Agenda
1. Mini-Review (Recorded)
2. Problems in Breakouts
3. Q + A (10 minutes)

The DFT

How fast does a signal change

\[ \sin(wx) \cos(wx) \]

\[ \omega = \omega \]

\[ W_N^k = e^{\frac{2\pi i k}{N}} \]

\[ w_N^N = 1 \]

\[ A e^{i\phi} \]

Properties of Roots of Unity

1. Periodicity

\[ W_N^k = W_N^{k+N} \]

2. \[ \sum_{k=0}^{N-1} W_N^k = 0 \]

3. Complex conjugacy

\[ W_N^k = W_N^{-k} \]

\[ N = 6 \]
\[ U_k = \left[ U_0^k, U_1^k, U_2^k, \ldots, U_N^k \right]^T \]

Properties of DFT basis vectors:

1. Periodicity: \( U_k = U_{k+N} \)
2. \( \| U_k \| = \sqrt{N} \)
3. \( U_k = \overline{U_{N-k}} \)
4. \( \langle U_k, U_i \rangle = 0 \) if \( k \neq i \)

How to determine how much a basis vector explains \( x \):

\[ \langle x, \frac{1}{\sqrt{N}} U_k \rangle = \frac{1}{\sqrt{N}} U_k^* x \]

\[
\frac{1}{\sqrt{N}} \begin{bmatrix}
- U_0^* \\
- U_1^* \\
\vdots \\
- U_{N-1}^*
\end{bmatrix} X = \begin{bmatrix}
x_0 \\
x_1 \\
\vdots \\
x_{N-1}
\end{bmatrix}
\]

1. It is a change of basis
2. Understand what the basis vectors represent
3. Be comfortable with the roots of unity