Discussion 3A

1. RL Circuit Solution Methods

Consider the following circuit:

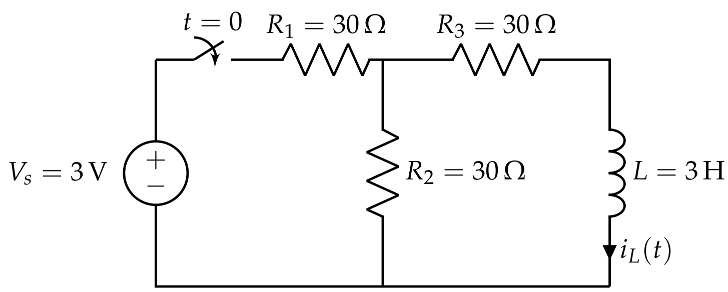


Figure 1

Before time $t = 0$, the circuit reaches a steady state. At time $t = 0$, the switch is closed. Our goal is to find the differential equation for the current through the inductor ($i_L(t)$). One method to approach this problem is to simply use Node Voltage Analysis (NVA). To start, we would define the node voltages in our circuit (including a ground node).

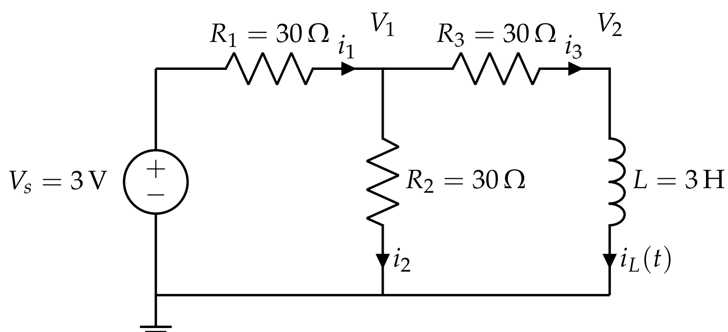


Figure 2

Then, we can set up a system of equations using KCL/KVL to find our desired differential equation.

First, let's perform KCL on the node with defined voltage V_1.

$$
\begin{aligned}
i_1 &= i_2 + i_3 \\
\frac{V_s - V_1}{R_1} &= \frac{V_1 - 0}{R_2} + \frac{V_1 - V_2}{R_3} \\
\frac{3 - V_1}{30} &= \frac{V_1 - 0}{30} + \frac{V_1 - V_2}{30} \\
V_1 &= 1 + \frac{V_2}{3}
\end{aligned}
$$

Now, let's perform KCL on the node with the defined voltage V_2.

Note that $V_2 - 0 = V_2$ is the voltage across the inductor so by the inductor I-V relationship, $V_2 = L\frac{\mathrm{d}i_L}{\mathrm{d}t} = 3\frac{\mathrm{d}i_L}{\mathrm{d}t}$.

$$
\begin{aligned}
i_3 &= i_L \\
\frac{V_1 - V_2}{R_3} &= i_L \\
\frac{V_1 - V_2}{30} &= i_L \\
\frac{V_1}{30} &= \frac{V_2}{30} + i_L \\
\frac{1}{30}\left(1 + \frac{V_2}{3}\right) &= \frac{V_2}{30} + i_L \\
\frac{1}{45}V_2 + i_L &= \frac{1}{30} \\
\frac{1}{45}\left(3\frac{\mathrm{d}i_L}{\mathrm{d}t}\right) + i_L &= \frac{1}{30} \\
\frac{\mathrm{d}i_L}{\mathrm{d}t} + 15i_L &= \frac{1}{2}
\end{aligned}
$$

Thus, we have found the differential equation! However, this method required solving a system of equations; is there another way?

(a) Another way to approach the problem is to use equivalence. Simplify the voltage source and resistor network into a voltage source and resistor using Thevenin equivalence. Then, reconnect the inductor and **find the differential equation for** $i_L(t)$.

For reference, here is the circuit that we want to simplify using Thevenin equivalence:

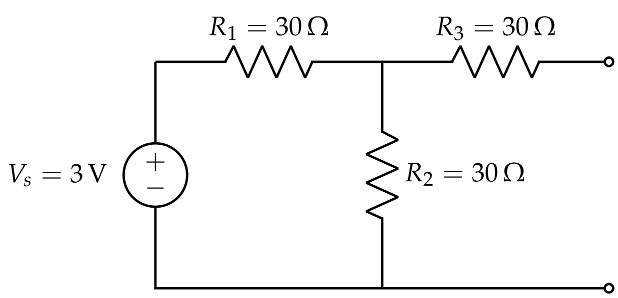


Figure 3

(HINT: Your final differential equation should be the same as the one from the problem introduction.)

(b) Now, let's start solving the differential equation. First, **find the initial condition $i_L(0)$ for our system**. Remember that the current through the inductor cannot change instantaneously (since this would correspond to infinite voltage through the inductor I-V relationship) so $i_L(0)$ will be the same as the steady state value from $t < 0$.

(HINT: If there is no voltage/current sources connected to this system, can there be any nonzero currents / voltage differences in the system during steady-state?)

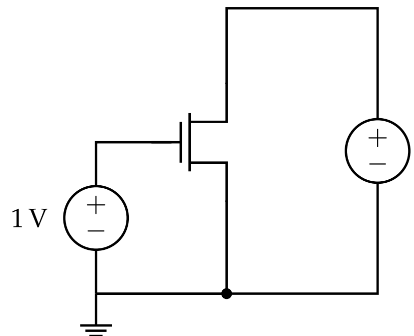
(c) **(OPTIONAL)** Now that we have our differential equation and initial condition, we can now solve for the current $i_L(t)$ as a function of time. **Solve the system for $i_L(t)$.** If you can, try to solve this by inspection. Otherwise, solve using the homogeneous and particular solution method.

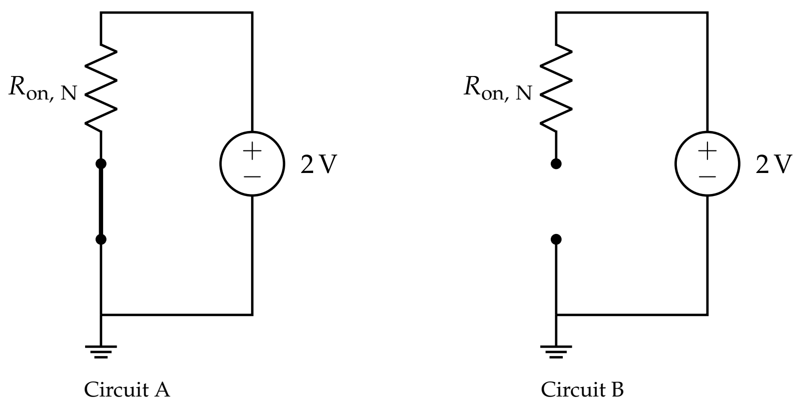
2. Transistor Behavior

Unlocked by Lectures 1 and 2

For all NMOS devices in this problem, $V_{tn} = 0.5\,\text{V}$. For all PMOS devices in this problem, $|V_{tp}| = 0.6\,\text{V}$. **Note: For this problem, we are also using the resistor-switch model for a transistor.**

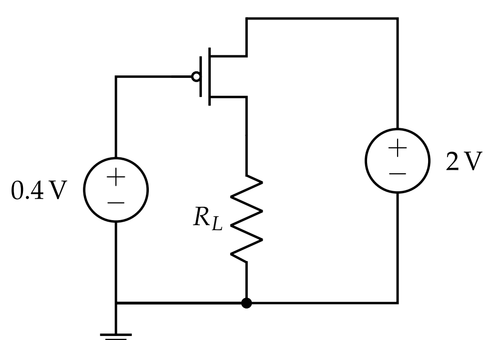
(a) Which is the equivalent circuit as seen from the voltage source on the right-hand side of the circuit? **Fill in the correct bubble.**

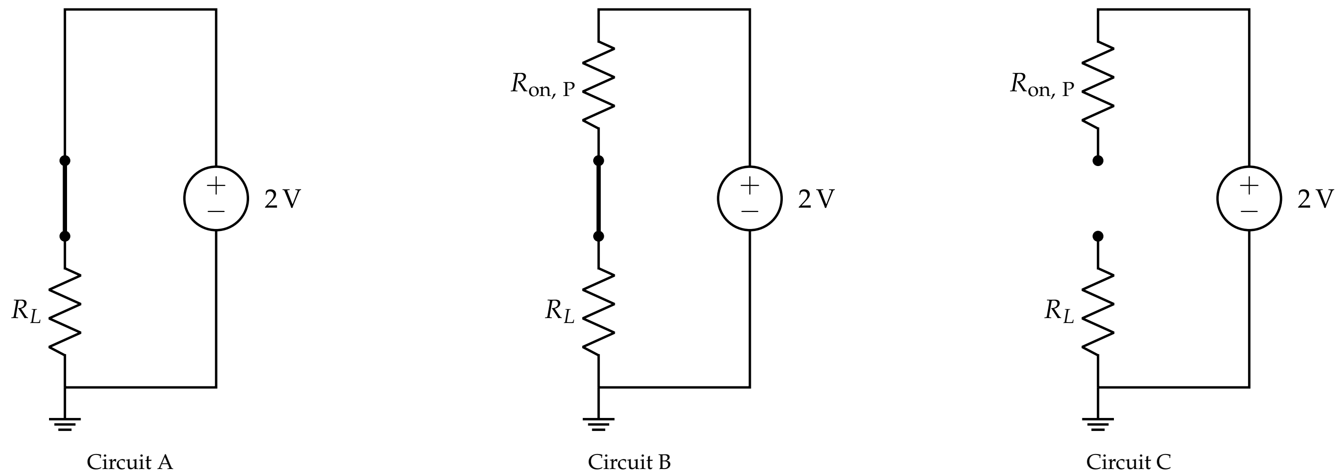




	A	B
Equivalent Circuit	◯	◯

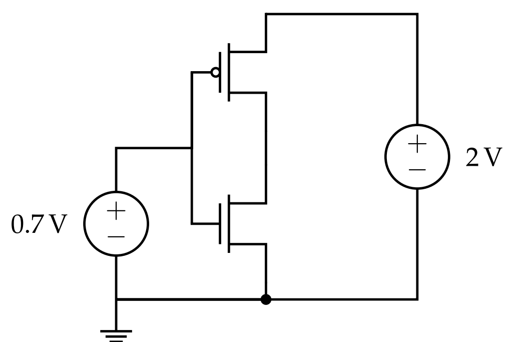
(b) Which is the equivalent circuit as seen from the voltage source on the right-hand side of the circuit? **Fill in the correct bubble.**

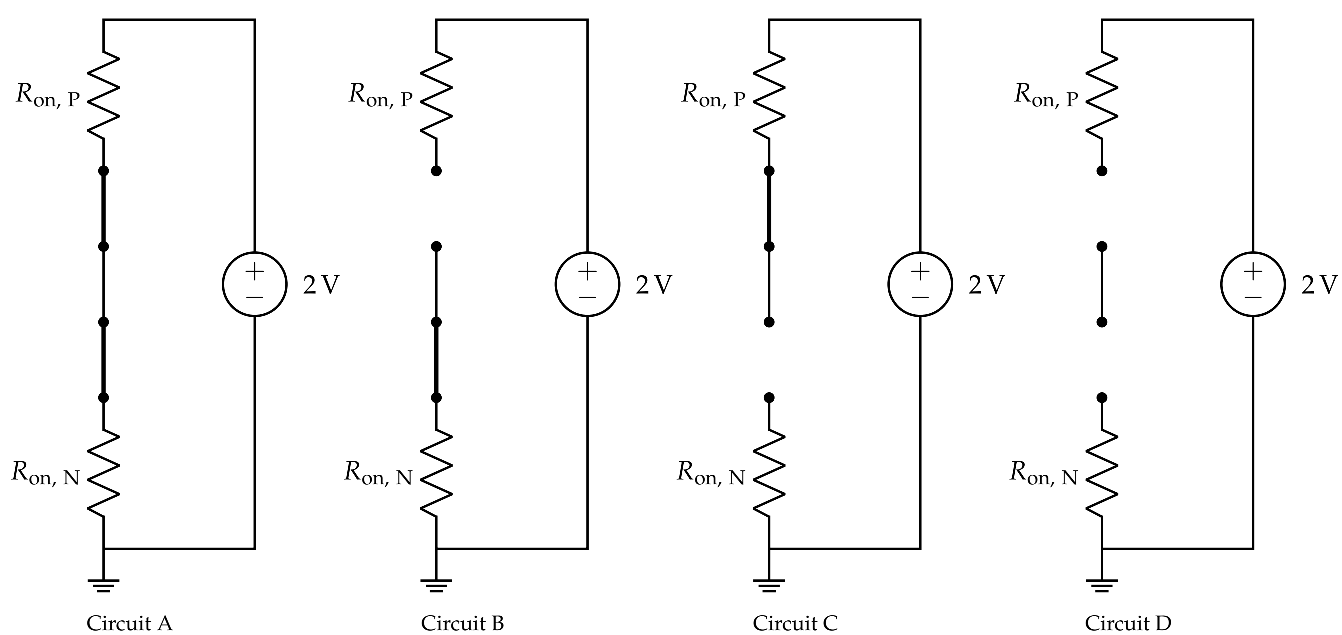




	A	B	C
Equivalent Circuit	◯	◯	◯

(c) Which is the equivalent circuit as seen from the voltage source on the right-hand side of the circuit? **Fill in the correct bubble.**





	A	B	C	D
Equivalent Circuit	◯	◯	◯	◯

Contributors:

- Nikhil Jain.
- Sidney Buchbinder.
- Anant Sahai.
- Chancharik Mitra.

