Homework 4

This homework is due on Friday, September 23, 2022 at 11:59PM. Self-grades and HW Resubmissions are due the following Friday, September 30, 2022 at 11:59PM.

1. **Hambley P5.46**

   Find the phasors for the voltage and the currents of the circuit shown in Figure 1. Construct a phasor diagram showing I_s, V, I_R, and I_L. What is the phase relationship between V and I_s?

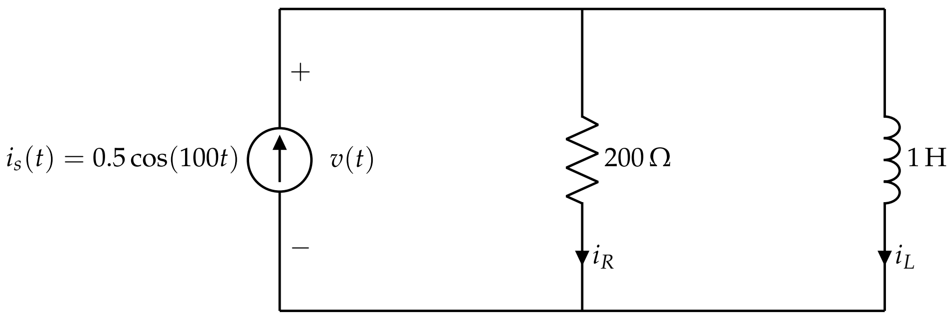


Figure 1: P5.46

Solution: We have that $I_s = 0.5\angle 0°$. The impedance of the resistor is 200 and the impedance of the inductor is j100. Hence, the combined, equivalent impedance is $\frac{1}{\frac{1}{200} - \frac{j}{100}}$. The angle is $-\arctan\left(\frac{-\frac{1}{100}}{\frac{1}{200}}\right) = 63.44°$ and the magnitude is $\frac{1}{\sqrt{(\frac{1}{200})^2 + (\frac{1}{100})^2}} = 89.44$. Hence,

$$V = I_s \frac{1}{\frac{1}{200} - \frac{j}{100}} = 44.72\angle 63.44° \tag{1}$$

Using Ohm's law, we have

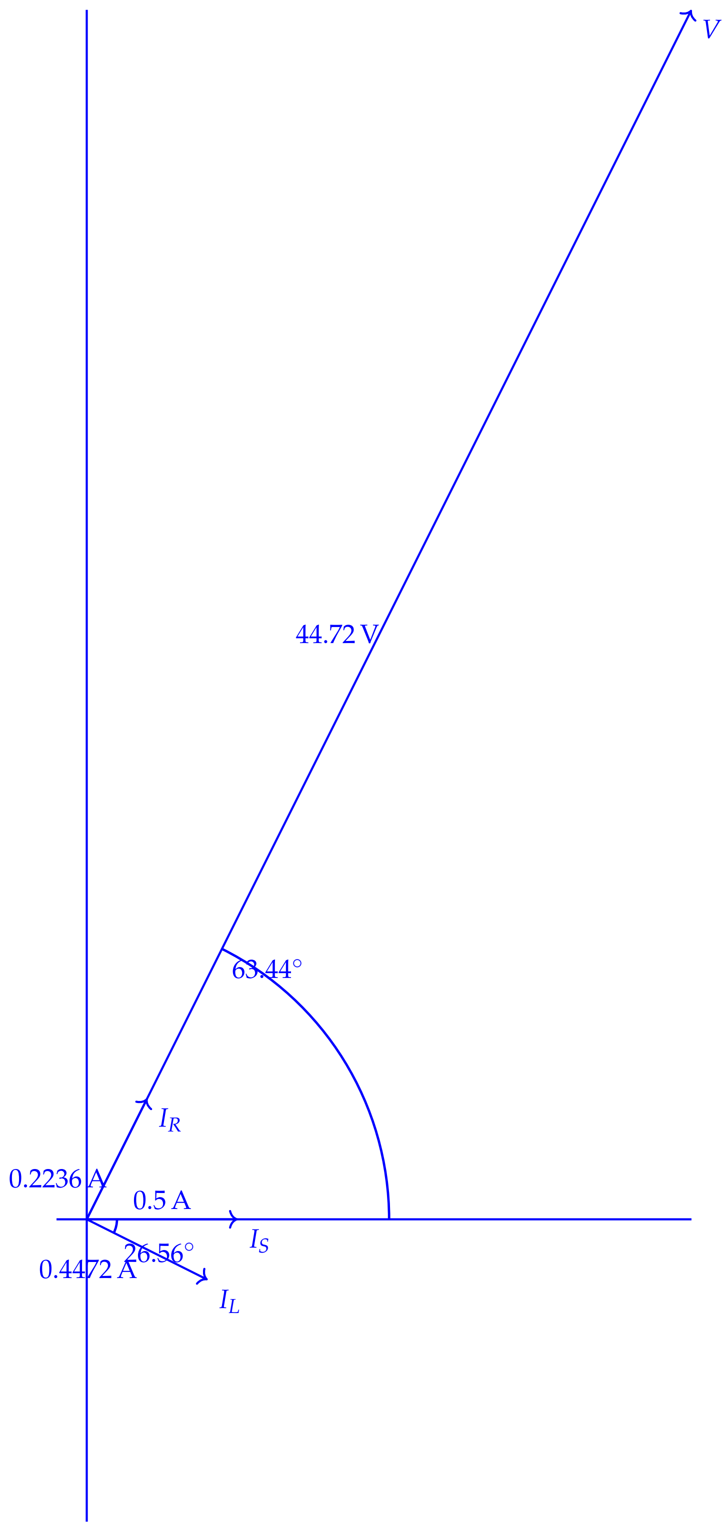
$$I_R = \frac{V}{R} = 0.2236\angle 63.44° \tag{2}$$

and

$$I_L = \frac{V}{j\omega L} \tag{3}$$

$$= 0.4472\angle(63.44 - 90)° = 0.4472\angle -26.56° \tag{4}$$

Hence, V leads I_s by 63.44°. A phase diagram is shown below:


V
44.72 V
63.44°
I_R
0.2236 A
0.5 A
I_S
26.56°
0.4472 A
I_L

2. Hambley P5.65

Consider a load that has an impedance given by $Z = 100 - \mathrm{j}50\Omega$. The current flowing through the load is $I = 15\sqrt{2}\angle 30^\circ\,\mathrm{A}$. Is the load inductive or capacitive? Determine the power factor, power, reactive power, and apparent power delivered to the load.

Solution: This is a capacitive load because the reactance (imaginary part of the impedance) is negative. The RMS current is $I_{\mathrm{RMS}} = \frac{I}{\sqrt{2}} = 15$. The power is

$$P = I_{\mathrm{RMS}}^2 R = (15^2)(100) = 22.5\,\mathrm{kW} \tag{5}$$

and the reactive power is

$$Q = I_{\mathrm{RMS}}^2 X = (15^2)(-50) = -11.25\,\mathrm{kV\,A} \tag{6}$$

The angle between the two is $\theta = \arctan\left(\frac{Q}{P}\right) = -26.57^\circ$, so the power factor is $\cos(\theta) = 89.44\%$ and the apparent power is $\sqrt{P^2 + Q^2} = 25.16\,\mathrm{kV\,A}$

3. Hambley P5.83

(a) Find the Thevenin and Norton equivalent circuits for the circuit shown in Figure 2.

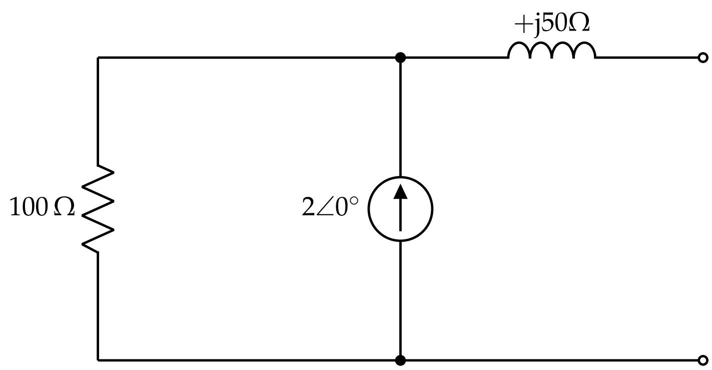
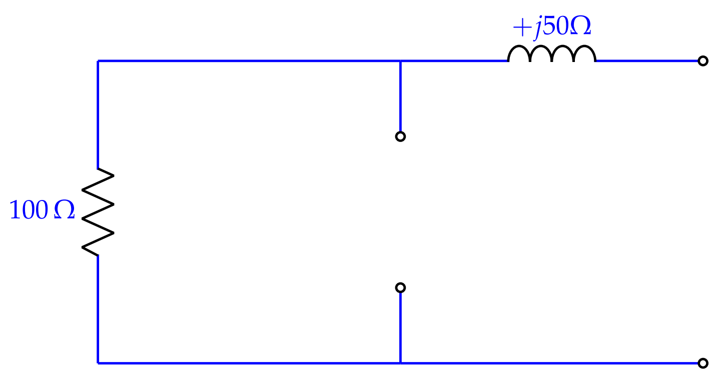
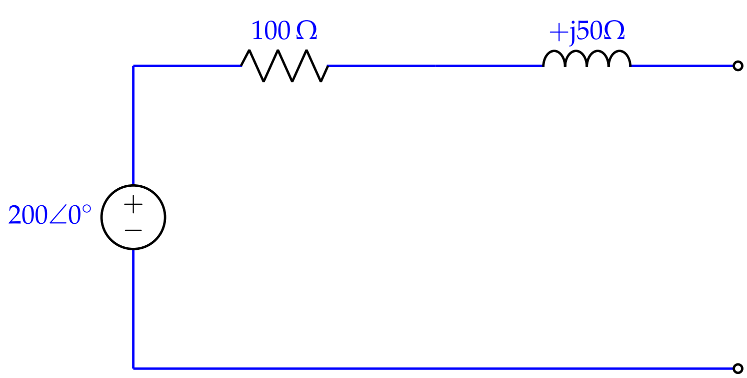


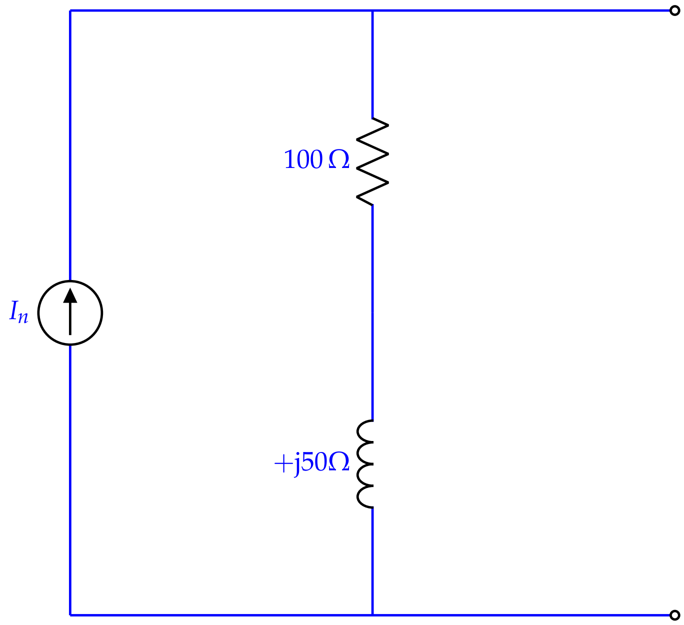
Figure 2: P5.83

Solution: Zeroing the current source, we have



Thus, the Thevenin impedance is $Z_t = 100 + \text{j}50 = 111.8\angle 26.57°$. Under open circuit conditions, there is zero voltage across the inductance, since the current flows through the resistance. Hence, the Thevenin voltage is $V_t = (2\angle 0°)(100) = 200\angle 0°$. From this, the Norton current is $I_n = \frac{V_t}{Z_t} = 1.789\angle -26.57°$. The Thevenin and Norton equivalent circuits are shown below respectively:





(b) Find the maximum power that this cicuit can deliver to a load if the load can have any complex impedance.

Solution: The load that would maximize power is $Z_{\text{load}} = (Z_t)^* = 100 - \text{j}50$. Using the Thevenin equivalent circuit, the current in this circuit would then be

$$I_{\text{load}} = \frac{V_t}{Z_t + Z_{\text{load}}} = \frac{200}{100 + \text{j}50 + 100 - \text{j}50} = 1 \tag{7}$$

so $I_{\text{load}-\text{RMS}} = \frac{1}{\sqrt{2}}$. Finally, the power would be $P_{\text{load}} = R_{\text{load}}(I_{\text{load}-\text{RMS}})^2 = 100\left(\frac{1}{\sqrt{2}}\right)^2 = 50\,\text{W}$.

(c) Repeat the previous part, but this time the load is purely resistive.

Solution: The purely resistive load that would maximize power is $Z_{\text{load}} = |Z_t| = 111.8$. Again, using the Thevenin equivalent circuit, we find that

$$I_{\text{load}} = \frac{V_t}{Z_t + Z_{\text{load}}} = \frac{200}{100 + \text{j}50 + 111.8} = 0.9190\angle - 13.28^\circ \tag{8}$$

so $I_{\text{load}-\text{RMS}} = \frac{0.9190}{\sqrt{2}}$. Finally, the power would be $P_{\text{load}} = R_{\text{load}}(I_{\text{load}-\text{RMS}})^2 = 100\left(\frac{0.9190}{\sqrt{2}}\right)^2 = 47.21\,\text{W}$.

4. Hambley P6.27

The input signal of a first-order lowpass filter with the transfer function given by Equation 6.9 on page 288 of the text and a half-power frequency of 400 Hz is

$$v_{\text{in}}(t) = 1 + 2\cos(800\pi t + 30^\circ) + 3\cos\left(20 \times 10^3 \pi t\right) \tag{9}$$

Find an expression for the output voltage.

Solution: The transfer function is given by Equation 6.9 in the text:

$$H(f) = \frac{1}{1 + \mathrm{j}\frac{f}{f_B}} \tag{10}$$

The given input signal is

$$v_{\text{in}}(t) = 1 + 2\cos(800\pi t + 30^\circ) + 3\cos\left(20 \times 10^3 \pi t\right) \tag{11}$$

which has components with frequencies of 0, 400, and 10000 Hz. Evaluating the transfer function for these frequencies yields

$$H(0) = \frac{1}{1 + \mathrm{j}\frac{0}{400}} = 1 \tag{12}$$

$$H(400) = 0.7071\angle - 45^\circ \tag{13}$$

$$H(10 \times 10^3) = 0.04\angle - 87.71^\circ \tag{14}$$

Applying the appropriate value of the transfer function to each component of the input signal yields the output:

$$v_{\text{out}}(t) = 1 + 1.4142\cos(800\pi t - 15^\circ) + 0.12\cos\left(20 \times 10^3 \pi t - 87.71^\circ\right) \tag{15}$$

5. Hambley P6.30

Sketch the magnitude of the transfer function $H(f) = \frac{V_{\text{out}}}{V_{\text{in}}}$ to scale versus frequency for the circuit shown in Figure 3. What is the value of the half-power frequency? *(HINT: Start by finding the Thevenin equivalent circuit seen by the capacitance.)*

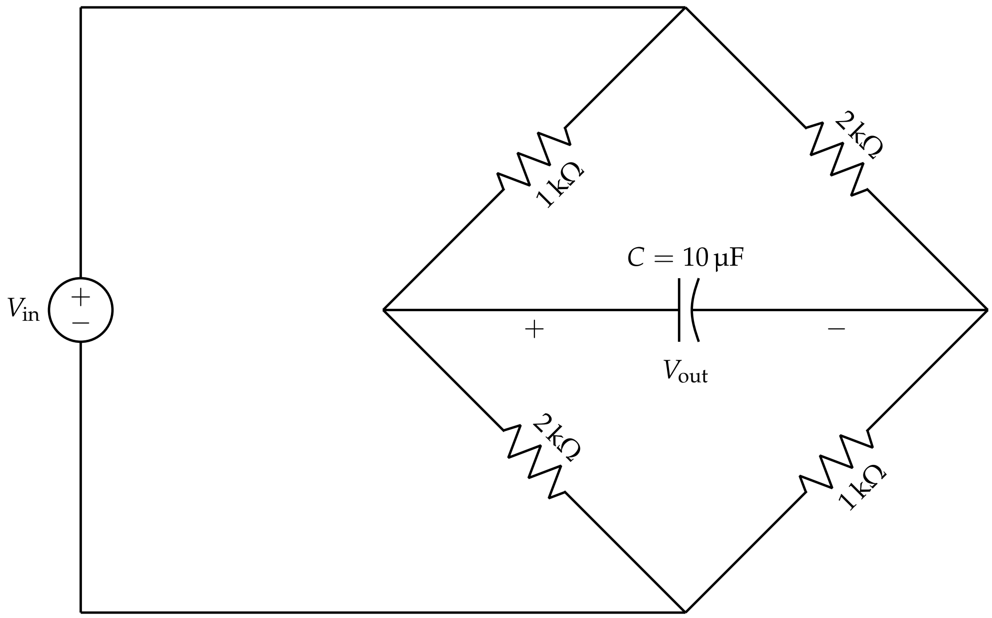
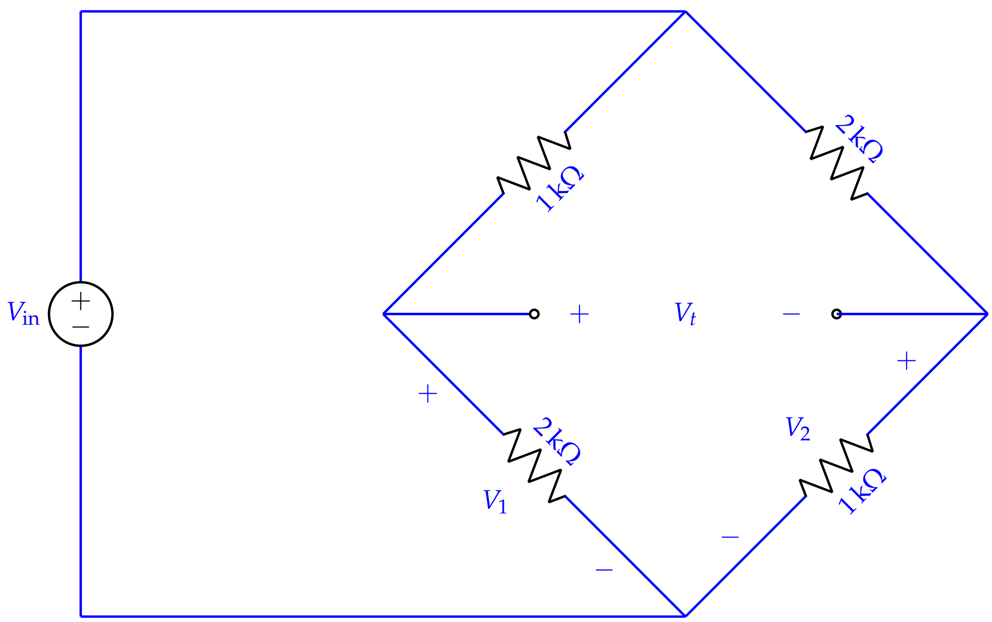


Figure 3: P6.30

Solution:

The circuit seen by the capacitance is

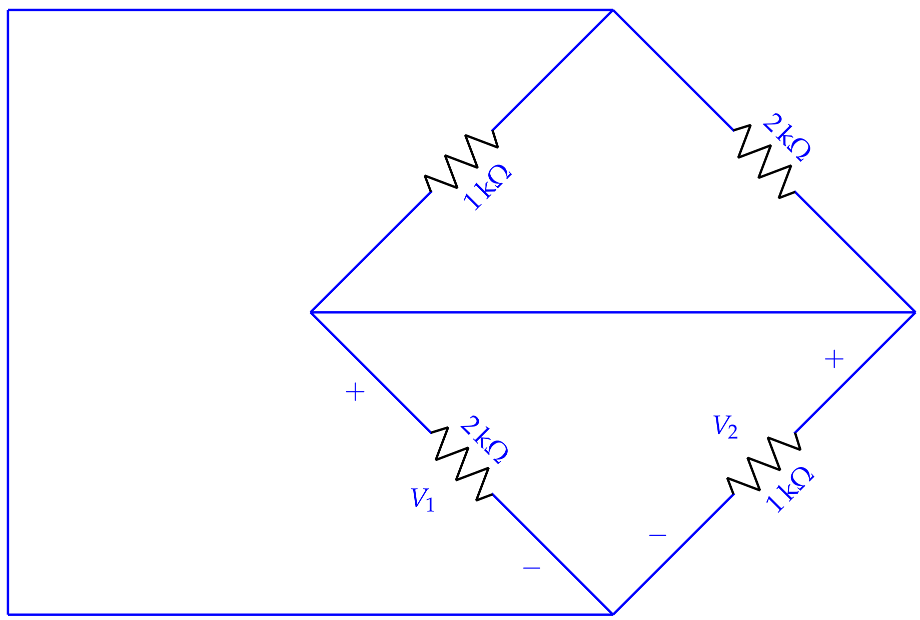


The open-circuit or Thevenin voltage is

$$V_t = V_1 - V_2 \tag{16}$$

$$= V_{\text{in}}\frac{2000}{2000+1000} - V_{\text{in}}\frac{1000}{1000+2000} \tag{17}$$
$$= \frac{1}{3}V_{\text{in}} \tag{18}$$

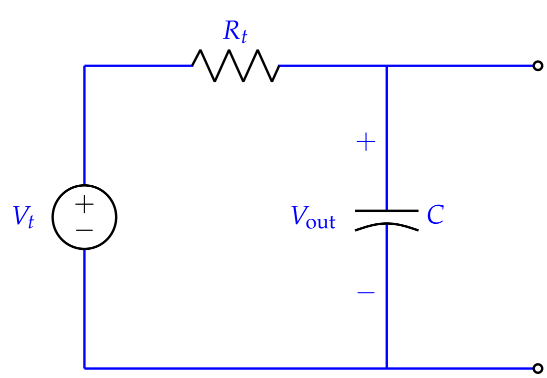
Zeroing the source, we have



The Thevenin resistance is

$$R_t = \frac{1}{\frac{1}{1000}+\frac{1}{2000}} + \frac{1}{\frac{1}{2000}+\frac{1}{1000}} \tag{19}$$
$$= 1333\,\Omega \tag{20}$$

Thus, the equivalent circuit is



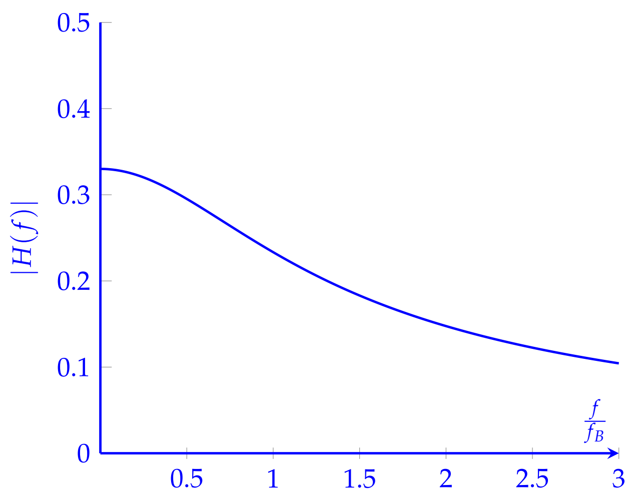
As in the text, this circuit has the transfer function

$$\frac{V_{\text{out}}}{V_t} = \frac{1}{1+\mathrm{j}\frac{f}{f_B}} \tag{21}$$

where $f_B = \frac{1}{2\pi R_t C} = \frac{1}{2\pi(1333)\times 10^{-5}} = 11.94\,\text{Hz}$. Using eq. (18) to substitute for V_t in eq. (21) and rearranging, we have

$$\frac{V_{\text{out}}}{V_t} = \frac{\frac{1}{3}}{1 + \mathrm{j}\frac{f}{f_B}} \tag{22}$$

A sketch of the transfer function magnitude is shown below:



6. Hambley P6.33

Consider the circuit shown in Figure 4. This circuit consists of a source having an internal resistance of R_s, an RC lowpass filter, and a load resistance of R_l.

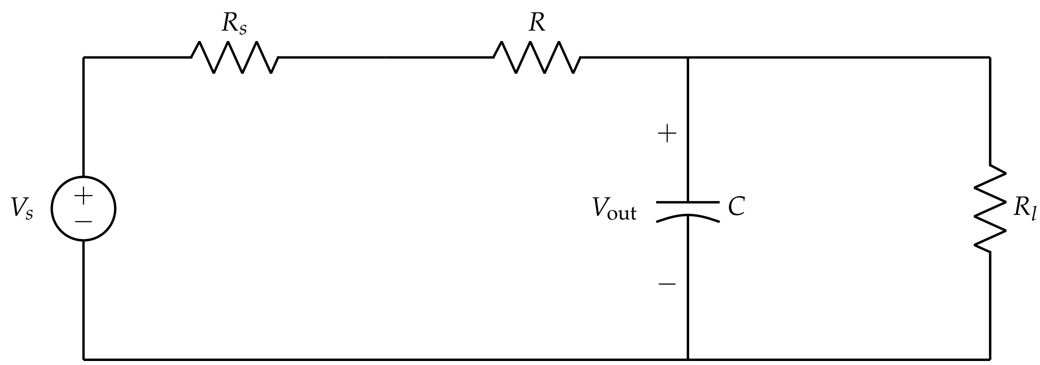


Figure 4: P6.33(a)

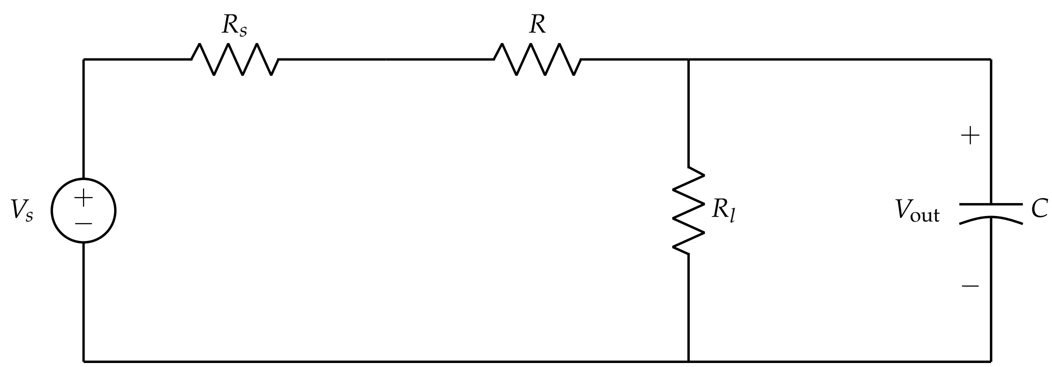


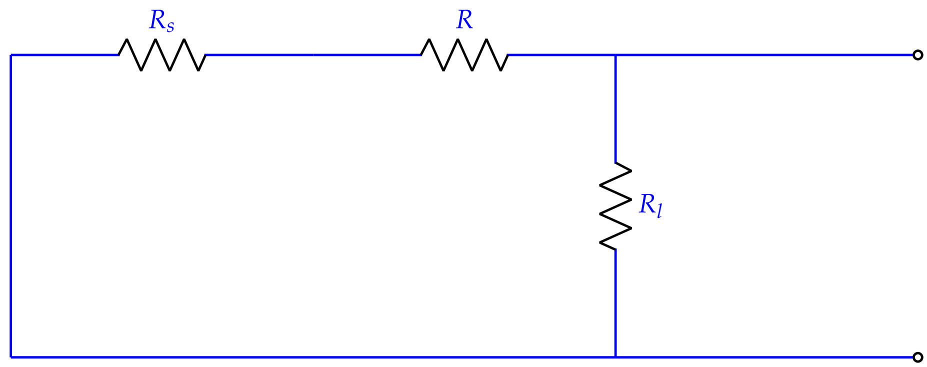
Figure 5: P6.33(b)

(a) Show that the transfer function of this circuit is given by

$$H(f) = \frac{V_{\text{out}}}{V_s} = \frac{R_l}{R_s + R + R_l} \times \frac{1}{1 + \mathrm{j}\frac{f}{f_B}} \tag{23}$$

in which the half-power frequency f_B is given by $f_B = \frac{1}{2\pi R_t C}$ where $R_t = \frac{R_L(R_s+R)}{R_l+R_s+R}$. Notice that R_t is the parallel combination of R_l and $(R_s + R)$. *(HINT: One way to make this problem easier is to rearrange the circuit as shown in Figure 5 and then to find the Thevenin equivalent for the source and resistances.)*

Solution: First, we find the Thevenin equivalent for the source and resistances.



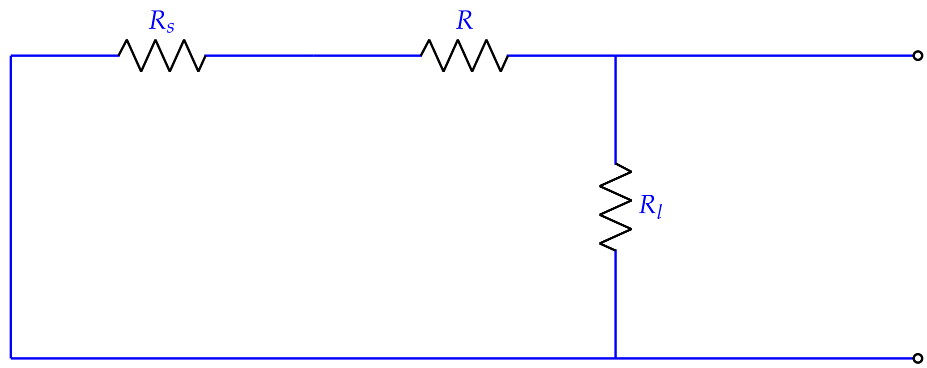
THe open-circuit voltage is given by

$$v_t(t) = v_s(t)\frac{R_l}{R_s + R + R_l} \tag{24}$$

In terms of phasors, this becomes

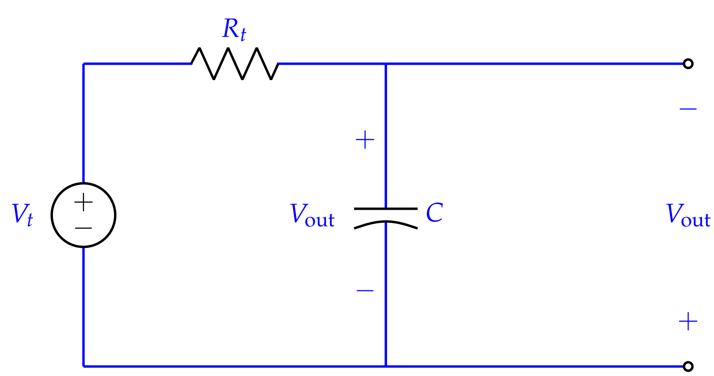
$$V_t = V_s\frac{R_l}{R_s + R + R_l} \tag{25}$$

Zeroing the source, we find the Thevenin resistance:



$$R_t = \frac{1}{\frac{1}{R_l} + \frac{1}{R+R_s}} \tag{26}$$

Thus, the original circuit has the equivalent:



The transfer function for this circuit is

$$\frac{V_{\text{out}}}{V_t} = \frac{1}{1 + \mathrm{j}\frac{f}{f_B}} \tag{27}$$

where $f_B = \frac{1}{2\pi R_t C}$. Using eq. (25) to substitute for V_t in eq. (27) and rearranging, we have

$$H(f) = \frac{V_{\text{out}}}{V_s} = \frac{R_l}{R_s + R + R_l} \times \frac{1}{1 + \mathrm{j}\frac{f}{f_B}} \tag{28}$$

(b) Given that $C = 0.2\,\mu\text{F}$, $R_s = 2\,\text{k}\Omega$, $R = 47\,\text{k}\Omega$, and $R_l = 1\,\text{k}\Omega$, sketch the magnitude of $H(f)$ to scale versus $\frac{f}{f_B}$ from 0 to 3.

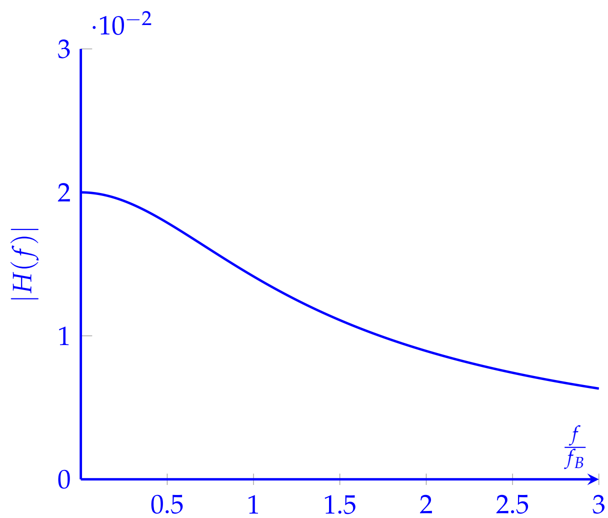
Solution: Evaluating for the circuit components given, we have

$$R_t = 980\,\Omega \tag{29}$$

$$f_B = 812\,\text{Hz} \tag{30}$$

$$H(f) = \frac{0.02}{1 + \mathrm{j}\frac{f}{f_B}} \tag{31}$$

The resulting plot is



7. Hambley P6.53

A transfer function is given by

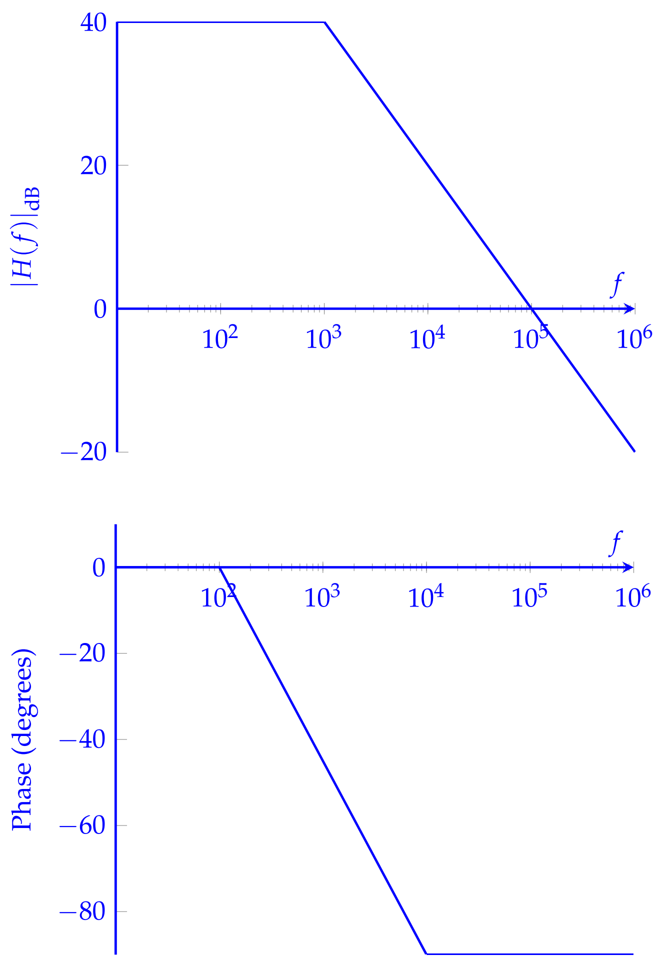
$$H(f) = \frac{100}{1 + j\frac{f}{1000}} \tag{32}$$

Sketch the asymptotic magnitude and phase Bode plots to scale. What is the value of the half-power frequency?

Solution: We have that

$$|H(f)| = \frac{100}{\sqrt{1 + \left(\frac{f}{1000}\right)^2}} \tag{33}$$

so $|H(f)|_{\text{dB}} = 20\log(100) - 20\log\left(\sqrt{1 + \left(\frac{f}{1000}\right)^2}\right) = 40 - 20\log\left(\sqrt{1 + \left(\frac{f}{1000}\right)^2}\right)$. This is similar to the transfer function treated in Section 6.4 in the text except for the additional 40 dB constant. The half-power frequency is $f_B = 1000$. The asymptotic Bode plots are:



8. Hambley P6.55

Sketch the asymptotic magnitude and phase Bode plots to scale for the transfer function

$$H(f) = 10\frac{1 - j\frac{f}{100}}{1 + j\frac{f}{100}} \tag{34}$$

Solution: We have that

$$|H(f)| = 10\frac{\sqrt{1 + \left(\frac{f}{100}\right)^2}}{\sqrt{1 + \left(\frac{f}{100}\right)^2}} = 10 \tag{35}$$

so $|H(f)|_{\text{dB}} = 20\log(10) = 20$. The phase is $-2\arctan\left(\frac{f}{100}\right)$. The asymptotic Bode plots are: