Discussion 10A

1. Uncontrollability

Recall that, for a n-dimensional, discrete-time linear system to be controllable, we require that the controllability matrix $\mathcal{C} = \begin{bmatrix} A^{n-1}B & A^{n-2}B & \dots & AB & B \end{bmatrix}$ to be rank n.

Consider the following discrete-time system with the given initial state:

$$\vec{x}[i+1] = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \vec{x}[i] + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u[i] \tag{1}$$

$$\vec{x}[0] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

(a) **Is the system controllable?**

(b) **Show that we can write the** i**th state as**

$$\vec{x}[i] = \begin{bmatrix} 2^i \\ -3x_1[i-1] + x_3[i-1] \\ x_2[i-1] + 2u[i-1] \end{bmatrix} \tag{3}$$

Is it possible to reach $\vec{x}[\ell] = \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}$ **for some** ℓ**? If so, for what input sequence** $u[i]$ **up to** $i = \ell - 1$**?**

(c) **Is it possible to reach** $\vec{x}[\ell] = \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix}$ **for some** ℓ**? For what input sequence** $u[i]$ **for** $i = 0$ **to** $i = \ell - 1$**?**

HINT: Use the result for $\vec{x}[i]$ *from the previous part.*

(d) **Find the set of all** $\vec{x}[2]$**, given that you are free to choose any** $u[0]$ **and** $u[1]$**.**

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