

EECS 16B Designing Information Devices and Systems II

Spring 2021 UC Berkeley

Homework 6

This homework is due on Friday, February 26, 2021, at 11:00PM. Self-grades and HW Resubmission are due on Tuesday, March 2, 2021, at 11:00PM.

1. Exam Policy and Practice

Your midterm is coming up on **March 15th, 2021**. To prepare for midterm proctoring, and to receive credit on this question, we ask you to read and complete the **Proctoring Document**. Make sure to **complete all action items in the document in order to receive credit for this question**.

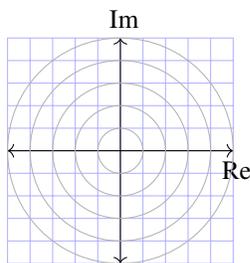
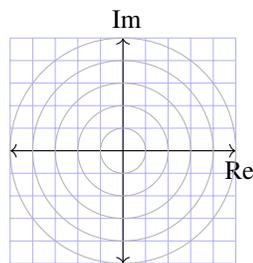
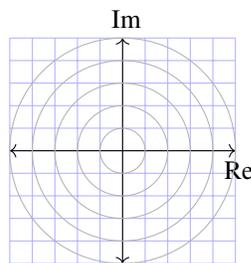
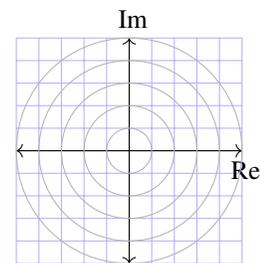
2. Reading Lecture Notes

Staying up to date with lectures is an important part of the learning process in this course. Here are links to the notes that you need to read for this week: [Note4](#), [Note5](#)

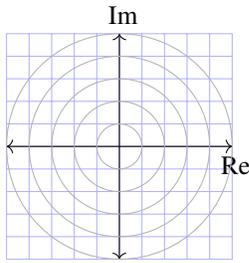
- Consider an RC circuit with a sinusoidal voltage input $V(t) = A \cos(\omega t)$. We are interested in finding the voltage on the capacitor in steady state (after a long time has passed). Can we solve this using our standard differential equation techniques? Can we solve this with phasors? Which one is more concise and why?
- There are two ways to make a low pass filter (discussed in the notes). What are they? Are there any major differences?

3. Phasors

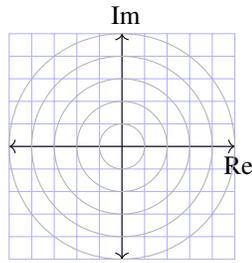
- Consider a resistor ($R = 1.5\Omega$), a capacitor ($C = 1F$), and an inductor ($L = 1H$) connected in series. **Give expressions for the impedances of Z_R, Z_C, Z_L for each of these elements as a function of the angular frequency ω .**
- Draw the individual impedances as “vectors” on the same complex plane for the case $\omega = \frac{1}{2}$ rad/sec. Also draw the combined impedance Z_{total} of their series combination. Then give the magnitude and phase of Z_{total} .**

a $Z_R(@\omega = 0.5)$ b $Z_C(@\omega = 0.5)$ c $Z_L(@\omega = 0.5)$ d $Z_{total}(@\omega = 0.5)$

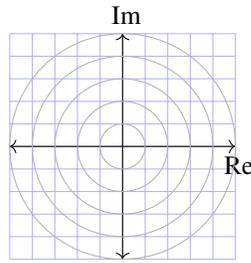
- Draw the individual impedances as “vectors” on the same complex plane for the case $\omega = 1$ rad/sec. Also draw the combined impedance Z_{total} of their series combination. Then give the magnitude and phase of Z_{total} .**



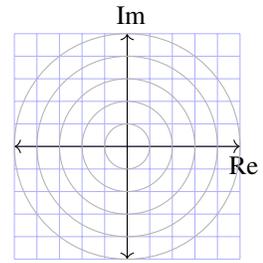
a $Z_R(@\omega = 1)$



b $Z_C(@\omega = 1)$

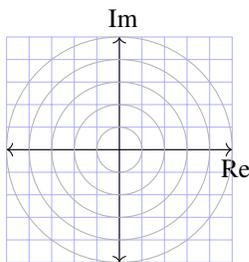


c $Z_L(@\omega = 1)$

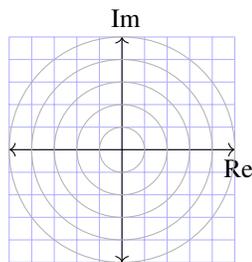


d $Z_{Total}(@\omega = 1)$

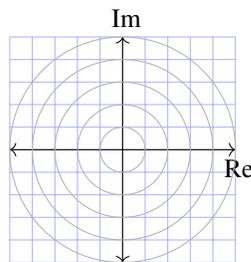
(d) Draw the individual impedances as “vectors” on the same complex plane for the case $\omega = 2$ rad/sec. Also draw the combined impedance Z_{total} of their series combination. Then give the magnitude and phase of Z_{total} .



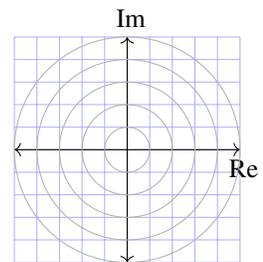
a $Z_R(@\omega = 2)$



b $Z_C(@\omega = 2)$



c $Z_L(@\omega = 2)$



d $Z_{Total}(@\omega = 2)$

(e) For the previous series combination of RLC elements, what is the “natural frequency” ω_n (i.e. the frequency ω_n where the series impedance is purely real). Fact: We call this the natural frequency since it is the frequency that the circuit will oscillate at if it was underdamped (if R was small enough).

4. Color Organ Filter Design

In the fourth lab, we will design low-pass, band-pass, and high-pass filters for a color organ. There are red, green, and blue LEDs. Each color will correspond to a specified frequency range of the input audio signal. The intensity of the light emitted will correspond to the amplitude of the audio signal.

- (a) First, you remember that you saw in lecture that you can build simple filters using a resistor and a capacitor. **Design a simple first-order passive low-pass filter with the following frequency range using 1 μ F capacitors.** (“Passive” means that the filter does not require any power supply to operate on the input signal. Passive components include resistors, capacitors, inductors, diodes, etc., while an example of an active component would be an op-amp).

- Low-pass filter: cut-off frequency $f_c = 2400\text{Hz}$, $\omega_c = 2\pi \cdot 2400 \frac{\text{rad}}{\text{sec}}$

Recall that the cutoff-frequency of such a filter is just where the magnitude of the filter is $\frac{1}{\sqrt{2}}$ of its peak value.

Show your work to find the resistor value that creates this low-pass filter. Draw the schematic-level representation of your design. Please mark V_{in} , V_{out} , and the ground node(s) in your schematic. Round your results to two significant figures.

- (b) **Now design a simple first-order passive high-pass filters with the following frequency range using 1 μ F capacitors.**

- High-pass filter: cut-off frequency $f_c = 100\text{Hz}$, $\omega_c = 2\pi \cdot 100 \frac{\text{rad}}{\text{sec}}$

Show your work to find the resistor value that creates this high-pass filter. Draw the schematic-level representation of your design. Please mark V_{in} , V_{out} , and the ground node(s) in your schematic. Round your results to two significant figures.

- (c) You can try to build a bandpass filter by cascading the first-order low-pass and high-pass filters you designed in parts (a) and (b). To do this, you might be tempted to connect the V_{out} node of your low-pass filter directly to the V_{in} node of your high-pass filter. However, if you did this, just as you saw in 16A for voltage dividers, the purported high-pass filter would “load” the low-pass filter and you might get some potentially complicated mess instead of what you wanted.

Show how you can use an ideal op-amp configured as a unity gain buffer to eliminate this loading effect to cascade the low-pass and high-pass filters, and write the resulting transfer function of the combined circuit. Draw the magnitude and phase transfer functions of the combined circuit. What kind of filter is this? You can optionally use the included Jupyter notebook `tf_gen.ipynb`. (HINT: Read Section 2.1 in *Note6*.)

(NOTE: In Python, use `1j` when your transfer function has a `j`.)

- (d) **Write down an expression for the time-domain output waveform $V_{\text{out}}(t)$ of this filter if the input voltage is $V_{\text{in}}(t) = 1 \sin(1000t)$ V.** Round your answer to 2 significant digits.

5. Phasors and Eigenvalues

Suppose that we have the two-dimensional system of differential equations expressed in matrix/vector form:

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + \vec{b}u(t) \quad (1)$$

where for this problem, the matrix A and the vector \vec{b} are both real.

- (a) **Give a necessary condition on the eigenvalues λ_k of A such that any impact of an initial condition will eventually completely die out.** (i.e. the system will reach steady-state.)

You don't have to prove this. The idea here is to make sure that you understand what kind of thing is required. (*HINT: Read Section 2 in Note4.*)

- (b) Now assume that $u(t)$ has a phasor representation \tilde{U} . In other words, $u(t) = \tilde{U}e^{j\omega t} + \overline{\tilde{U}}e^{-j\omega t}$.

Assume that the vector solution $\vec{x}(t)$ to the system of differential equations (1) can also be written in phasor form as

$$\vec{x}(t) = \tilde{X}e^{j\omega t} + \overline{\tilde{X}}e^{-j\omega t}. \quad (2)$$

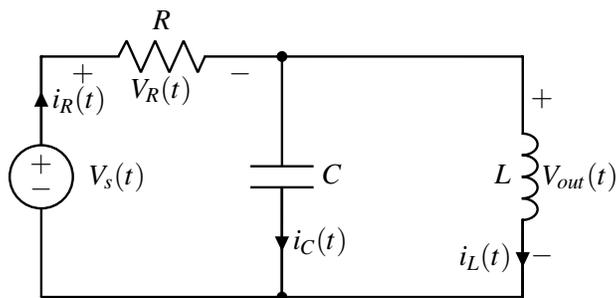
Derive an expression for \tilde{X} involving $A, \vec{b}, j\omega, \tilde{U}$, and the identity matrix I .

(*HINT: Plug (2) into (1) and simplify, using the rules of differentiation and grouping terms by which exponential $e^{\pm j\omega t}$ they multiply.*)

6. Phasor-Domain Circuit Analysis

The analysis techniques you learned previously in 16A for resistive circuits are equally applicable for analyzing circuits driven by sinusoidal inputs in the phasor domain. In this problem, we will walk you through the steps with a concrete example.

Consider the following circuit where the input voltage is sinusoidal. The end goal of our analysis is to find an equation for $V_{out}(t)$.



The components in this circuit are given by:

$$V_s(t) = 10\sqrt{2}\cos\left(100t - \frac{\pi}{4}\right)$$

$$R = 5\ \Omega$$

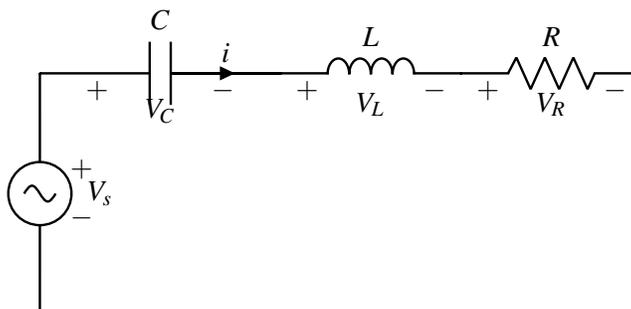
$$L = 50\text{mH}$$

$$C = 2\text{mF}$$

- Give the amplitude V_0 , oscillation frequency ω , and phase ϕ of the input voltage V_s .
- Transform the circuit into the phasor domain. **What are the impedances of the resistor, capacitor, and inductor? What is the phasor \tilde{V}_s of the input voltage $V_s(t)$?**
- Use the circuit equations to **solve for \tilde{V}_{out}** , the phasor representing the output voltage.
- Convert the phasor \tilde{V}_{out} back to get the time-domain signal $V_{out}(t)$.**

7. RLC filter

Consider the following RLC circuit:



- Write down the impedance of a series RLC circuit in the form $Z_{RLC}(\omega) = A(\omega) + jX(\omega)$, where $X(\omega)$ is a real valued function of ω .
- Write the transfer function from V_S to V_R — the voltage drop across the resistor.
- For the different specific values for R, L, C given by different cases (underdamped, overdamped, and critically damped) in the previous HW, use a computer to sketch plots of the magnitude and phase of the transfer function above. You can optionally use the included Jupyter notebook `tf_gen.ipynb`. (NOTE: In Python, use `1j` when your transfer function has a `j`.)
- To see how the values of R, L, C impact the impedance at different frequencies, run the included Jupyter notebook `hw6rlc_transfer.ipynb`. The script will generate two plots, the transfer function of the circuit as a function of frequency and the location of the eigenvalues in the imaginary, real plane. **Explain what happens at the following sets of values, and why it is interesting:**

	R	L	C
I	1	2.5E-5	1E-8
II	10	2.5E-5	1E-8
III	10	2.5E-5	2E-9
IV	500	.0001	2E-8

Table 1: Values for RLC Bandwidth problem, part d

8. Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student!

We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

(a) **What sources (if any) did you use as you worked through the homework?**

(b) **If you worked with someone on this homework, who did you work with?**

List names and student ID's. (In case of homework party, you can also just describe the group.)

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