Homework 0

This homework is due on Tuesday, August 30th, at 11:59PM.
Self-Grades, HW Resubmissions, and HW Resubmission Self-Grades are due on Friday, September 2, at 11:59PM.

NOTE: All other homeworks follow a Friday-to-Friday cycle, in which the homework is due on a given Friday and the Self-Grades/Resubmission/Resubmission Self-Grades are due the following Friday.

1. **Group Formation Survey**

   Please fill out this group formation survey if you are interested in getting matched up in a study group. We highly recommend joining a study group in order to foster a sense of community in the course and learn from others. EECS 16B is a pretty fast-paced course, and you can benefit quite a bit from your peers' perspectives on the material. Within a few weeks, you should get an email informing you of the group you have been matched with. It is respectful and professional behavior to follow up with your group members; completing this survey suggests you are interested in joining a group, after all – we hope you stay true to your word! Special shoutout to Prof. Ranade's group formation research team for making this possible! Just so you have an answer to put down for this question, write down whether you filled out the survey or not.

2. **Policy Quiz**

   Please take the following policy quiz and attach a screenshot of your score: link to quiz. The goal is to ensure that everyone is familiar with the course policies, which you can read about here.

   If you have a problem accessing the quiz, try using your UC Berkeley account.

   Take your score on the Google form, divide by 2 and round up to either 2, 5, 8, or 10. This is your self-grade score.

3. **Videos**

   We will use a variety of online tools and websites this semester. Please watch the following video tutorials about how to use them.

   - Gradescope:
     - Submitting an online assignment. (This is especially pertinent for the Sim lab option.)
     - Submitting PDF homework.
     - Viewing feedback and requesting regrades.
   - Office Hour Queue.

   If you have a problem accessing any of these videos, try viewing them using your UC Berkeley account.

   After watching the videos, please write down for your answer to this problem that you understand how the tools work; if you have any questions about the videos, please post them on the corresponding Ed thread for this problem.

4. Hambley P2.23

Find the values of i_1 and i_2 in Figure 1.

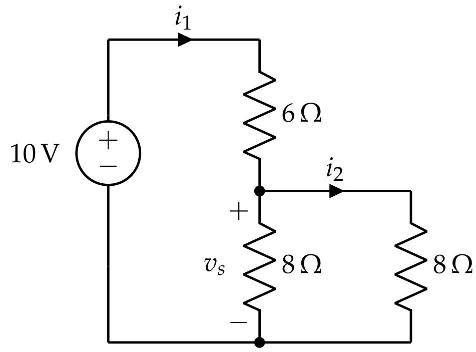

Figure 1: P2.23

Solution: The equivalent resistance of the parallel resistors is

$$R_{\text{eq}} = \frac{1}{\frac{1}{8} + \frac{1}{8}} = \frac{8}{2} = 4 \tag{1}$$

so the total resistance in the circuit is $R_{\text{total}} = 6 + 4 = 10\,\Omega$. Hence, we have

$$i_1 = \frac{10\,\text{V}}{R_{\text{total}}} = 1\,\text{A} \tag{2}$$

Using the voltage divider formula, we have

$$v_s = \frac{R_{\text{eq}}}{R_{\text{eq}} + 6} 10\,\text{V} = 4\,\text{V} \tag{3}$$

so

$$i_2 = \frac{v_s}{8\,\Omega} = 0.5\,\text{A} \tag{4}$$

5. Hambley P2.24

Find the values of i_1 and i_2 in Figure 2. Find the power for each element in the circuit, and state whether each is absorbing or delivering energy. Verify that the total power absorbed equals the total power delivered.

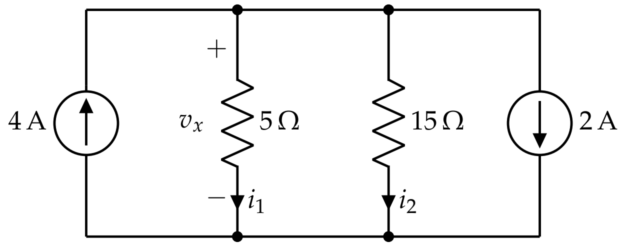

Figure 2: P2.24

Solution: From KCL,

$$i_1 + i_2 + 2 = 4 \tag{5}$$

The equivalent resistance of the parallel resistors is

$$R_{\text{eq}} = \frac{5 \times 15}{5 + 15} = \frac{15}{4}\Omega \tag{6}$$

so

$$v_x = (i_1 + i_2)R_{\text{eq}} = \frac{15}{2}\text{V} \tag{7}$$

Thus, $i_1 = \frac{v_x}{5} = 1.5\,\text{A}$ and $i_2 = \frac{v_x}{15} = 0.5\,\text{A}$. The $4\,\text{A}$ current source delivers $4\,\text{A} \times v_x = 30\,\text{W}$ of power and the $2\,\text{A}$ current source dissipates $2\,\text{A} \times v_x = 15\,\text{W}$ of power. The $5\,\Omega$ resistor dissipates $1.5\,\text{A} \times v_x = \frac{45}{4}\text{W}$ of power and the $15\,\Omega$ resistor dissipates $0.5\,\text{A} \times v_x = \frac{15}{4}\text{W}$ of power. The total amount of power delivered is the total amount of power absorbed.

6. Hambley P2.40

We want to design a voltage-divider circuit to provide an output voltage $v_0 = 5\,\text{V}$ from a 9 V battery as shown in Figure 3. The current taken from the 9 V source with no load connected is to be 10 mA.

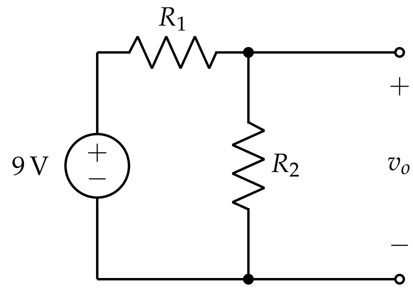

Figure 3: P2.40

(a) **Find the values of R_1 and R_2.**

Solution: By the current requirement, we have

$$R_1 + R_2 = \frac{9}{10 \times 10^{-3}} = 900\,\Omega \tag{8}$$

and by the voltage divider requirement, we have

$$\frac{R_2}{R_1 + R_2} \times 9 = 5\,\text{V} \tag{9}$$

Combining these two equations, we have $R_2 = 500\,\Omega$ and $R_1 = 400\,\Omega$.

(b) Now suppose that a load resistance of $1\,\mathrm{k}\Omega$ is connected across the output terminals (i.e., in parallel with R_2). **Find the loaded value of v_0.**

Solution: With the load equivalent resistance, $R_{\text{eq}} = \frac{0.5\times 1}{0.5+1} = \frac{1}{3}\,\mathrm{k}\Omega = \frac{1000}{3}\,\Omega$. Thus,

$$v_0 = \frac{\frac{1000}{3}}{\frac{1000}{3}+400}9 \approx 4.09\,\mathrm{V} \tag{10}$$

(c) **How would we change the design so the voltage remains closer to $5\,\mathrm{V}$ when the load is connected? How would this affect the life of the battery?**

Solution: To remain close to $5\,\mathrm{V}$ in the output, one will need to substantially reduce R_1. This will mean that the total current taken from the source will go up, leading to a reduction in battery lifetime.

7. It's Finally Raining

A lettuce farmer in Salinas Valley has grown tired of weather.com's imprecise rain measurements. Therefore, they decided to take matters into their own hands by building a rain sensor. They placed a rectangular tank outside and attached two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.

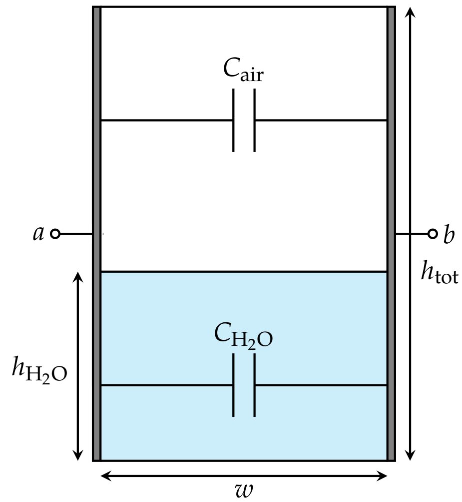

Figure 4: Water tank

The width and length of the tank are both w (i.e., the base is square) and the height of the tank is h_{tot}.

(a) **What is the capacitance between terminals a and b when the tank is full? What about when it is empty?** *Note:* the permittivity of air is ϵ, and the permittivity of rainwater is 81ϵ.

Solution: Capacitance of parallel plates is governed by the equation:

$$C = \frac{\epsilon A}{d} \tag{11}$$

where ϵ is the *permittivity* of the dielectric material, A is the area of the plates, and d is the distance between the plates. If we apply this to our physical structure, we find that the area of the plates are $h_{\text{tot}} \cdot w$, and the distance between the plates is w. The only difference here between a full and empty tank is the permittivity of the material between the two plates.

$$C_{\text{empty}} = \frac{\epsilon_{\text{air}} \, h_{\text{tot}} w}{w} = \epsilon \, h_{\text{tot}} \tag{12}$$

$$C_{\text{full}} = \frac{\epsilon_{H_2O} \, h_{\text{tot}} w}{w} = 81\epsilon \, h_{\text{tot}} \tag{13}$$

(b) Suppose the height of the water in the tank is $h_{\mathrm{H_2O}}$. **Modeling the tank as a pair of capacitors in parallel, find the total capacitance between the two plates.** Call this capacitance C_{tank}.

Solution: We can break the total capacitance into two parts. First, let's calculate the capacitance of the two plates separated by water:

$$C_{\mathrm{water}} = \frac{\epsilon_{\mathrm{H_2O}}\, h_{\mathrm{H_2O}} w}{w} = 81\epsilon\, h_{\mathrm{H_2O}} \tag{14}$$

And now we can calculate the capacitance of the two plates separated by air:

$$C_{\mathrm{air}} = \frac{\epsilon_{\mathrm{air}} \left(h_{\mathrm{tot}} - h_{\mathrm{H_2O}}\right) w}{w} = \epsilon \left(h_{\mathrm{tot}} - h_{\mathrm{H_2O}}\right) \tag{15}$$

Because these two capacitors appear in parallel, we can simply add our two previous results to find the total equivalent capacitance:

$$C_{\mathrm{tank}} = C_{\mathrm{water}} + C_{\mathrm{air}} = \epsilon \left(h_{\mathrm{tot}} + 80 h_{\mathrm{H_2O}}\right) \tag{16}$$

(c) After building this capacitor, the farmer consults the internet to assist them with a capacitance-measuring circuit. A fellow internet user recommends the following:

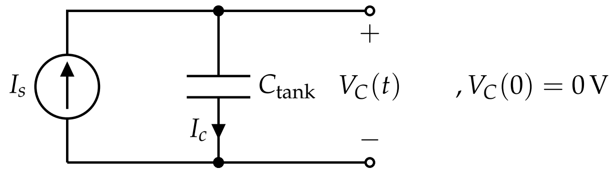

Figure 5: Circuit Model

In this circuit, C_{tank} is the total tank capacitance that you calculated earlier. I_s is a known current supplied by a current source.

The suggestion is to measure V_C for a brief interval of time, and then use the difference to determine C_{tank}.

Determine $V_C(t)$, where t is the number of seconds elapsed since the start of the measurement. You should assume that before any measurements are taken, the voltage across C_{tank}, i.e. V_C, is initialized to $0\,\text{V}$, i.e. $V_C(0) = 0$.

Solution: The element equation for the capacitor is:

$$I_C = C_{\text{tank}} \frac{\mathrm{d}V_C}{\mathrm{d}t} \tag{17}$$

We also know from KCL that:

$$I_C = I_s \tag{18}$$

Thus, we get the following differential equation for V_C:

$$\frac{\mathrm{d}V_C}{\mathrm{d}t} = \frac{I_s}{C_{\text{tank}}} \tag{19}$$

We recall that I_s and C_{tank} are constant values and the initial value of V_C is zero ($V_C(0) = 0$). Applying these facts and integrating the differential equation, we get the following equation for V_C:

$$V_C(t) = \frac{I_s}{C_{\text{tank}}} t \tag{20}$$

(d) **If we can measure $V_C(t)$ and knowing the result of part (c), how could we derive the value of C_{tank}? Then, using the result from part (b), write h_{H_2O} as a function of C_{tank} and other constants.**

Solution: We connect the current source providing I_sA to the capacitor C_{tank}. At the same time, we can measure $V_C(t)$. After some time passes, we measure $V_C(t)$ and plug it into the following equation (assuming, as before, that $V_C(0) = 0$):

$$C_{\text{tank}} = \frac{I_s}{V_C(t)} t \tag{21}$$

If we know C_{tank}, we can determine h_{H_2O}. Using the equation derived in part (b), we see that

$$h_{H_2O} = \frac{C_{\text{tank}} - h_{\text{tot}}\epsilon}{80\epsilon} \tag{22}$$