
EECS 16B Designing Information Devices and Systems II
Spring 2021 Discussion Worksheet Discussion 3B

The relevant notes for this discussion are the sections 1 and 2 of [Note 3](#).

1. Changing Coordinates and Systems of Differential Equations

Suppose we have the pair of differential equations (valid for $t \geq 0$)

$$\frac{d}{dt}x_1(t) = -9x_1(t)$$

$$\frac{d}{dt}x_2(t) = -2x_2(t)$$

with initial conditions $x_1(0) = -1$ and $x_2(0) = 3$.

- (a) Solve for $x_1(t)$ and $x_2(t)$ for $t \geq 0$.

Suppose we are actually interested in a different set of variables with the following differential equations:

$$\frac{d}{dt}z_1(t) = -5z_1(t) + 2z_2(t)$$

$$\frac{d}{dt}z_2(t) = 6z_1(t) - 6z_2(t).$$

- (b) Write out the above system of differential equations in matrix form. Assuming that the initial state $\vec{z}(0) = \begin{bmatrix} 7 & 7 \end{bmatrix}^T$, can we solve this system directly?

- (c) Consider that in our frustration with the previous system of differential equations, we start hearing voices. These voices whisper to us that that we should try the following change of variables:

$$z_1(t) = -y_1(t) + 2y_2(t)$$

$$z_2(t) = 2y_1(t) + 3y_2(t).$$

Write out this transformation in matrix form ($\vec{z} = V\vec{y}$).

For each of the parts (d) - (f), solve the questions two ways: 1. using direct substitution, and 2. using matrices and vectors .

- (d) How do the initial conditions for $z_i(t)$ translate into the initial conditions for $y_i(t)$?

1. By direct substitution:

2. Using matrices and vectors:

(e) Rewrite the differential equations in terms of $y_i(t)$. Can we solve this system of differential equations?

1. By direct substitution:

2. Using matrices and vectors:

(f) What are the solutions for $z_i(t)$?

1. Solve this with direct substitution:

2. Solve this with matrices and vectors:

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