# This homework is due on Saturday, January 20th, at 11:59PM.

We are aware that these questions have appeared as past EECS 16A questions. Nevertheless, we recommend that you do the problems in earnest (think through the problems and show all work) as they are meant to reflect concepts that will serve as foundation for the material in EECS 16B.

# 1. Administration

- (a) Please fill out our introductory survey: link to survey. This survey will help us understand our students' prerequisite knowledge for content creation purposes.
- (b) Please complete the Administrative Policy Quiz assignment on Gradescope. The goal is to ensure that everyone is familiar with the course policies, which you can read about here. Take your percent score on the Gradescope assignment, multiply by 10 and round up to either 2, 5, 8, or 10. This is your self-grade score.

#### 2. Modeling Weird Capacitors

For parts (a) - (c) of this problem, **pick the circuit option from below** that <u>best</u> models the given physical capacitor.



(a) A parallel plate capacitor with plate dimensions *L* and *W*, separated by a gap *D*, is filled with an insulator of permittivity  $\epsilon_1$ , with the bottom plate displaced with overlap *W* as shown below. You can assume W < L and  $D \ll W$ .



- i. What is the circuit option that best models the physical capacitor?
  - (A) Option 1
  - (B) Option 2
  - (C) Option 3
  - (D) Option 4
- ii. What is the total capacitance, *C*, for this capacitor? Express your answer in terms of  $\epsilon_1$ , *D*, *L*, and *W*.
- (b) A parallel plate capacitor with plate dimensions *L* and *W*, separated by a gap  $2 \cdot D$ , is filled with two insulators of permittivities  $\epsilon_1$  and  $\epsilon_2$  as shown below. You can assume there's a plate between the two dielectrics. What is the circuit option that best models the physical capacitor?



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- (C) Option 3
- (D) Option 4
- (c) A parallel plate capacitor with plate dimensions *L* and *W*, separated by a gap  $2 \cdot D$ , is filled with three different materials with permittivities  $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$  as shown in the figure below. You can assume there's a plate between the two dielectrics on the right. What is the circuit option that best models the physical capacitor?



- (A) Option 1
- (B) Option 2
- (C) Option 3
- (D) Option 4
- (d) For this final part, please express the equivalent capacitance, C<sub>eq</sub>, between the top and bottom node for each of the following circuits from the previous parts. Feel free to include the *parallel operator* ("||") in your answer.



iii. Option 3



#### 3. Op-Amp Analysis!

(a) We want to find a relationship between the output voltage, *V*<sub>out</sub>, and the input current, *I*<sub>s</sub>, in the circuit below.



- i. Determine the node voltage  $V_a$  in terms of  $I_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .
- ii. Determine the node voltage  $V_b$  in terms of  $I_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .
- iii. Choose the correct expression for the output voltage  $V_{out}$  in terms of  $I_s$ ,  $V_b$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .
  - (A)  $V_{out} = \left(1 \frac{R_3}{R_2}\right) \cdot V_b I_s \cdot R_1$ (B)  $V_{out} = V_b$ (C)  $V_{out} = \left(1 + \frac{R_3}{R_2}\right) \cdot V_b - I_s \cdot R_3$ (D)  $V_{out} = \frac{R_3 + R_2}{R_2} V_b$

(E) 
$$V_{out} = \left(1 - \frac{R_3}{R_2}\right) \cdot V_b - I_s \cdot (R_1 + R_3)$$

(b) Now, we will connect a set of capacitors to our previous circuit with an initially open switch *S*<sub>1</sub>, as follows:



Now assume the output voltage is  $V_{out} = 5$  V. Also, assume the capacitors  $C_1 = 4\mu F$ ,  $C_2 = 2\mu F$ , and  $C_3 = 3\mu F$  are initially discharged. In steady-state after switch  $S_1$  is closed, determine the following quantities. Please provide **numerical** values for your answers.

- i. What is the energy stored in **capacitor**  $C_1$ ?
- ii. What is the charge accumulated on **capacitor**  $C_3$ ?
- iii. What is the voltage across **capacitor** *C*<sub>3</sub>?

### 4. Finding Mr. Thevenin

For the following circuits, find the Thevenin and Norton equivalent resistance, voltage, and current between the nodes *a* and *b*.

(a) Consider the circuit below:



- i. Can you turn off  $V_s$  (5V voltage source) and  $I_s$  (2A current source) to find  $R_{th}$ ?
  - (A) Yes
  - (B) No
- ii. What is  $R_{th}$ ?
  - (A)  $R_{th} = 2 \Omega$
  - (B)  $R_{th} = 3 \Omega$
  - (C)  $R_{th} = 4.5 \,\Omega$
  - (D)  $R_{th} = 6 \Omega$
  - (E)  $R_{th} = 9 \Omega$
- iii. What is  $V_{th}$ ?
  - (A)  $V_{th} = 0 V$
  - (B)  $V_{th} = 2 V$
  - (C)  $V_{th} = 3 V$
  - (D)  $V_{th} = 4 V$
  - (E)  $V_{th} = 6 \,\mathrm{V}$
- iv. What is *I*<sub>no</sub>?
  - (A)  $I_{no} = 0 \text{ A}$
  - (B)  $I_{no} = 0.67 \,\mathrm{A}$
  - (C)  $I_{no} = 1 \text{ A}$
  - (D)  $I_{no} = 2 \mathrm{A}$
  - (E)  $I_{no} = 3 \text{ A}$
- (b) Consider this new circuit with a current-dependent voltage source (that depends on  $I_x$ , the current through the 3  $\Omega$  resistor):  $V_x = 3\Omega \cdot I_x$  [V].

(HINT: To find  $R_{th}$ , you will need to use a test voltage  $V_{test}$  (or test current) and find the relationship to its current  $I_{test}$  (or voltage).)



- i. Should you turn off  $V_x$  to find  $R_{th}$ ?
  - (A) Yes
  - (B) No
- ii. What is  $R_{th}$ ?
  - (A)  $R_{th} = 2 \Omega$
  - (B)  $R_{th} = 3 \Omega$
  - (C)  $R_{th} = 4.5 \Omega$
  - (D)  $R_{th} = 6 \Omega$
  - (E)  $R_{th} = 9 \Omega$
- iii. What is  $V_{th}$ ?
  - (A)  $V_{th} = 0 V$
  - (B)  $V_{th} = 2 V$
  - (C)  $V_{th} = 3 V$
  - (D)  $V_{th} = 4 \,\mathrm{V}$
  - (E)  $V_{th} = 6 \text{ V}$
- iv. What is  $I_{no}$ ?

$$(A) I_{no} = 0 A$$

- (B)  $I_{no} = 0.67 \,\mathrm{A}$
- (C)  $I_{no} = 1 \text{ A}$
- (D)  $I_{no} = 2 A$
- (E)  $I_{no} = 3 \text{ A}$

## 5. Please don't burn your fingers

One day, hidden somewhere deep within Cory 140, you discover an ancient capacitive circuit.



- (a) Calculate the equivalent capacitance  $C_e$  between  $E_1$  and  $E_2$  given  $C_0 = C_{E_1\_F_1} = C_{F_1\_E_2} = C_{E_1\_F_2} = C_{F_2\_E_2} = 40 \text{ pF}.$ 
  - (A) 20 pF
  - (B) 40 pF
  - (C) 60 pF
  - (D) 80 pF
  - (E) 120 pF
- (b) What you found was in fact a multi-finger touchscreen that forms different capacitive circuits depending on how many fingers we place.





To figure out how this multi-finger touchscreen works, you decide to connect it to your op-amp setup from the Touch 3 labs. The circuit between terminals  $E_1$  and  $E_2$  is modeled as equivalent capacitance  $C_e$ , and  $V_{in}$  is a function generator with alternating square wave voltage between  $V_{in} = 0$  V and  $V_{in} = 2V_r$ .



Assume an ideal op-amp and the circuit is in negative feedback.

- i. After experimenting with the circuit for a bit, you notice a sudden increase in the positive peaks of  $V_{out}$ . How must the equivalent capacitance  $C_e$  have changed?
  - (A)  $C_e$  increased
  - (B)  $C_e$  decreased
- ii. How are the equivalent capacitance  $C_e$  and the number of fingers touching related?
  - (A) More fingers increases  $C_e$
  - (B) More fingers decreases  $C_e$
  - (C)  $C_e$  does not depend on the number of fingers
- (c) Oops! Instead of a function generator, we accidentally used a constant voltage source V<sub>in</sub> instead. We will find out how long it will take before the circuit breaks! Here is the circuit with the new voltage source V<sub>in</sub>.



For the following problems, assume the circuit is in negative feedback.

- i. First, what is the current flowing in the  $1 k\Omega$  resistor ( $I_{1 k\Omega}$  in the circuit)? Assume  $V_{in} = 2 V$ ,  $V_r = 1 V$ . Express your answer in mA (numerical value), and make sure your sign is correct (according to the labeled current in the circuit.).
- ii. Now assume a *constant* current source  $I_s$  (instead of  $V_{in}$  and the 1 k $\Omega$  resistor), as shown in the circuit below.



If the initial voltage across the capacitor is zero at time t = 0, what is the value of  $V_{out}$  over time? Assume the output does not saturate (i.e.,  $V_{DD} > V_{out} > V_{SS}$ ). Express your answer in terms of the variables  $I_s$ ,  $V_r$ ,  $C_e$ , and t by simplifying any integrals or derivatives (i.e. your final answer should not have any integrals or derivatives in it.)

iii. If the op-amp is connected to supply sources  $V_{DD} = -V_{SS}$ , **1**) how long does it take for  $V_{out}$  to saturate the op-amp? and **2**) what is the value of  $V_{out}$  in saturation? (Assume  $I_s > 0$ ,  $V_r > 0$  and  $V_{DD} > V_r > V_{SS}$ )

$$V_r > 0$$
, and  $V_{DD} > V_r > V_{SS}$   
(A)  $t = C_s \frac{-V_{SS} + V_r}{V_{SS}}$   $V_{sut} = V_{SS}$ 

(B) 
$$t = C_e \frac{V_{DD} - V_r}{L}$$
  $V_{out} = V_{DD}$   
 $V_{out} = V_{DD}$ 

(C) 
$$t = \frac{-V_{SS} + V_r}{C_e I_s}$$
  $V_{out} = V_{SS}$   
(D)  $t = \frac{V_{DD} - V_r}{C_e I_s}$   $V_{out} = V_{DD}$ 

# 6. Ask Opamps Anything

We've decided to design a 1D resistive touch-screen using an ideal opamp. The resistive touchscreen has a total length of L, a cross sectional area of A and resistivity of  $\rho$ .



- (a) First, we want to find  $V_1$ , because we will use this block in a larger design.
  - i. What are the values for the resistance between the touch point and ground  $(R_d)$  and between the touch point and  $V_1$   $(R_{rest})$ ?

(A) 
$$R_d = \rho \frac{A}{d}$$
  $R_{rest} = \rho \frac{A}{L-d}$   
(B)  $R_d = \rho \frac{d}{A}$   $R_{rest} = \rho \frac{L-d}{A}$   
(C)  $R_d = \rho \frac{L-d}{A}$   $R_{rest} = \rho \frac{d}{A}$   
(D)  $R_d = \rho \frac{A}{L-d}$   $R_{rest} = \rho \frac{A}{d}$ 

ii. Identify a correct equivalent topology for this scenario:



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iii. What is the value of  $V_1$  if the resistive touch screen, as a function of  $R_d$  and  $R_{rest}$ ?

(A) 
$$V_1 = V_{ref} \frac{R_d}{R_{rest}}$$
  
(B)  $V_1 = V_{ref} \frac{R_{rest}}{R_d}$   
(C)  $V_1 = V_{ref} \left(1 + \frac{R_d}{R_{rest}}\right)$   
(D)  $V_1 = V_{ref} \left(1 + \frac{R_{rest}}{R_d}\right)$ 

(b) Next, an LED indicator driven by a comparator is added to the output of the prior circuit.



i. You are provided the curve for the voltage  $V_1$  as a function of the touch distance d. What should the value of  $V_{comp}$  be to ensure the LED turns on when  $d > \frac{L}{2}$ ?



- (A)  $V_{comp} = +V_{ref}$
- (B)  $V_{comp} = -V_{ref}$
- (C)  $V_{comp} = +2 V_{ref}$
- (D)  $V_{comp} = -2 V_{ref}$
- (E)  $V_{comp} = +4 V_{ref}$
- (F)  $V_{comp} = -4 V_{ref}$
- ii. When the LED shown in the diagram is turned on the voltage across it is  $V_{\text{LED}} = 1 \text{ V}$ , what is the current,  $i_{\text{LED}}$ , through it? Consider the load resistance  $R_L = 1 \text{ k}\Omega$ , and voltages supplies  $V_{DD} = 5 \text{ V}$  and  $V_{SS} = 0 \text{ V}$ . Your answer should be a numerical value.
- iii. Now, assume  $i_{\text{LED}} = 1 \text{ mA}$ ,  $V_{\text{LED}} = 2 \text{ V}$ ,  $R_L = 3 \text{ k}\Omega$ ,  $V_{DD} = 5 \text{ V}$ , and  $V_{SS} = 0 \text{ V}$ . How much power  $P_{out}$  is delivered by the output of the comparator? Your answer should be a **numerical** value.