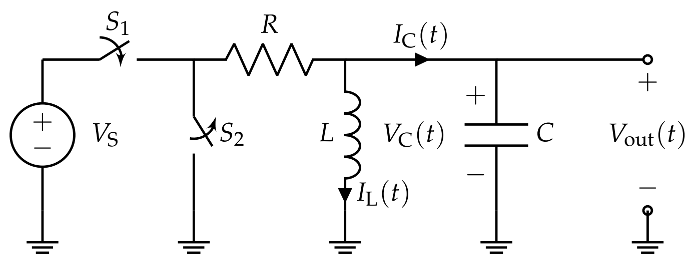
Discussion 8A

1. RLC Circuit with Vector Differential Equations

Consider the following circuit fed by a constant voltage source V_S.



The switch S_1, open for $t < 0$, closes at $t = 0$, and the switch S_2, closed for $t < 0$, opens at $t = 0$. Assume $V_C(0) = 0$ and $I_L(0) = 0$.

(a) **Derive a set of two differential equations, one for $I_L(t)$, the current through the inductor, and one for $V_C(t)$, the voltage across the capacitor.** Write your answer in terms of R, L, C, V_S, and constants.

(b) **Using your answers from the previous part, create a vector differential equation with the state vector being $\vec{x}(t) = \begin{bmatrix} V_C(t) \\ I_L(t) \end{bmatrix}$.** Write your answers in terms of R, L, C, V_S, and constants.

(c) Regardless of your answer to the previous part, suppose the vector differential equation is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = \underbrace{\begin{bmatrix} -4 & -6 \\ \frac{1}{2} & 0 \end{bmatrix}}_{A} \vec{x}(t) + \underbrace{\begin{bmatrix} 4 \\ 0 \end{bmatrix}}_{\vec{b}} V_S \tag{1}$$

First, find the eigenvalues of the matrix A.

(d) **Next, find the eigenvectors that will form your V basis.**

(e) **Now, in order to diagnolize the system, write A in terms of V, V^{-1}, and Λ.** *(HINT: For a 2×2 real matrix, the inverse of that matrix is $V^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.)*

(f) **With $\vec{x}(0) = \vec{0}$, solve for $\vec{x}(t)$ and find the asymptotic/steady-state behavior as $t \to \infty$.** *(HINT: Use the information from the previous part to perform a change of basis that simplifies the state equations.)*