallel inductor and capacitor combine to form infinite impedance at the resonant frequency ω . Then the circuit becomes 'open', so the resistor R is floating and hence has no voltage drop, i.e.

$$\tilde{V}_R = 0 \implies \tilde{V}_{out} = \tilde{V}_S$$

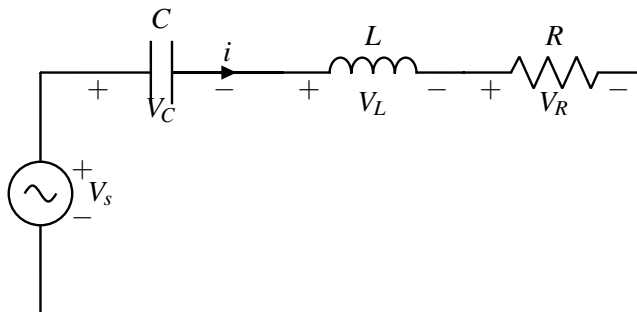
(d) Convert the phasor \tilde{V}_{out} back to get the time-domain signal $V_{out}(t)$.

Solution: Since $\tilde{V}_{out} = \tilde{V}_S$,

$$v_{out}(t) = 10\sqrt{2} \cos\left(100t - \frac{\pi}{4}\right)$$

7. RLC filter

Consider the following RLC circuit:



- (a) Write down the impedance of a series RLC circuit in the form $Z_{RLC}(\omega) = A(\omega) + jX(\omega)$, where $X(\omega)$ is a real valued function of ω .

Solution: Since the capacitor, resistor and inductor are in series, the equivalent impedance is given by,

$$Z_{RLC}(\omega) = R + Z_L(\omega) + Z_C(\omega)$$

$$\implies Z_{RLC}(\omega) = R + j\omega L + \frac{1}{j\omega C}$$

Since,

$$\frac{1}{j} = -j$$

$$Z_{RLC}(\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Hence,

$$A(\omega) = R$$

and

$$X(\omega) = \omega L - \frac{1}{\omega C}$$

- (b) Write the transfer function from V_S to V_R — the voltage drop across the resistor.

Solution: For an impedance divider, we know that:

$$V_{out}(\omega) = \frac{R}{Z_{RLC}(\omega)} V_{in}(\omega) = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} V_{in}(\omega)$$

Giving:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)}$$

Then, divide through by R to get a simpler expression to work with:

$$H(\omega) = \frac{1}{1 + j\left(\omega \frac{L}{R} - \frac{1}{\omega RC}\right)}$$

- (c) For the different specific values for R, L, C given by different cases (underdamped, overdamped, and critically damped) in the previous HW, use a computer to sketch plots of the magnitude and phase of the transfer function above. You can optionally use the included Jupyter notebook `tf_gen.ipynb`. (NOTE: In Python, use `1j` when your transfer function has a j .)

Solution:

From the last part, we have the magnitude and phase of the transfer function as:

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\omega \frac{L}{R} - \frac{1}{\omega RC}\right)^2}}$$

$$\angle(H(\omega)) = -\text{atan2}\left(\omega \frac{L}{R} - \frac{1}{\omega RC}, 1\right)$$

Now we will plot all the values.

Overdamped $R = 1k\Omega, C = 10nF, L = 25\mu H$

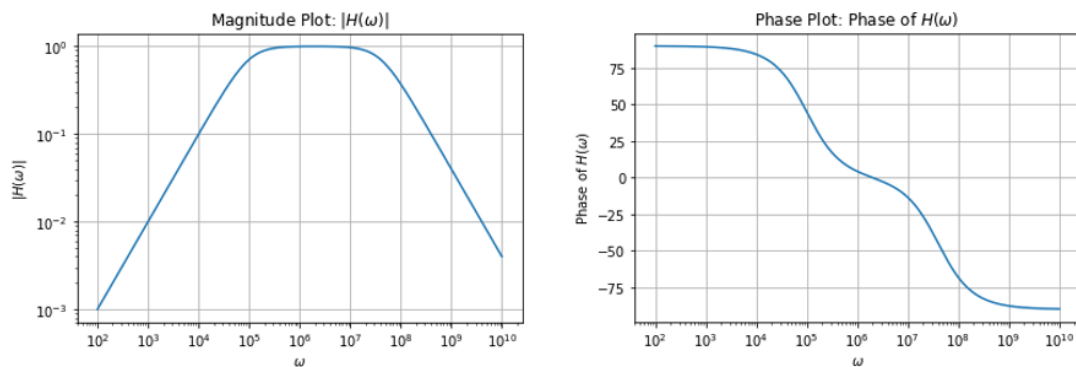


Figure 8: Magnitude and Phase transfer functions for overdamped case

Underdamped $R = 1\Omega, C = 10nF, L = 25\mu H$

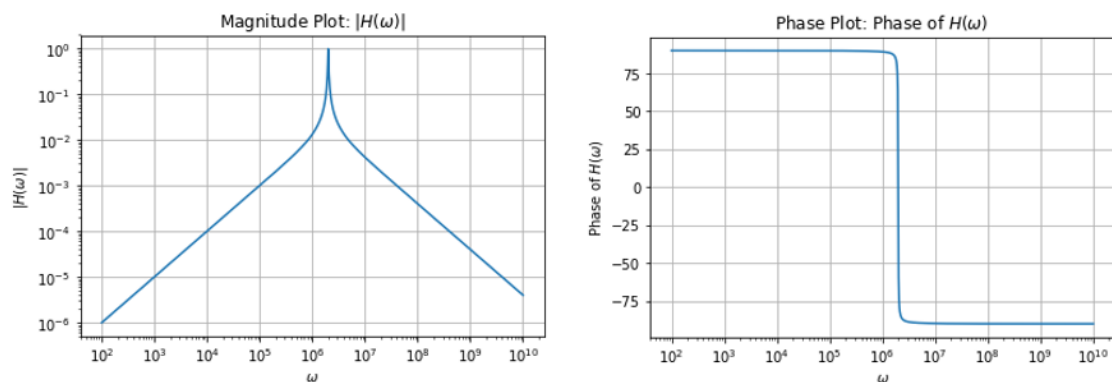


Figure 9: Magnitude and Phase transfer functions for underdamped case

critically damped $R = 2\sqrt{\frac{L}{C}} = 100, C = 10\text{nF}, L = 25\mu\text{H}$

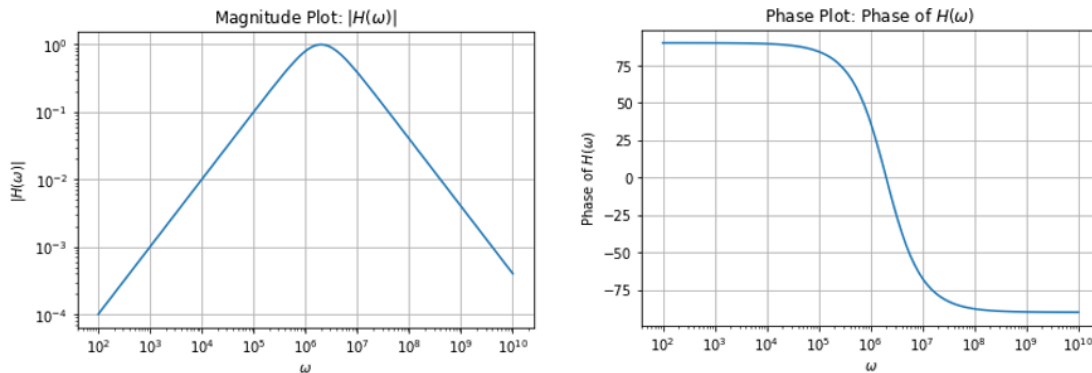


Figure 10: Magnitude and Phase transfer functions for critically damped case

- (d) To see how the values of R, L, C impact the impedance at different frequencies, run the included Jupyter notebook `hw6rlc_transfer.ipynb`. The script will generate two plots, the transfer function of the circuit as a function of frequency and the location of the eigenvalues in the imaginary, real plane. **Explain what happens at the following sets of values, and why it is interesting:**

	R	L	C
I	1	2.5E-5	1E-8
II	10	2.5E-5	1E-8
III	10	2.5E-5	2E-9
IV	500	.0001	2E-8

Table 1: Values for RLC Bandwidth problem, part d

Solution: In this part, the values for *I* are default, then parts *II, III, IV* show how the impedance peak can change location and magnitude. Please see the figure below, then explanatory text.

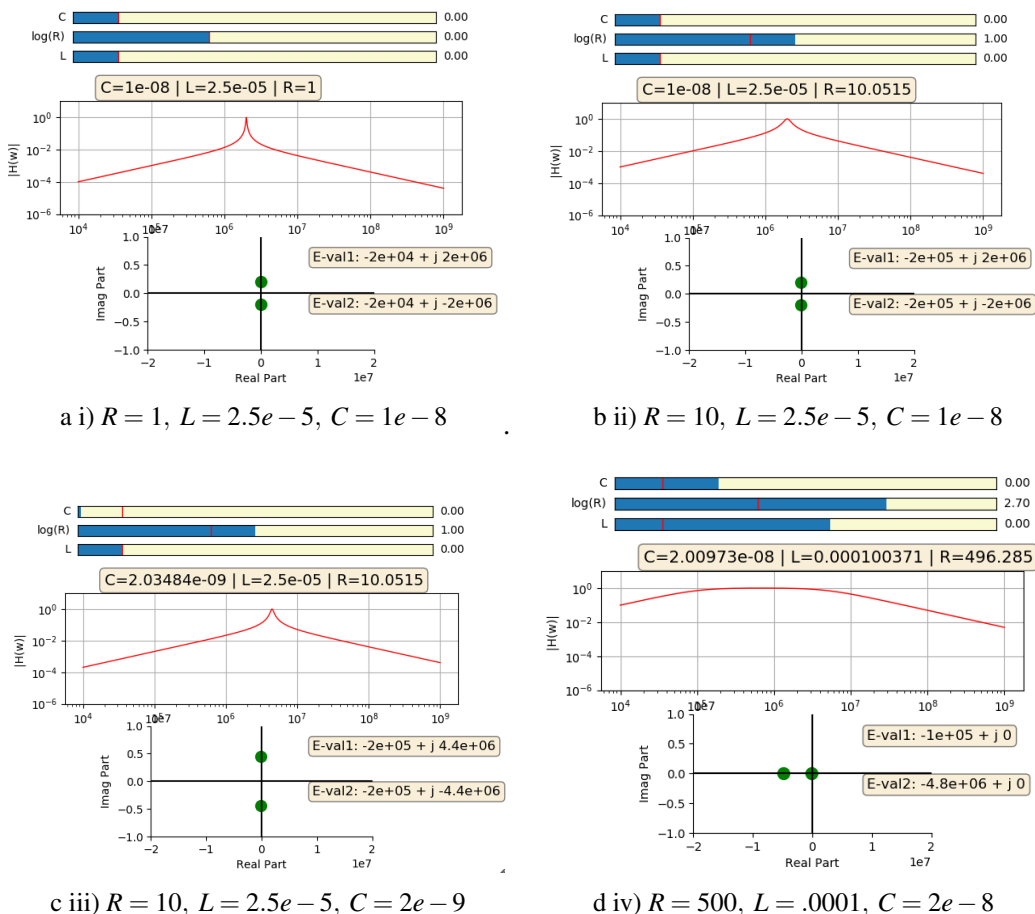


Figure 11: Here you can see how change L, C changes the resonant frequency, and increasing R increases the width of the resonant peak.

8. Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student!

We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

- (a) **What sources (if any) did you use as you worked through the homework?**
- (b) **If you worked with someone on this homework, who did you work with?**
List names and student ID's. (In case of homework party, you can also just describe the group.)

Contributors:

- Jaijeet Roychowdhury.
- Anant Sahai.
- Nathan Lambert.
- Kyoungtae Lee.
- Nikhil Shinde.

- Ayan Biswas.
- Geoffrey Négier.
- Regina Eckert.
- Sanjit Batra.
- Kris Pister.
- Daniel Abraham.