EECS 16B Designing Information Devices and Systems II
Fall 2021 Discussion Worksheet Discussion 1A

For this discussion, Note 1 is helpful for background in transistors and RC circuits.

1. NAND Circuit

Let us consider a NAND logic gate. This circuit implements the boolean function $\overline{(A \cdot B)}$. The $\cdot$ stands for the AND operation, and the $\overline{}$ stands for NOT; combining them, we get NAND!

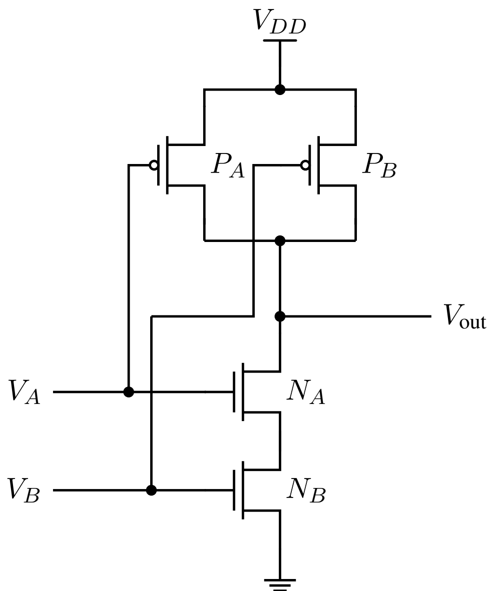

Figure 1: NAND gate transistor-level implementation.

V_{tn} and V_{tp} are the threshold voltages for the NMOS and PMOS transistors, respectively. Assume that $V_{DD} > V_{tn}$ and $|V_{tp}| > 0$.

(a) Label the gate, source, and drain nodes for the NMOS and PMOS transistors above.

Solution: In an NMOS, the terminal at the higher potential is always the drain, and the terminal at the lower potential is always the source. Therefore, the drains are at the top of N_A (connected to V_{out}) and the top of N_B (connected to N_A). The sources are at the bottom of N_A (connected to N_B) and the bottom of N_B (connected to ground). The gate terminal of N_A is connected to V_A; the gate of N_B is connected to V_B.

In a PMOS, the terminal at the higher potential is always the source, and the terminal at the lower potential is always the drain. Therefore, the source is at the top of P_A and P_B (connected to V_{DD}). The drain is at the bottom of P_A and P_B (connected to V_{out}). The gate terminal of P_A is connected to V_A; the gate of P_B is connected to V_B.

(b) If $V_A = V_{DD}$ and $V_B = V_{DD}$, which transistors act like open switches? Which transistors act like closed switches? What is V_{out}?

Solution: P_A and P_B are off (open switches). N_B and N_A are on (closed switches). $V_{\text{out}} = 0\text{V}$ because it is connected to ground through a closed circuit consisting of P_A and P_B (and detached from V_{DD}).

(c) If $V_A = 0V$ and $V_B = V_{DD}$, what is V_{out}?

Solution: P_B and N_A are off (open switches). P_A and N_B are on (closed switches). $V_{\text{out}} = V_{DD}$ because it is connected to V_{DD} through a closed circuit consisting of P_A (and detached from ground, since *both* N_A and N_B must be closed for V_{out} to be connected to ground).

(d) If $V_A = V_{DD}$ and $V_B = 0V$, what is V_{out}?

Solution: P_A and N_B are off (open switches). P_B and N_A are on (closed switches). But, since N_B is open, N_A being closed doesn't connect V_{out} to ground. So, $V_{\text{out}} = V_{DD}$ because it is connected to V_{DD} through a closed switch.

Note that with the simplest transistor model, one cannot to determine V_{GS} for N_A, since we don't know the source voltage for that transistor. V_{out} is still high, because regardless of whether N_A is on, there is an open (or very high resistance) between V_{out} and ground while there is a short to V_{DD}.

(e) If $V_A = 0V$ and $V_B = 0V$, what is V_{out}?

Solution: N_B is off, creating an open circuit. P_A and P_B are on, creating a closed circuit. $V_{\text{out}} = V_{DD}$ because it is connected by closed circuit to V_{DD}.

Like above, the source of N_A has an ambigous value and we cannot determine whether N_A is on or off. However, this doesn't affect the output because the path to ground is an open (since N_B is definitely off, $V_{GS,N_A} = 0 \leq V_{tn}$.

(f) Write out the truth table for this circuit.

V_A	V_B	V_{out}
0	0	
0	V_{DD}	
V_{DD}	0	
V_{DD}	V_{DD}	

Solution:

V_A	V_B	V_{out}
0	0	V_{DD}
0	V_{DD}	V_{DD}
V_{DD}	0	V_{DD}
V_{DD}	V_{DD}	0

2. RC Circuits - Part I

In this problem, we will find the voltage across a capacitor over time in an RC circuit. In this part, we set up our problem by first defining four functions over time: $I(t)$ is the current at time t, $V(t)$ is the voltage across the circuit at time t, $V_R(t)$ is the voltage across the resistor at time t, and $V_C(t)$ is the voltage across the capacitor at time t.

Recall from 16A that the voltage across a resistor is defined as $V_R = RI_R$ where I_R is the current across the resistor. Also, recall that the voltage across a capacitor is defined as $V_C = \frac{Q}{C}$ where Q is the charge across the capacitor.

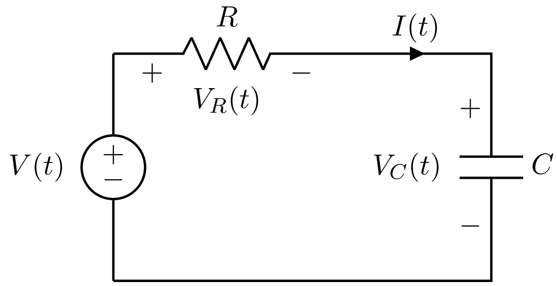

Figure 2: Example Circuit

(a) First, find an equation that relates the current across the capacitor $I(t)$ with the voltage across the capacitor $V_C(t)$.

Solution: We start from the Q-V relationship of the capacitor:

$$Q(t) = CV_C(t).$$

Differentiating $V_C(t) = \frac{Q(t)}{C}$ in terms of t, we get

$$\frac{dV_C(t)}{dt} = \frac{dQ(t)}{dt}\frac{1}{C}.$$

By definition, the change in charge is the current across the capacitor, so

$$\frac{d}{dt}V_C(t) = I(t)\frac{1}{C} \quad (1)$$

(b) Write a system of equations that relates the functions $I(t)$, $V_C(t)$, and $V(t)$.

Solution: From KCL, we have

$$\frac{V(t) - V_C(t)}{R} - I(t) = 0$$

$$RI(t) + V_C(t) = V(t) \quad (2)$$

(c) So far, we have two relations between I(t) and $V_C(t)$. To solve this system of equations, we can remove $I(t)$ from the equation using what we found in part (a). Rewrite the previous equation in part (b) in the form of a differential equation.

Solution:

From part (a), we have

$$I(t) = \frac{dV_C(t)}{dt}C$$

Substituting this into Equation 2 gives us

$$RC\frac{dV_C(t)}{dt} + V_C(t) = V(t)$$

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