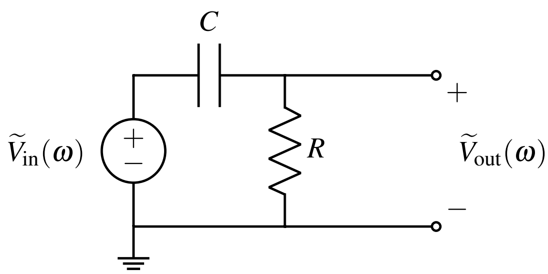
EECS 16B Designing Information Devices and Systems II
Spring 2021 Discussion Worksheet Discussion 6A

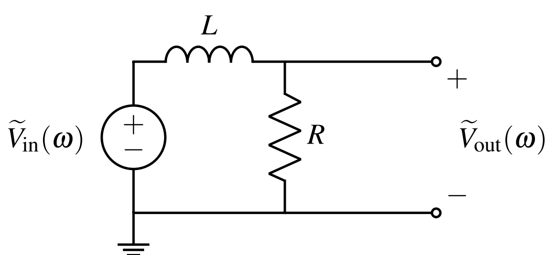
The relevant notes for this discussion are Note 5 and Note 6.

1. Transfer function practice

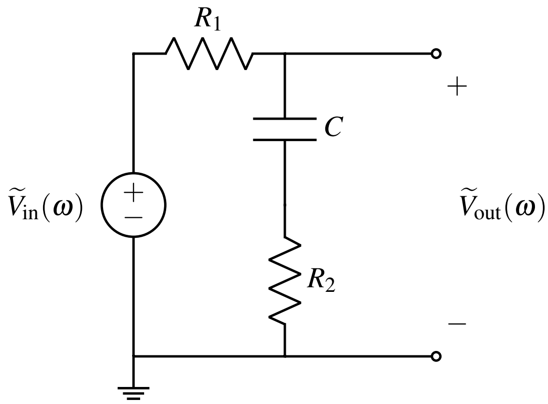
In this problem, you'll be deriving some transfer functions on your own. For each circuit, determine $H(\omega) = \frac{\widetilde{V}_{\text{out}}(\omega)}{\widetilde{V}_{\text{in}}(\omega)}$. How does each circuit respond as $\omega \to 0$ (low frequencies)? as $\omega \to \infty$ (high frequencies)?



(a)



(b)

(c) 

2. Plotting and combining transfer functions

Recall that any transfer function can be written in polar form as

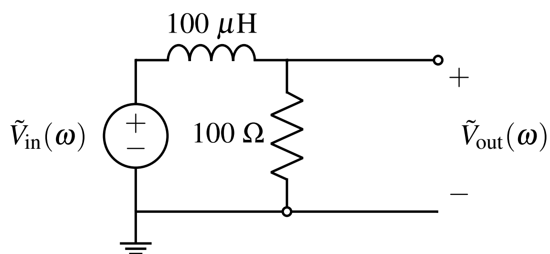
$$H(\omega) = A(\omega)e^{j\alpha(\omega)}$$

where $A(\omega)$ and $\alpha(\omega)$ are real functions of omega giving the magnitude and phase of the transfer function, respectively. To see how transfer functions combine, consider two $H_1(\omega)$ and $H_2(\omega)$.

$$\begin{aligned} H_1(\omega) &= Ae^{j\alpha} \\ H_2(\omega) &= Be^{j\beta} \\ H_1(\omega) \cdot H_2(\omega) &= Ae^{j\alpha}Be^{j\beta} = ABe^{j(\alpha+\beta)} \\ \frac{H_1(\omega)}{H_2(\omega)} &= \frac{Ae^{j\alpha}}{Be^{j\beta}} = \frac{A}{B}e^{j(\alpha-\beta)} \end{aligned}$$

As you can see, magnitudes of transfer functions multiply and divide while the phases add and subtract.

In this problem we will try to plot the transfer function of the following circuit:



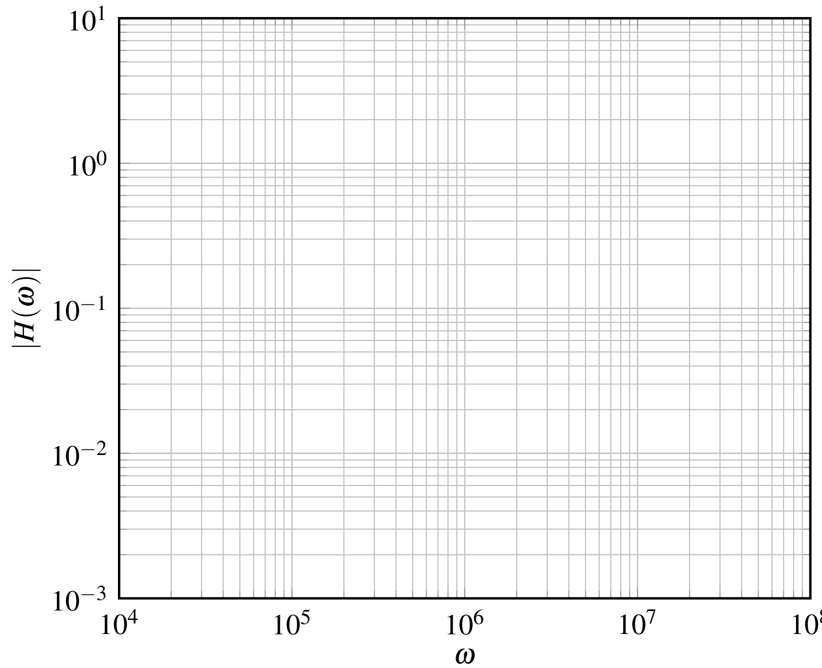
(a) Write expressions for $|H(\omega)|$ and $\angle H(\omega)$. For now, you can keep it in terms of R and L.

(b) What is the cutoff frequency for this circuit? Mark it on the log-log plot with a vertical line. Recall that a transfer function of the form $H(\omega) = \frac{k}{1+j\omega/\omega_c}$ has a cutoff frequency of ω_c.

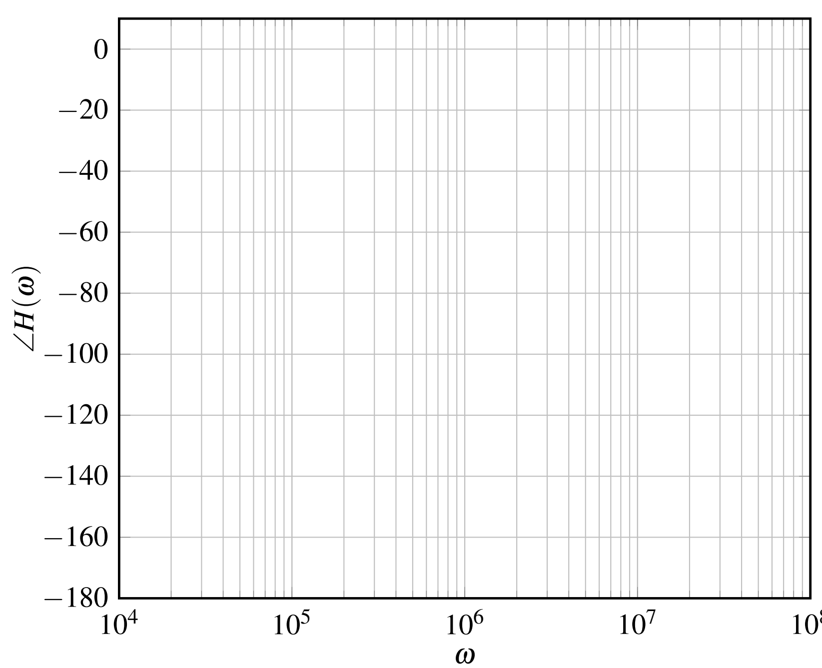
(c) Sketch plots of the magnitude and phase of this transfer function. We have provided a table with the transfer function evaluated at a few representative points around the cutoff frequency.

ω	10^4	10^5	10^6	10^7	10^8
$\lvert H(\omega)\rvert$	1.00	0.995	0.707	0.100	0.01
$\angle H(\omega)$	-0.6	-6	-45	-84	-89

Plot of $|H(\omega)|$ (for **you** to draw).



Plot of $\angle H(\omega)$ (for **you** to draw).



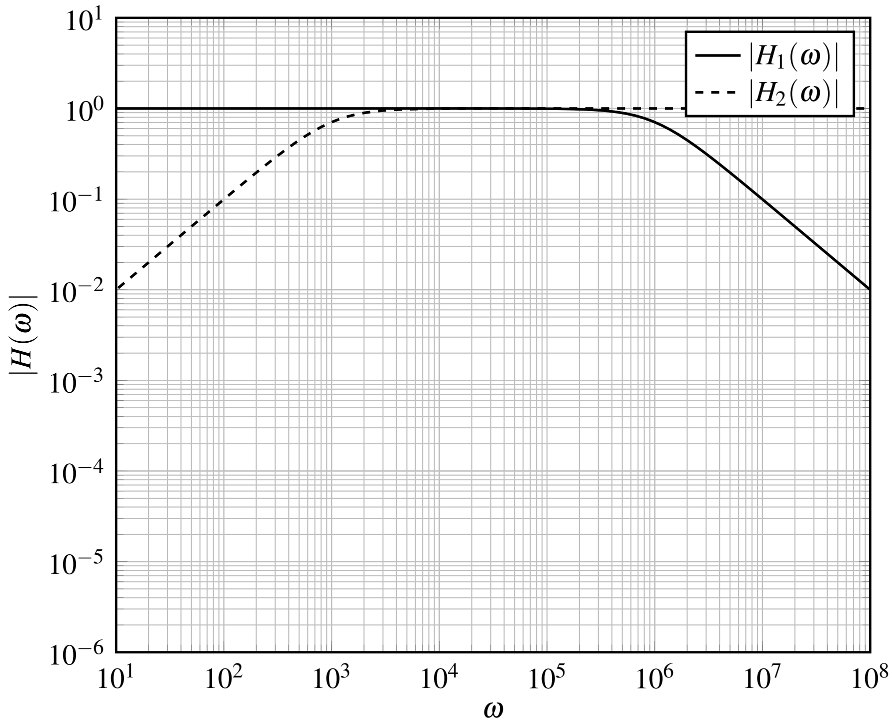
(d) Now suppose we want to compose the filter from part (1.a) with this new circuit. Use $R = 1\text{k}\Omega$ and $C = 1\mu\text{F}$ for the filter from part (1.a). We can compose two circuits by connecting the output of the first circuit into the second circuit, through a unity gain buffer. For this problem, call the circuit from this system with an inductor H_1, and the filter from (1.a) H_2. The transfer function of the composed circuit is:

$$H(\omega) = H_1(\omega) \cdot H_2(\omega)$$

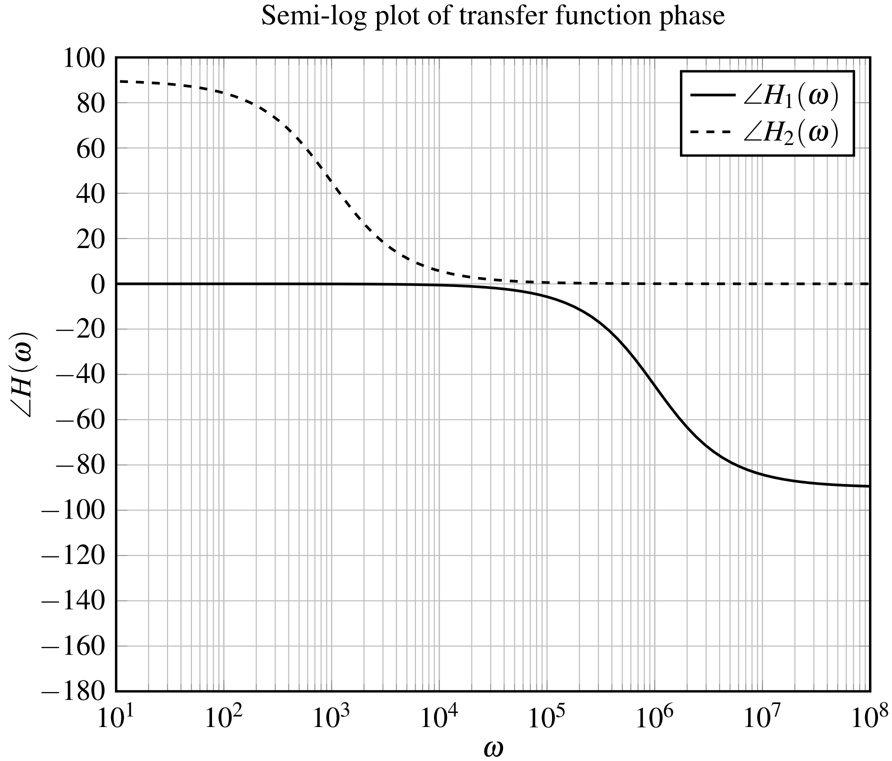
i. Draw this circuit.

ii. Plot the magnitude of the composed circuit. Below is a log-log plot with the magnitudes of $|H_1(\omega)|$ and $|H_2(\omega)|$ drawn to assist you. Recall that $|AB| = |A| \cdot |B|$ for complex numbers A and B.

Plots of $|H_i(\omega)|$.



ii. Plot the phase of the composed circuit. Below is a semi-log plot with the phases $\angle H_1(\omega)$ and $\angle H_2(\omega)$ drawn to assist you.



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