Phasors, Filters, Bode Plots

16B Review Session
12-1 PM
Neelesh Ramachandran
Phasors [not a core topic by itself; unlocks filters + circuit analysis/design]

Motivation:

→ often, \( V_{in}(t) = A \cos (ωt + \varphi) \) for circuits [shorthand input]
→ linear circuits (RLC): frequencies don't change.
→ use phasor to analyze circuit; retain non-free into.

Formulas:

\[
\begin{align*}
\text{Time Domain} & \quad \rightarrow \quad \text{Phasor Domain}: \\
A \cos (ωt + \varphi) & \leftrightarrow \frac{A}{2} \left( e^{jωt} + e^{-jωt} \right) \\
V_{in}(t) & \quad \rightarrow \quad V_{in} \\
\end{align*}
\]

\[
\begin{align*}
\text{Phasor Domain} & \quad \rightarrow \quad \text{Time Domain}: \\
B e^{j\varphi} & \leftrightarrow 2B \cos (ωt + \varphi)
\end{align*}
\]

Impedances

\[
\begin{align*}
R: Z_r &= R \\
L: Z_L &= jωL \\
C: Z_C &= \frac{1}{jωC} = -\frac{j}{ωC}
\end{align*}
\]

Euler's Formulas

\[
\begin{align*}
\sin (x) &= \frac{1}{2j} \left( e^{jx} - e^{-jx} \right) \\
\cos (x) &= \frac{1}{2} \left( e^{jx} + e^{-jx} \right)
\end{align*}
\]

References:

→ Note 4: Sections 5/6/7
→ Lecture 4B: page 9 onwards
→ Lecture 5A

Problems:

→ Midterm 1: C 3b)
→ HW 6: Q3, 5, 6
→ Dis 5B

Examples

1) \( V_{in}(t) = \cos (t) \) \quad \Rightarrow \quad \hat{V}_{in} = 0.5
2) \( V_{in}(t) = 4 \cos (5t + 7/3) \) \quad \Rightarrow \quad \hat{V}_{in} = 2e^{j5/3}
3) \( V_{in}(t) = 4 \cos (20t + 5/18) \) \quad \Rightarrow \quad \hat{V}_{in} = 2e^{j5/3}
4) \( V_{in}(t) = 3 \sin (6t - 5/6) = 3 \cos (6t + 4\pi/3) \) \quad \Rightarrow \quad \hat{V}_{in} = \frac{3}{2} e^{j4\pi/3}
5) \( V_{in}(t) = 9 \cos (5/3) \) \quad \Rightarrow \quad \hat{V}_{in} = \frac{9}{2} e^{j5/3}

Don't be scared of complex numbers! → they make computations a lot easier!

\[
\begin{align*}
\text{Formulas:} \\
\text{Time Domain} & \quad \rightarrow \quad \text{Phasor Domain}: \\
A \cos (ωt + \varphi) & \leftrightarrow \frac{A}{2} \left( e^{jωt} + e^{-jωt} \right) \\
V_{in}(t) & \quad \rightarrow \quad V_{in} \\
\end{align*}
\]

\[
\begin{align*}
\text{Phasor Domain} & \quad \rightarrow \quad \text{Time Domain}: \\
B e^{j\varphi} & \leftrightarrow 2B \cos (ωt + \varphi)
\end{align*}
\]
Motivation:
→ signals exist in real world: not "pure sinusoids" (contain noise)
→ want to cancel noise but keep signal!

Considerations:
→ is signal freq. > or < noise freq? 
→ what components are available? 
  RC ↔ LR but maybe no inductors in lab!
→ cutoff frequency: what frequency do we begin to attenuate (decrease amplitude) by more than 1/2 ?

Overview / Categories

<table>
<thead>
<tr>
<th>Type</th>
<th>Topology</th>
<th>Overview</th>
<th>Filter</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-Pass</td>
<td>CR, RL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-Pass</td>
<td>CR, RL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Band-Pass</td>
<td>RC, L</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Band-Stop</td>
<td>RLC</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**KEY Formula:** if circuit TF is

\[ H(w) = \frac{|H(w)| \cdot e^{j \cdot \phi(w)}}{\text{magnitude}} \]

\[ V_{in}(t) = A \cos (w_0 t + \phi) \implies A \cdot |H(w)| \cos (w_0 t \phi + \phi \cdot H(w)) \]

\[ V_{out}(t) = \text{filtering} \]

- Referenced:
  - Note 5
  - Lectures 5A/5B/6A

Problems:
→ Midterm 1 64/8
→ Dis 6A/6B
→ HW 6, Q7 [HW 5 is entirely about RLC variants]
→ HW 7, Q 2/3/4
→ Note 5: worked example at start / Note 4: worked example at end
Bode Plots

Motivation:
- Exact transfer functions: need graphingcalc to plot
- Want an approximation method
- Bode plots summarize effect of a circuit on input at any frequency
- Given input signal, use bode plot to near-instantly get output signal \( H(jw) \)

Main Concepts
- Bode Plot Summary: Note 6

\[
H(jw) = \frac{K}{(\frac{1}{w_p})^N_p \left( \frac{1+j\frac{w}{w_1}}{1+j\frac{w}{w_2}} \right)^N_z} \]

Example: \( H(w) = 100 \frac{1}{\left( \frac{1+j\frac{w}{w_{1000}}} {1+j\frac{w}{w_{1000}}} \right)^2} \)

Problems:
- Midterm 1, Q3c / 8
- Dis 6A/6B/9A (Q1)
- HW 6/7
HW 6, Problem 6  (like Note 5, first example)

Note 4, last example

6. Phasor-Domain Circuit Analysis

The analysis techniques you learned previously in 16A for resistive circuits are equally applicable for analyzing circuits driven by sinusoidal inputs in the phasor domain. In this problem, we will walk you through the steps with a concrete example.

Consider the following circuit where the input voltage is sinusoidal. The end goal of our analysis is to find an equation for \( V_{out}(t) \).

![Phasor-Domain Circuit Diagram]

The components in this circuit are given by:

\[
V_i(t) = 10\sqrt{2}\cos \left( 100\pi t - \frac{\pi}{4} \right)
\]

- \( R = 5\Omega \)
- \( L = 50\text{mH} \)
- \( C = 2\text{mF} \)

(a) Give the amplitude \( V_i \), oscillation frequency \( \omega \), and phase \( \varphi \) of the input voltage \( V_i \).

(b) Transform the circuit into the phasor domain. What are the impedances of the resistor, capacitor, and inductor? What is the phasor \( V_s \) of the input voltage \( V_i(t) \)?

(c) Use the circuit equations to solve for \( \tilde{V}_{out} \), the phasor representing the output voltage.

(d) Convert the phasor \( \tilde{V}_{out} \) back to get the time-domain signal \( V_{out}(t) \).

\[ \tilde{V}_s = \frac{10\sqrt{2}}{2} \text{ e}^{-j\frac{\pi}{4}} \]

\[ = 5\sqrt{2} \text{ e}^{-j\frac{\pi}{4}} \]

\[ V_{out} = \tilde{V}_s \]

\[ V_{in}(t) = 10\sqrt{2} \cos \left( 100\pi t - \frac{\pi}{4} \right) \]

\[ \Rightarrow V_{out}(t) = \frac{1}{R} \left[ \frac{1}{j\omega C} \right] \]

\[ H(\omega) = \frac{1}{R} \left( 1 + \frac{j\omega L}{R} \right) \]

\[ H(\omega) = 1 \text{ @ } \omega = 100 \]

\[ V_{out}(t) = A \left[ H(\omega) \right] \cos \left( 100\pi t - \frac{\pi}{4} \right) + B \cos \left( 100\pi t - \frac{\pi}{4} \right) \]

\[ = 10\sqrt{2} \cos \left( 100\pi t - \frac{\pi}{4} \right) \]