

1. Introduction to Discrete Time Difference Equations

In this question, we discuss how to think about systems represented in discrete time, as well as the similarities and differences with systems in continuous time.

(a) Consider the scalar system:

$$x[i + 1] = 3x[i] + 2u[i] \tag{1}$$

where $x[i]$ is the state parameter of the system and $u[i]$ is the input to the system, each at time i . In this question, we will solve this equation to obtain the closed form solution, ie. an expression for $x[i]$ that only depends on i and $u[i]$. Suppose that the initial condition of this system is $x[0] = 1$.

i. The solution method is very similar to first order ODEs. **First, solve for the homogeneous solution of the difference equation. What kind of solution should $x[i]$ be?** (*HINT: You should guess something similar to $e^{\lambda t}$ which is what we guessed for ODEs.*)

ii. Now that we have the homogeneous solution, **solve for the particular solution.** (*HINT: Start by solving for $x[1]$, then $x[2]$ using your value of $x[1]$, and so on. Do you see a pattern emerging?*)

iii. Using your answers for the previous parts and the initial condition given in part (a), **find the solution for $x[i]$.**

iv. Now, suppose that $u[i] = b$ for all time. **What happens to $x[i]$ as i goes to infinity?**

(b) In this part, we will solve for the impulse and step response of the scalar first order difference equation. Suppose the system equation is:

$$x[i + 1] = ax[i] + bu[i] \quad (2)$$

Suppose that the state system's initial condition is $x[0] = 0$; that is, suppose the system starts at rest.

i. **Solve for the impulse response of the system, $h[i]$, by setting $u[i] = \delta[i]$.** The impulse is defined as

$$\delta[i] = \begin{cases} 1 & i = 0 \\ 0 & i \neq 0 \end{cases}$$

ii. **Solve for the step response of the system, $f[i]$, by setting $u[i] = \text{step}[i]$.** The step function is defined as

$$\text{step}[i] = \begin{cases} 1 & i \geq 0 \\ 0 & i < 0 \end{cases}$$

Determining the system response to a certain input is an important topic in signals and systems, which you will study if you take EE 120.

(c) Now suppose that we have a second order difference equation.

$$x_1[i] - ax_1[i-1] - bx_1[i-2] = cu[i] \quad (3)$$

We could solve this equation using the same method that we used in part (a), but we'll convert it into a vector difference equation since we already know how to solve first order difference equations.

Convert the above equation into a system of equations by setting $x_2[i] = x_1[i-1]$ (HINT: We want the state variables, x_1 and x_2 , on the left hand side of the vector difference equation to depend on time i and the right hand side to depend on time $i-1$.)

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