# EECS 16B Designing Information Devices and Systems II
# Spring 2021 Discussion Worksheet Discussion 8B

The relevant note for this discussion is Note 9.

**1. Eigenvalues Placement in Discrete Time**

Consider the following linear discrete time system

$$\vec{x}[t+1] = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \vec{x}[t] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[t] + \vec{w}[t] \tag{1}$$

(a) Is the system given in eq. (1) stable?

(b) Derive a state space representation of the resulting closed loop system using state feedback of the form $u[t] = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \vec{x}[t]$.

*Hint*: If you're having trouble parsing this expression for $u[t]$, note that $\begin{bmatrix} k_1 & k_2 \end{bmatrix}$ is a *row vector*, while $\vec{x}[t]$ is a *column vector*. What happens when we multiply a row vector with a column vector like this?

(c) Find the appropriate state feedback constants, $k_1, k_2$, that place the eigenvalues of the state space representation matrix at $\lambda_1 = -\frac{1}{2}, \lambda_2 = \frac{1}{2}$.

(d) Is the system now stable?

(e) Suppose that instead of $\begin{bmatrix} 1 \\ 0 \end{bmatrix} u[t]$ in eq. (1), we had $\begin{bmatrix} 1 \\ 1 \end{bmatrix} u[t]$ as the way that the discrete-time control acted on the system. As before, we use $u[t] = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \vec{x}[t]$ to try and control the system. What would the eigenvalues be? Can you move all the eigenvalues to where you want? Give an intuitive explanation of what is going on.

**2. Controlling states by designing sequences of inputs**

This is something that you saw in 16A in the Segway problem. In that problem, you were given a semi-realistic model for a segway. Here, we are just going to consider the following matrix chosen for ease of understanding what is going on:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Let's assume we have a *discrete-time* system defined as follows:

$$\vec{x}[t+1] = A\vec{x}[t] + \vec{b}u[t].$$

(a) We are given the initial condition $\vec{x}[0] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Let's say we want to achieve $\vec{x}[T] = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ for some specific $T \geq 0$. We don't need to stay there, we just want to be in this state at that time. What is the smallest $T$ such that this is possible? What is our choice of sequence of inputs $u[t]$?

(b) What if we started from $\vec{x}[0] = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$? What is the smallest $T$ and what is our choice of $u[t]$?

(c) What if we started from $\vec{x}[0] = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$? What is the smallest $T$ and what is our choice of $u[t]$?

**3. Uncontrollability**

Consider the following discrete-time system with the given initial state:

$$\vec{x}[t+1] = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \vec{x}[t] + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u[t]$$

$$\vec{x}[0] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(a) Is the system controllable?

(b) Is it possible to reach $\vec{x}[T] = \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}$ for some $t = T$? For what input sequence $u[t]$ up to $t = T - 1$?

(c) Is it possible to reach $\vec{x}[T] = \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix}$ for some $t = T$? For what input sequence $u[t]$ up to $t = T - 1$?

(d) Find the set of all possible states reachable after two timesteps.

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