Homework 1

This homework is due on Friday, September 2, 2022, at 11:59PM. Self-grades and HW Resubmissions are due on the following Friday, September 9, 2022, at 11:59PM.

1. **Hambley P3.7**

   A constant (dc) current $i(t) = 3\,\text{mA}$ flows into a $50\,\mu\text{F}$ capacitor. The voltage at $t = 0$ is $v(0) = -20\,\text{V}$. The references for $v(t)$ and $i(t)$ have the passive configuration. Find the power at $t = 0$ and state whether the power flow is into or out of the capacitor. Repeat for $t = 1$.

   Solution: We can use the capacitor differential equation and integrate both sides as follows:

$$v(t) = \frac{1}{C}\int_0^t i(\theta)\,\mathrm{d}\theta + v(0) \tag{1}$$

$$= (2 \times 10^6)\int_0^t (3 \times 10^{-3})\,\mathrm{d}\theta - 20 \tag{2}$$

$$= 60t - 20 \tag{3}$$

   and $p(t) = i(t)v(t) = (3 \times 10^{-3})(60t - 20)$. Evaluating at $t = 0$, we have $p(0) = -60\,\text{mW}$. Because the power has a negative value, the capacitor is delivering energy. At $t = 1\,\text{s}$, we have $p(1) = 120\,\text{mW}$. Because the power is positive, we know that the capacitor is absorbing energy.

2. Hambley P3.16

A capacitance and the current through it are shown in Figure 1 and Figure 2 respectively. At $t = 0$, the voltage is $v_C(0) = 10\,\mathrm{V}$. Sketch the voltage, power, and stored energy to scale versus time.

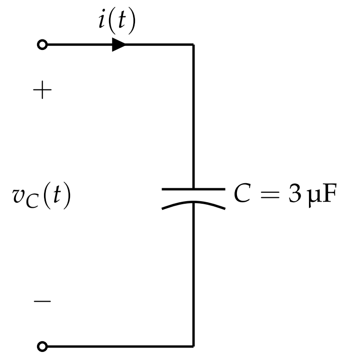

Figure 1: Circuit for P3.16

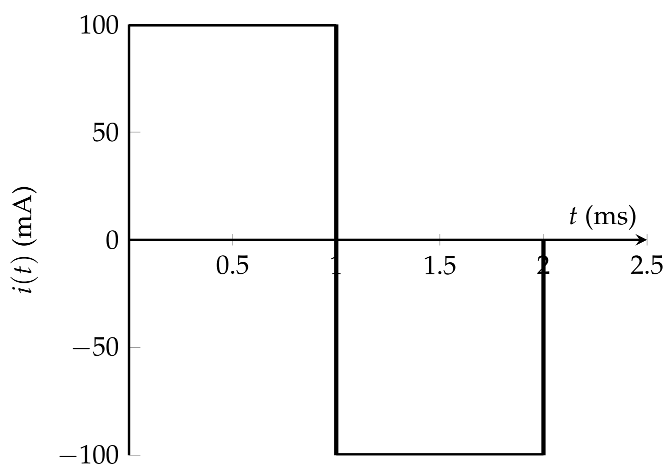

Figure 2: Current vs Time for P3.16

Solution: We can apply the capacitor differential equation and integrate both sides, yielding

$$v_C(t) = \frac{1}{C}\int_0^t i(t')\,\mathrm{d}t' + v(0) \tag{4}$$

$$= \left(\frac{1}{3}\times 10^6\right)\int_0^t i(t')\,\mathrm{d}t' + 10 \tag{5}$$

The plot of this is as follows:

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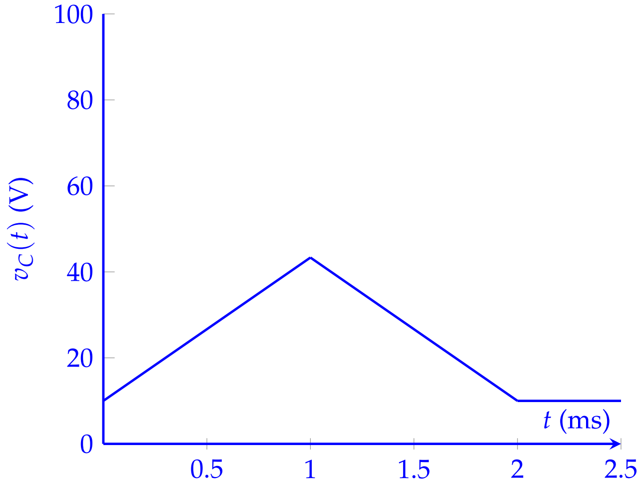
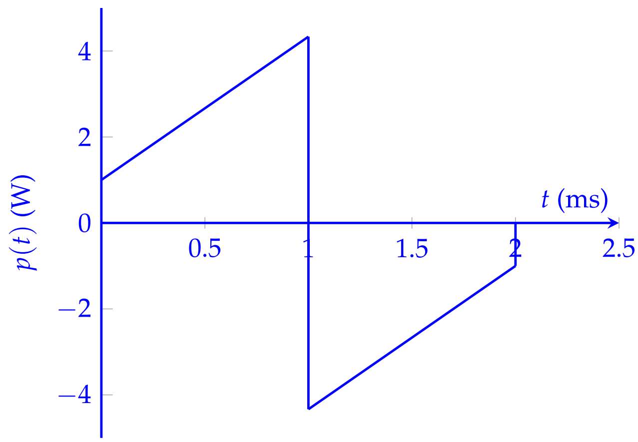
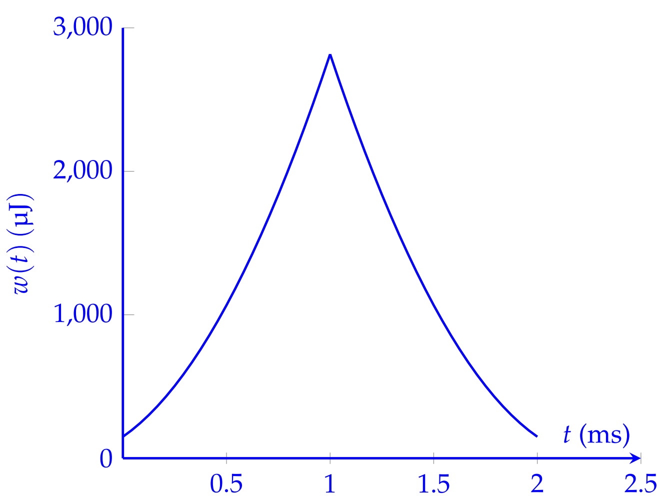


Next, power is the pointwise multiplication of the above graph with Figure 2, so we have



Lastly, the stored energy is given by $\frac{1}{2}C(v_C(t))^2 = 1.5 \times 10^{-6} \times (v_C(t))^2$, so we have the following plot



3. Hambley P3.34

We have a parallel-plate capacitor, with each plate having a width w and a length ℓ. The plates are separated by air with a distance d. Assume that ℓ and w are both much larger than d. The maximum voltage that can be applied is limited to $V_{\max} = Kd$, in which K is called the breakdown strength of the dielectric. Derive an expression for the maximum energy that can be stored in the capacitor in terms of K and the volume of the dielectric. If we want to store the maximum energy per unit volume, does it matter what values are chosen for ℓ, w, and d? What parameters are important?

Solution: Using $C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon_r \varepsilon_0 w\ell}{d}$ and $V_{\max} = Kd$ to substitute into $W_{\max} = \frac{1}{2}CV_{\max}^2$, we have $W_{\max} = \frac{1}{2}\frac{\varepsilon_r \varepsilon_0 w\ell}{d}K^2 d^2 = \frac{1}{2}\varepsilon_r \varepsilon_0 K^2 w\ell d$. However, the voluem of the dielectric is $\text{Vol} = w\ell d$, so we have

$$W_{\max} = \frac{1}{2}\varepsilon_r \varepsilon_0 K^2 \cdot \text{Vol} \tag{6}$$

Thus, we conclude that the maximum energy stored is independent of w, ℓ, and d if the voluem is constant and if both w and ℓ are much larger than d. To achieve large energy storage per unit volume, we should look for a dielectric having a large value for $\varepsilon_r K^2$. The dielectric should have high relative dielectric constant and high breakdown strength.

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4. Hambley P3.43

The current flowing through a 2 H inductance is shown in Figure 3. Sketch the voltage, power, and stored energy vs time.

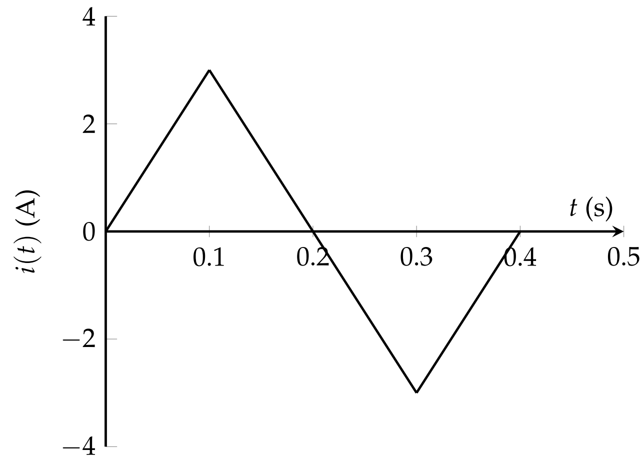
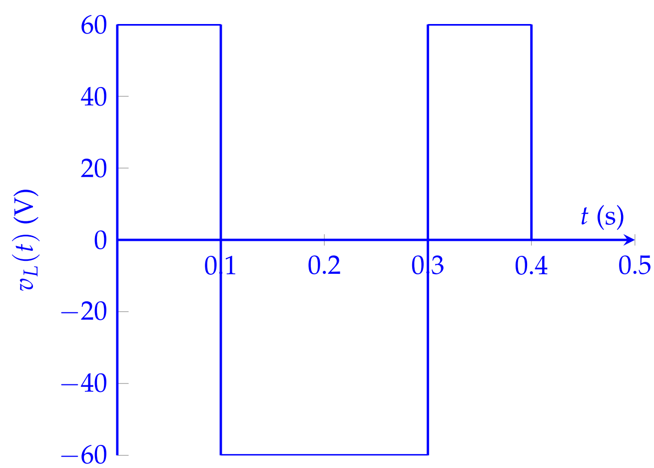
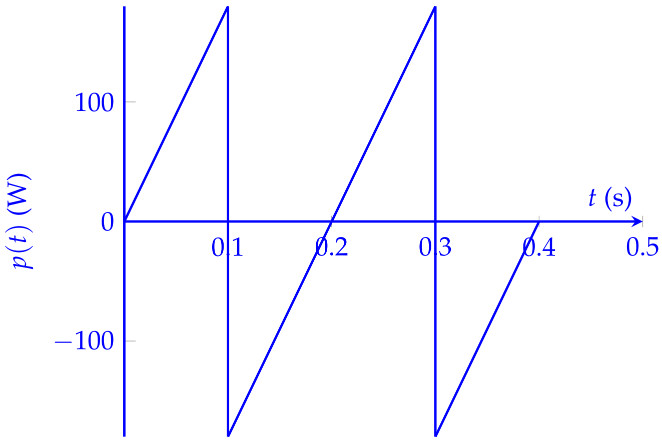


Figure 3: Current vs Time for P3.43

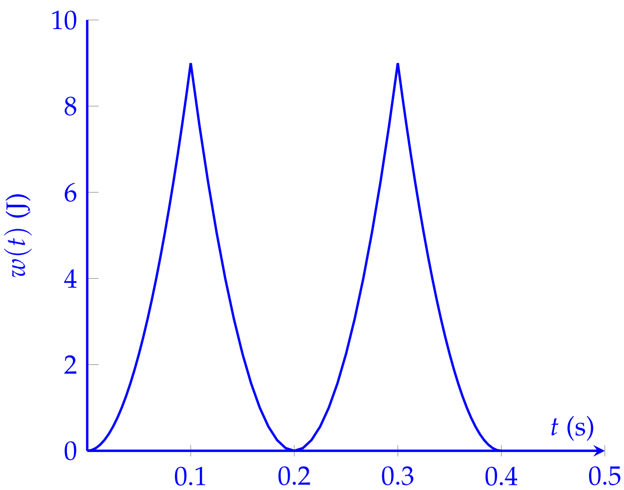
Solution: The voltage is $v_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t}$, so we take the slopes of Figure 3 and multiply by L. This gives the following:



The power is $p(t) = v_L(t)i_L(t)$, so we multiply the above graphs pointwise and obtain



Lastly, the stored energy is $\frac{1}{2}L(i_L(t))^2$. The plot for this is



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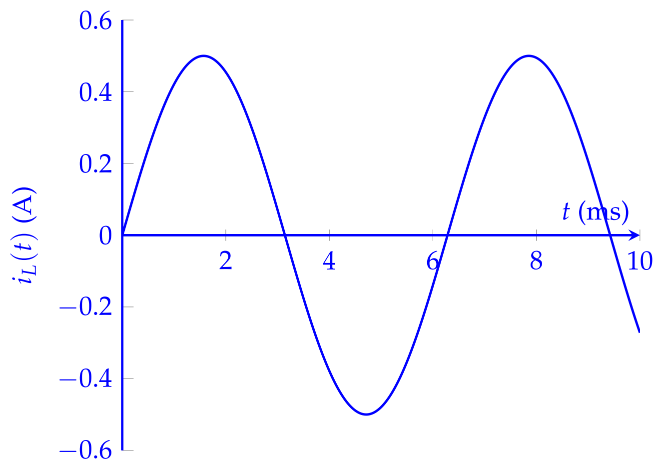
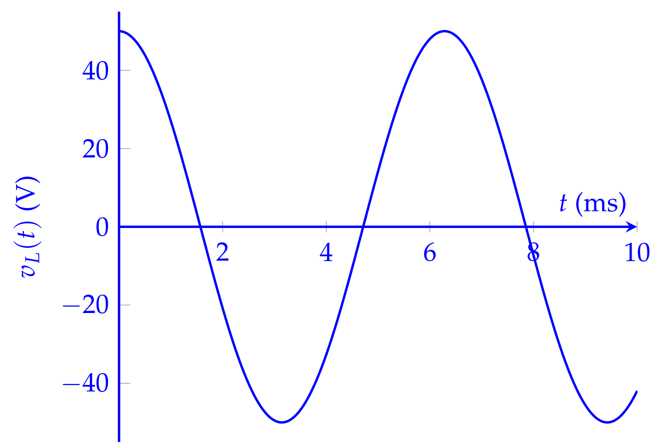
5. Hambley P3.7

The current in a 100 mH inductance is given by $0.5\sin(1000t)$A. Find expressions and sketch the waveforms to scale for the voltage, power, and stored energy, allowing t to range from 0 to 3π ms. The argument of the sine function is in radians.

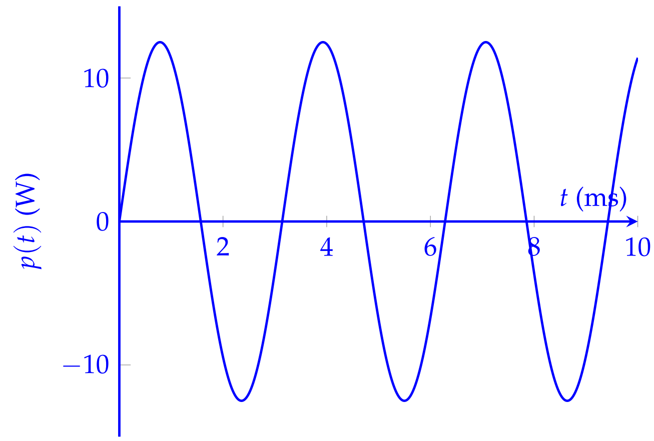
Solution: Below is the plot for $i_L(t)$:



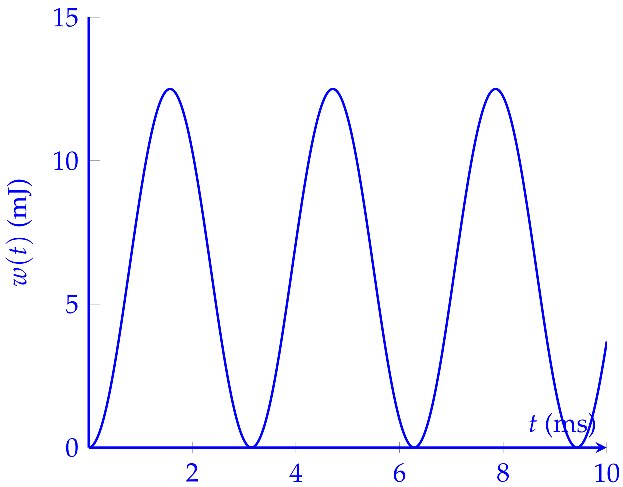
Using this and the fact that $v_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t} = 50\cos(1000t)$, we have the following plot for voltage:



Next, we have $p(t) = v_L(t)i_L(t) = 25\cos(1000t)\sin(1000t) = 12.5\sin(2000t)$, which is plotted below:



Lastly, we have $w(t) = \frac{1}{2}L(i_L(t))^2 = 0.0125\sin^2(1000t)$, which is plotted below:



6. Hambley P3.55

What value of inductance corresponds to an open circuit, assuming zero initial current? Explain your answer. Repeat for a short circuit.

Solution: In an open circuit, $i = 0$, we have that

$$i = \frac{1}{L}\int_{t_0}^{t} v(t')\,\mathrm{d}t' + i(t_0) \tag{7}$$

and $i(t_0) = 0$. Hence, it must be the case that $L \to \infty$.
In a short circuit, $v = 0$, so we have

$$v = L\frac{\mathrm{d}i}{\mathrm{d}t} = 0 \tag{8}$$

so $L = 0$.

7. Hambley P3.74

A pair of mutually coupled inductances has $L_1 = 2\,\mathrm{H}$, $L_2 = 1\,\mathrm{H}$, $i_1 = 2\cos(1000t)\mathrm{A}$, $i_2 = 0$, and $v_2 = 2000\sin(1000t)\mathrm{V}$. (The arguments of the sine and cosine functions are in radians.) Find $v_1(t)$ and the magnitude of the mutual inductance.

Solution: In general, we have

$$v_1(t) = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} \pm M\frac{\mathrm{d}i_2}{\mathrm{d}t} \tag{9}$$

$$v_2(t) = \pm M\frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} \tag{10}$$

Substituting the given information, we have

$$v_1(t) = -4 \times 10^3 \sin(1000t) \tag{11}$$

$$2000\sin(1000t) = \mp M \times 2000\sin(1000t) \tag{12}$$

We deduce that $M = 1\,\mathrm{H}$. Furthermore, because the lower of the two algebraic signs applies, we know that the currents are referenced into unlike terminals.

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