Discussion 9B

1. Changing behavior through feedback

In this question, we discuss how *feedback control* can be used to change the effective behavior of a system.

(a) Consider the scalar system:

$$x[i+1] = 0.9x[i] + u[i] + w[i] \tag{1}$$

where $u[i]$ is the control input we get to apply based on the current state and $w[i]$ is the external disturbance, each at time i.

Is the system stable? If $|w[i]| \leq \epsilon$, what can you say about $|x[i]|$ at all times i if you further assume that $u[i] = 0$ and the initial condition $x[0] = 0$? How big can $|x[i]|$ get?

(b) Suppose that we decide to choose a control law $u[i] = fx[i]$ to apply in feedback. **Given a specific λ, you want the system to behave like:**

$$x[i+1] = \lambda x[i] + w[i]? \tag{2}$$

To do so, how would you pick f?

NOTE: In this case, $w[i]$ can be thought of like another input to the system, except we can't control it.

(c) **For the previous part, which f would you choose to minimize how big $|x[i]|$ can get?**

(d) **What if instead of a 0.9, we had a 3 in the original eq. (1). Would system stability change? Would our ability to control λ change?**

2. Eigenvalue Placement in Discrete Time

Recall that, for a discrete linear system to be stable, we require that all of the eigenvalues of A in $\vec{x}[i+1] = A\vec{x}[i] + B\vec{u}[i]$ must have magnitude less than 1.

Consider the following linear discrete time system

$$\vec{x}[i+1] = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \vec{x}[i] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[i] + \vec{w}[i] \tag{3}$$

(a) **Is the system given in eq. (3) stable?**

(b) We can attempt to stabilize the system by implementing closed loop feedback. That is, we choose our input $u[i]$ so that the system is stable. **If we were to use state feedback as in eq. (4), what is an equivalent representation for this system? Write your answer as $\vec{x}[i+1] = A_{\text{CL}}\vec{x}[i]$ for some matrix A_{CL}.**

$$u[i] = \begin{bmatrix} f_1 & f_2 \end{bmatrix} \vec{x}[i] \tag{4}$$

HINT: If you're having trouble parsing the expression for $u[i]$, note that $\begin{bmatrix} f_1 & f_2 \end{bmatrix}$ is a row vector, while $\vec{x}[i]$ is column vector. What happens when we multiply a row vector with a column vector like this?)

(c) **Find the appropriate state feedback constants, f_1, f_2, that place the eigenvalues of the state space representation matrix at $\lambda_1 = -\frac{1}{2}, \lambda_2 = \frac{1}{2}$.**

(d) **Is the system now stable in closed-loop, using the control feedback coefficients f_1, f_2 that we derived above?**

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