EECS 16B Designing Information Systems and Devices II UC Berkeley Spring 2024

Discussion 5A

1. Transfer Function Practice

Transfer functions take an input phasor and "transform" it into an output phasor. Most of the work we will do with transfer functions is analyzing how it will "respond" to a specific kind of input. We will also design our own transfer functions using common circuit components such as resistors, inductors, and capacitors to achieve some specified behavior. A block diagram of a transfer function is represented below. In this discussion, we will learn how to derive $H(\mathrm{j}\omega)$ from a given circuit, and we will analyze how it behaves for certain values of ω.

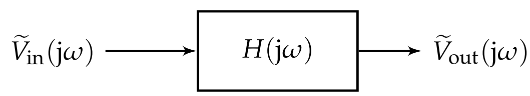


Figure 1: Transfer Function Block Diagram

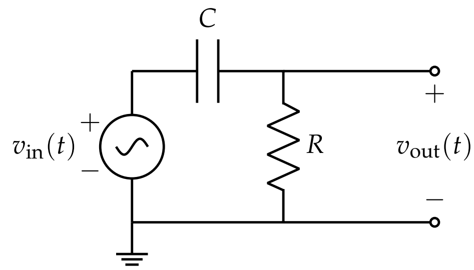
Recall that $Z_L = \mathrm{j}\omega L$ and $Z_C = \frac{1}{\mathrm{j}\omega C}$. For large ω, $|Z_L| = \omega L$ becomes large (and becomes small for small ω). On the other hand, for large ω, $|Z_C| = \frac{1}{\omega C}$ becomes small (and becomes large for small ω).

In this problem, you'll be deriving some transfer functions. For each circuit:

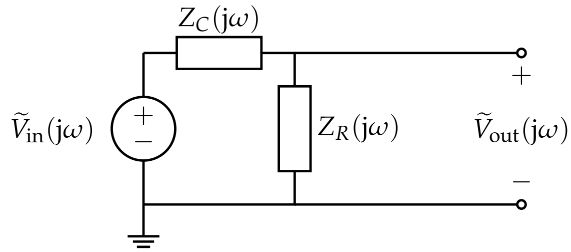
- Determine the **transfer function** $H(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out}}(\mathrm{j}\omega)}{\widetilde{V}_{\text{in}}(\mathrm{j}\omega)}$.
- How does $|H(\mathrm{j}\omega)|$ respond as $\omega \to 0$ **(low frequencies)** and as $\omega \to \infty$ **(high frequencies)**?
- Is the circuit a **high-pass filter, low-pass filter, or band-pass filter**?
- **For parts (a) and (b)**, find the **cutoff frequency** ω_c, which is the frequency such that

$$|H(\mathrm{j}\omega_c)| = \frac{|H(\mathrm{j}\omega)|_{\max}}{\sqrt{2}} \tag{1}$$

(a) **RC circuit** ($R = 1\,\text{k}\Omega$, $C = 1\,\mu\text{F}$):

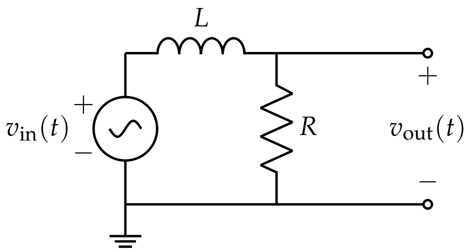


(a) Circuit in “time domain”

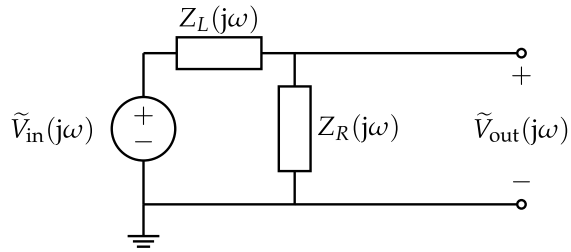


(b) Circuit in “phasor domain”

(b) **LR circuit** ($L = 5\,\mathrm{H}$, $R = 500\,\Omega$):

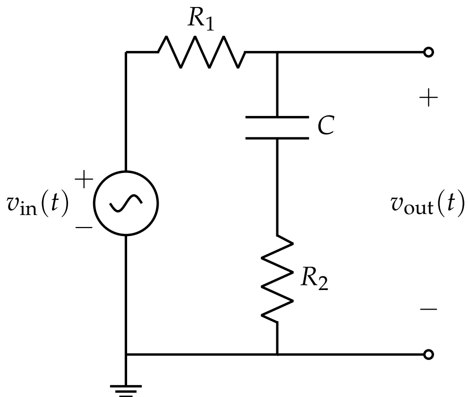


(a) Circuit in "time domain"

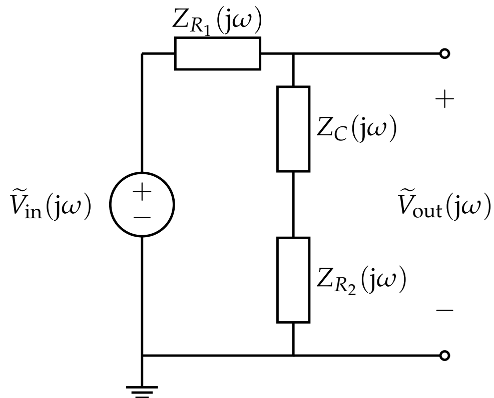


(b) Circuit in "phasor domain"

(c) **RCR circuit** ($R_1 = 9\,\mathrm{k\Omega}$, $R_2 = 1\,\mathrm{k\Omega}$, $C = 1\,\mathrm{\mu F}$):



(a) Circuit in "time domain"

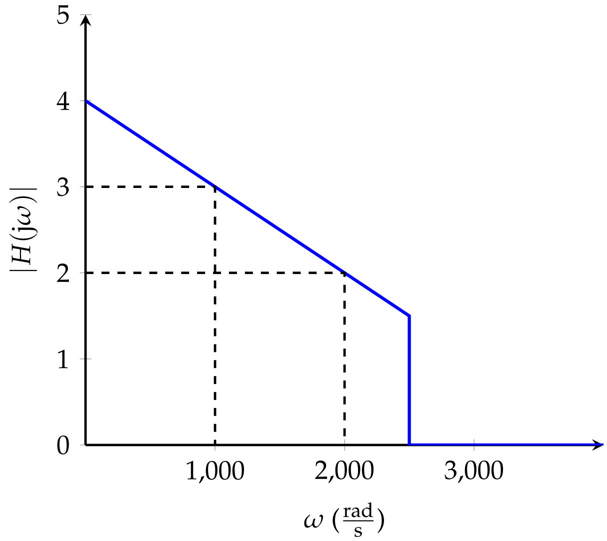


(b) Circuit in "phasor domain"

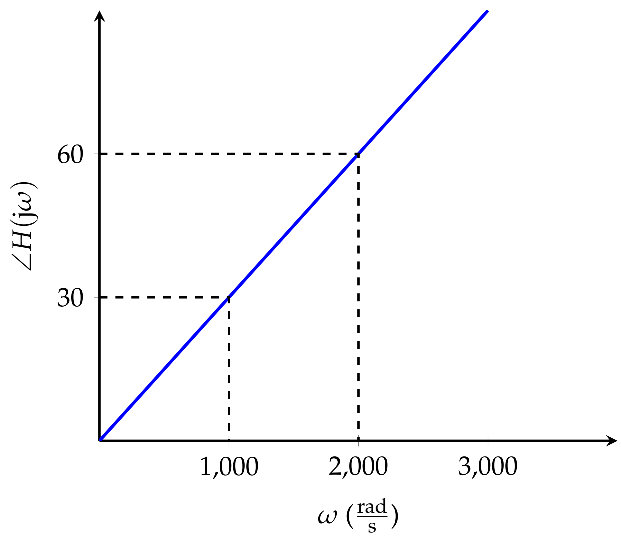
(d) **Assuming $v_{\text{in}}(t) = 12\sin(\omega_{\text{in}} t)$ compute the $v_{\text{out}}(t)$ using the transfer function computed in part 1.a.** Remember that $R = 1\,\text{k}\Omega$ and $C = 1\,\mu\text{F}$ for this circuit, and assume $\omega_{\text{in}} = 1000\,\frac{\text{rad}}{\text{s}}$. In words, what is the effect of the transfer function in part 1.a on the magnitude and phase of the input signal?

2. Linearity of Transfer Functions (Adapted from Hambley Example 6.1)

The transfer function $H(j\omega)$ of a filter is shown in Figure 5.



(a) Transfer Function Magnitude



(b) Transfer Function Phase

Figure 5: Transfer Function $H(j\omega)$

If the input signal is given by

$$v_{\text{in}}(t) = 2\cos\left(1000t + \frac{\pi}{6}\right) + 2\cos(2000t) \tag{2}$$

find an expression for the output of the filter $v_{\text{out}}(t)$.

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