1. Complex Algebra (Review)

(a) Express the following values in polar forms: \(-1, j, -j, (j)^{\frac{1}{2}}, \text{ and } (-j)^{\frac{1}{2}}\). Recall \(j^2 = -1\), and the complex conjugate of a complex number is denoted with a bar over the variable. The complex conjugate is defined as follows: for a complex number \(z = x + jy\), the complex conjugate \(\bar{z} = x - jy\).

(b) Represent \(\sin(\theta)\) and \(\cos(\theta)\) using complex exponentials. (Hint: Use Euler’s identity \(e^{j\theta} = \cos(\theta) + j\sin(\theta)\).)

For the next parts, let \(a = 1 - j\sqrt{3}\) and \(b = \sqrt{3} + j\).

(c) Show the number \(a\) in complex plane, marking the distance from origin and angle with real axis.
(d) Show that multiplying $a$ with $j$ is equivalent to rotating the complex number by $\frac{\pi}{2}$ or $90^\circ$ in the complex plane.

(e) For complex number $z = x + jy$ show that $|z| = \sqrt{\overline{z}z}$, where $\overline{z}$ is the complex conjugate of $z$. 
(f) Express $a$ and $b$ in polar form.

(g) Find $ab$, $a\overline{b}$, $\frac{a}{b}$, $a + \overline{a}$, $a$, $\overline{ab}$, $\overline{a\overline{b}}$, and $(b)^{\frac{1}{2}}$. 
2. Introduction to Inductors

An inductor is a circuit element analogous to a capacitor; its voltage is proportional to the derivative of the current across it. That is:

\[ V_L(t) = L \frac{dI_L(t)}{dt} \]  

When first studying capacitors, we analyzed a circuit where a current source was directly attached to a capacitor. In Figure 1, we form the counterpart circuit for an inductor:

![Figure 1: Inductor in series with a voltage source.](image)

(a) **What is the current through an inductor as a function of time?** If the inductance is \( L = 3 \text{ H} \), what is the current at \( t = 6 \text{ s} \)? Assume that the voltage source turns from 0 V to 5 V at time \( t = 0 \text{ s} \), and there’s no current flowing in the circuit before the voltage source turns on, i.e \( I_L(0) = 0 \text{ A} \).

(b) Now, we add some resistance in series with the inductor, as in Figure 2.

![Figure 2: Inductor in series with a voltage source.](image)
Solve for the current $I_L(t)$ and voltage $V_L(t)$ in the circuit over time, in terms of $R, L, V_S, t$. Note that $I_L(0) = 0$ A. Try to solve this equation by inspection. Otherwise, you can use the following integral for the particular solution (with the proper values and functions):

$$e^{-st} \int e^{st} b(t) dt$$

(c) Suppose $R = 500 \, \Omega, L = 1 \, \text{mH}, V_S = 5 \, \text{V}$. Plot the current through and voltage across the inductor ($I_L(t), V_L(t)$), as these quantities evolve over time.
$V_L(t)$ (V)

$t$ (µs)

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