EECS 16B Designing Information Devices and Systems II
Spring 2021 Discussion Worksheet

Discussion 12A

In this discussion, we practice computing the SVD for a "wide" matrix (more columns than rows) and for a "tall" matrix (more rows than columns). There is also an associated jupyter notebook on Datahub that will serve useful to confirm the numerical calculations (specifically for performing Gram-Schmidt).

Also, note that the techniques and insights communicated in this discussion are conveyed in Note 13, sec. 3.

1. Computing the SVD: A "Tall" Matrix Example

Define the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}.$$

(a) In this part, we will find the full SVD of A in steps.

(i) Compute $A^\top A$ and find its eigenvalues.

(ii) Find orthonormal eigenvectors $\vec{v}_i$ (right singular vectors, columns of V).

(iii) Find singular values, $\sigma_i = \sqrt{\lambda_i}$.

(iv) Use $\vec{v}_i$ to find orthonormal $\vec{u}_i$ (for nonzero σ).

(v) Use the previous parts to write the full SVD of A.

(vi) Use the Jupyter notebook to run the code cell that calls `numpy.linalg.svd` on A. What is the result? Does it match our result above?

(b) Find the rank of A.

(c) Find a basis for the range (or column space) of A.

(d) Find a basis for the null space of A.

(e) We now want to create the SVD of $A^\top$. Rather than repeating all of the steps in the algorithm, feel free to use the jupyter notebook for this subpart (which defines a `numpy.linalg.svd` command). What are the relationships between the matrices composing A and the matrices composing $A^\top$?

2. Computing the SVD: A "Wide" Matrix Example

Define the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}.$$

(a) In this part, we will find the full SVD of A in steps.

(i) Compute $AA^\top$ and find its eigenvalues.

(ii) Find orthonormal eigenvectors $\vec{u}_i$ (left singular vectors, columns of U). Feel free to use the associated Jupyter notebook to perform Gram-Schmidt for this part, if needed.

(iii) Find the singular values, $\sigma_i = \sqrt{\lambda_i}$.

(iv) Use $\vec{u}_i$ to find orthonormal $\vec{v}_i$ (for nonzero σ). Feel free to use the associated Jupyter notebook to perform Gram-Schmidt for this part, if needed.

(v) Use the previous parts to write the full SVD of A.

(b) Find the rank of A, using the computed full SVD.

(c) Find a basis for the range (or columnspace) of A.

(d) Find a basis for the nullspace of A.

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