Homework 2

This homework is due on Friday, September 9, 2022 at 11:59PM. Self-grades and HW Resubmissions are due the following Friday, September 16, 2022 at 11:59PM.

1. **Capacitor Physics**

   The two metallic plates of a parallel plate capacitor is separated by glass (permittivity $\varepsilon = 3.9 \times 8.854 \times 10^{-12}\,\frac{\text{F}}{\text{m}}$) whose thickness is 1 µm.

   (a) What is the capacitance per unit area?

   (b) It is known that the resistance of a material can be expressed as $R = \frac{\rho L}{A}$ where ρ is called resistivity (a material dependent constant), L is the length and A is the area of the resistor. Show the time constant in a R-C circuit with the same area for a resistor and capacitor is independent of the area.

   (c) If $\rho = 1.72 \times 10^{-8}\,\Omega\,\text{m}$ (this is the value for copper), and the length of the resistor used is 1 meter, what is the time constant of this R-C circuit?

2. Hambley P4.3 and P4.4

(a) The initial voltage across the capacitor shown in Figure 1 is $v_c(0+) = 0$. Find an expression for the voltage across the capacitor as a function of time, and sketch it to scale versus time.

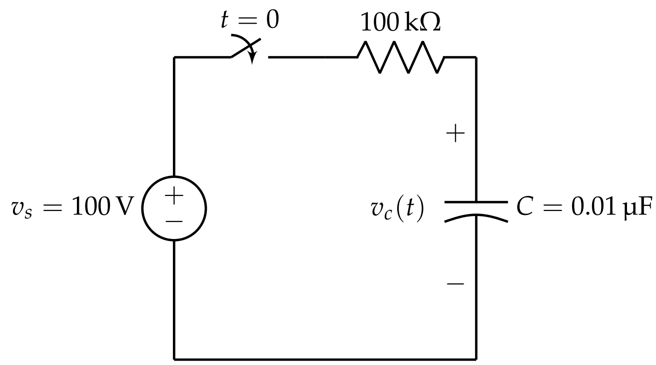

Figure 1: P4.3

(b) Repeat part (a) for an initial voltage $v_c(0+) = -50\,\text{V}$.

3. Hambley P4.7

The capacitor shown in Figure 2 is charged to a voltage of 50 V prior to $t = 0$.

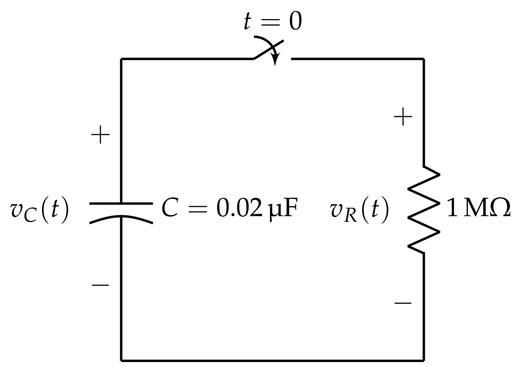

Figure 2: P4.7

(a) Find expressions for the voltage across the capacitor $v_C(t)$ and the voltage across the resistor $v_R(t)$.

(b) Find an expressino for the power delivered to the resistor.

(c) Integrate the power from $t = 0$ to $t = \infty$ to find the energy delivered.

(d) Show that the energy delivered to the resistor is equal to the energy stored in the capacitor prior to $t = 0$.

4. Hambley P4.15

A capacitance C is charged to an initial voltage V_i. At $t = 0$, a resistance R is connected across the capacitance. Write an expression for the current. Then, integrate the current from $t = 0$ to $t = \infty$, and show that the result is equal to the initial charge stored on the capacitance.

5. Hambley P4.18

Consider the circuit shown in Figure 3. Prior to $t = 0$, $v_1 = 100\,\text{V}$ and $v_2 = 0$.

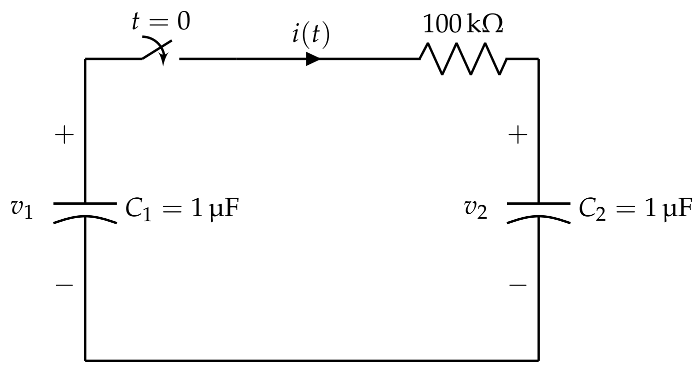

Figure 3: P4.18

(a) Immediately after the switch is closed, what is the value of the current (i.e., what is the value of $i(0+)$)?

(b) Write the KVL equation for the circuit in terms of the current and initial voltages. Take the derivative to obtain a differential equation.

(c) What is the value of the time constant in this circuit?

(d) Find an expression for the current as a function of time.

(e) Find the value that v_2 approaches as t becomes very large.

6. Capacitor Energy

Say a series R-C circuit is supplied by a constant voltage V. At $t = 0$, voltage across the capacitor was 0. We know how to find the expression for capacitor voltage as a function of time. Using this expression,

(a) Find the expression for total stored energy at $t = \infty$, $w_s = \int_0^\infty v(t)i(t)\,\mathrm{d}t$ (i.e., at steady state).

(b) We know that when a current flows through a resistor, we dissipate energy at a rate of i^2R. Using this relation, find the total dissipated energy at $t = \infty$, $w_d = \int_0^\infty i^2R\,\mathrm{d}t$ (i.e., at steady state).

(c) Find the total energy taken from the source, noting that some of it was stored and some of it was dissipated.

(d) Does the result in part (b) vary if $R = 100\,\Omega$ vs $R = 1\,\mathrm{k}\Omega$? What about $R = 0\,\Omega$? *OPTIONAL: Can you explain this result?*

7. Hambley P4.21

Solve for the steady-state values of i_1, i_2, and i_3 in the circuit shown in Figure 4.

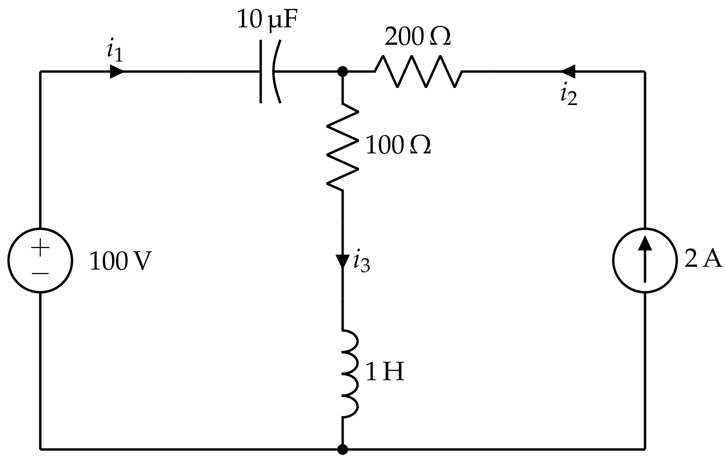

Figure 4: P4.21

8. Hambley P4.41

Determine expressions for and sketch $v_R(t)$ to scale versus time for the circuit of Figure 5. The circuit is operating in steady state with the switch closed prior to $t = 0$. Consider the time interval $-0.2 \leq t \leq 1\,\text{ms}$.

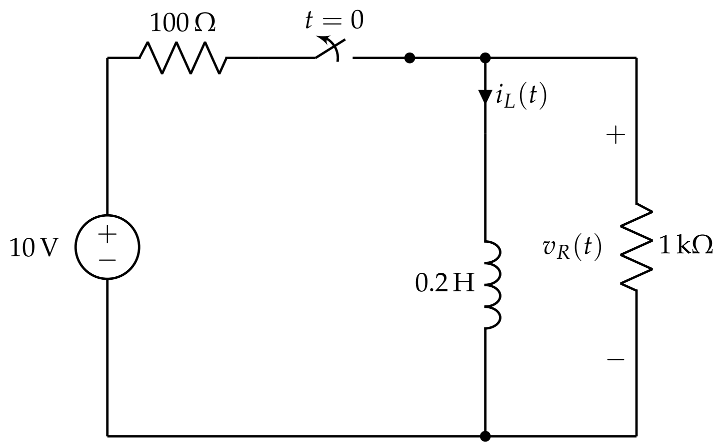

Figure 5: P4.41

9. Hambley P4.46

Consider the circuit shown in Figure 6. The voltage source is known as a **ramp function**, which is defined by

$$v(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases} \tag{1}$$

Assume that $v_C(0) = 0$. Derive an expression for $v_C(0)$ for $t \geq 0$. Sketch $v_C(t)$ to scale versus time.

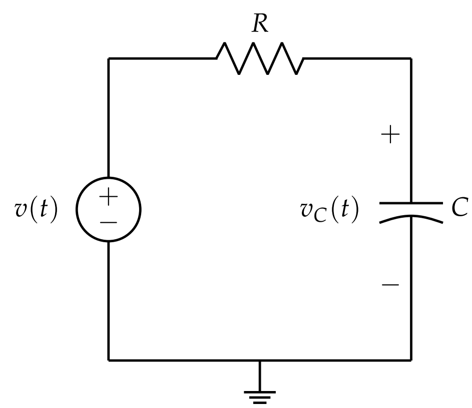

Figure 6: P4.46

