Discussion 2B

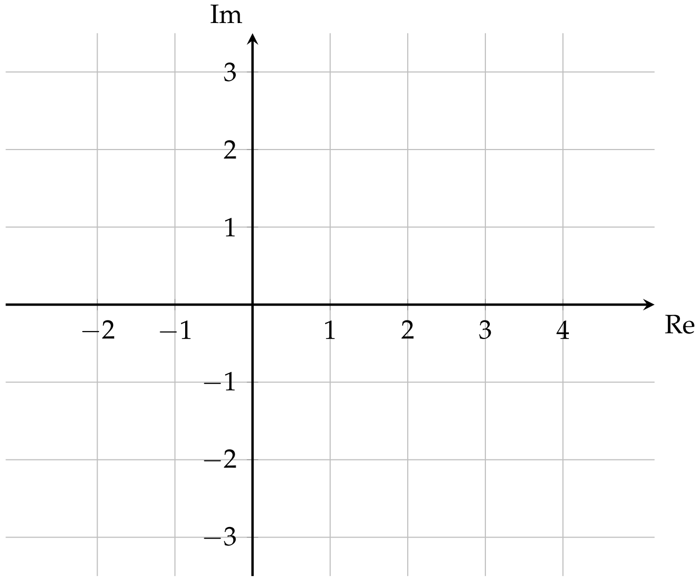
1. **Complex Algebra (Review)**

   (a) **Express the following values in polar forms: -1, j, $-$j, $(\mathrm{j})^{\frac{1}{2}}$, and $(-\mathrm{j})^{\frac{1}{2}}$.** Recall $\mathrm{j}^2 = -1$, and the complex conjugate of a complex number is denoted with a bar over the variable. The complex conjugate is defined as follows: for a complex number $z = x + \mathrm{j}y$, the complex conjugate $\overline{z} = x - \mathrm{j}y$.

   (b) Represent $\sin(\theta)$ and $\cos(\theta)$ using complex exponentials. (*Hint:* Use Euler's identity $\mathrm{e}^{\mathrm{j}\theta} = \cos(\theta) + \mathrm{j}\sin(\theta)$.)

   For the next parts, let $a = 1 - \mathrm{j}\sqrt{3}$ and $b = \sqrt{3} + \mathrm{j}$.

   (c) Show the number a in complex plane, marking the distance from origin and angle with real axis.



(d) Show that multiplying a with j is equivalent to rotating the complex number by $\frac{\pi}{2}$ or 90° in the complex plane.

(e) For complex number $z = x + \mathrm{j}y$ show that $|z| = \sqrt{z\bar{z}}$, where $\bar{z}$ is the complex conjugate of z.

(f) Express a and b in polar form.

(g) Find ab, $a\overline{b}$, $\frac{a}{b}$, $a+\overline{a}$, $a-\overline{a}$, $\overline{ab}$, $\overline{a}\overline{b}$, and $(b)^{\frac{1}{2}}$.

2. Introduction to Inductors

An inductor is a circuit element analogous to a capacitor; its voltage is proportional to the derivative of the current across it. That is:

$$V_L(t) = L\frac{\mathrm{d}I_L(t)}{\mathrm{d}t} \tag{1}$$

When first studying capacitors, we analyzed a circuit where a current source was directly attached to a capacitor. In Figure 1, we form the counterpart circuit for an inductor:

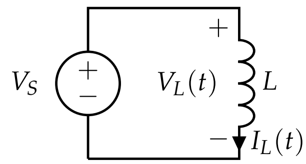


Figure 1: Inductor in series with a voltage source.

(a) **What is the current through an inductor as a function of time? If the inductance is $L = 3\,\mathrm{H}$, what is the current at $t = 6\,\mathrm{s}$?** Assume that the voltage source turns from $0\,\mathrm{V}$ to $5\,\mathrm{V}$ at time $t = 0\,\mathrm{s}$, and there's no current flowing in the circuit before the voltage source turns on, i.e $I_L(0) = 0\,\mathrm{A}$.

(b) Now, we add some resistance in series with the inductor, as in Figure 2.

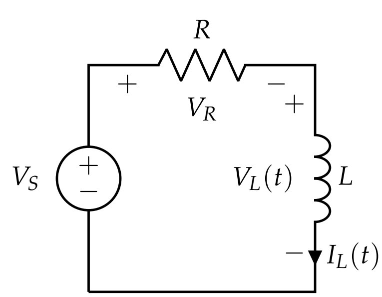
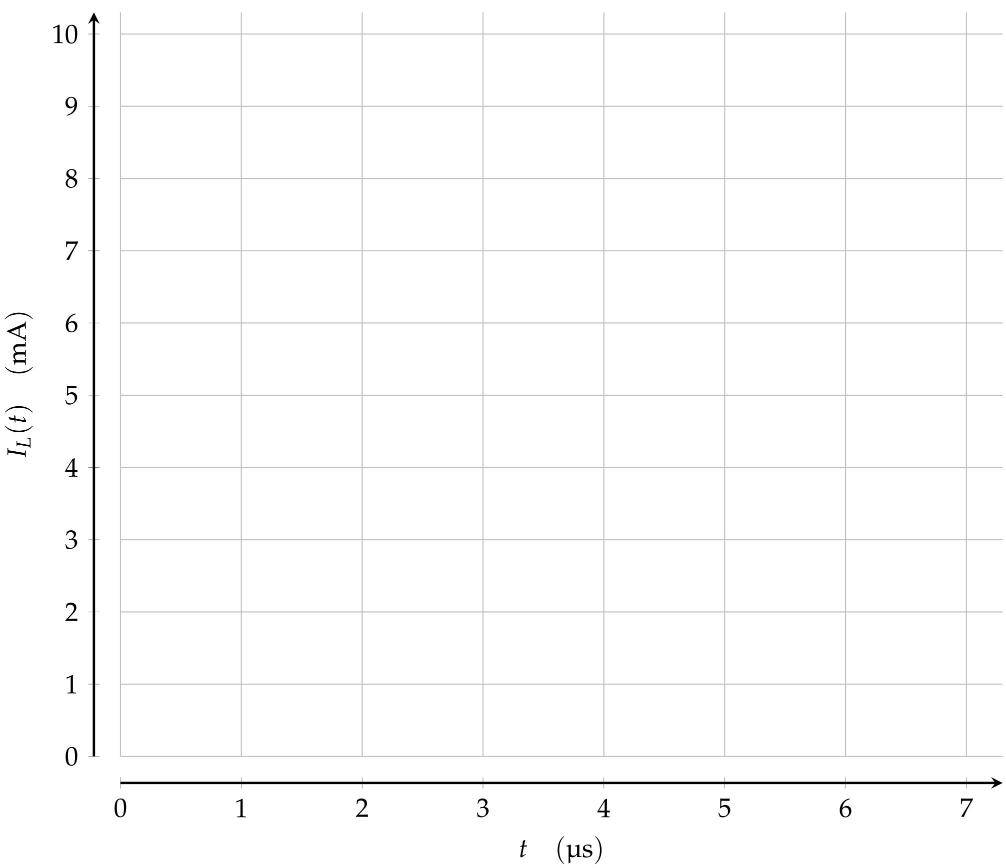


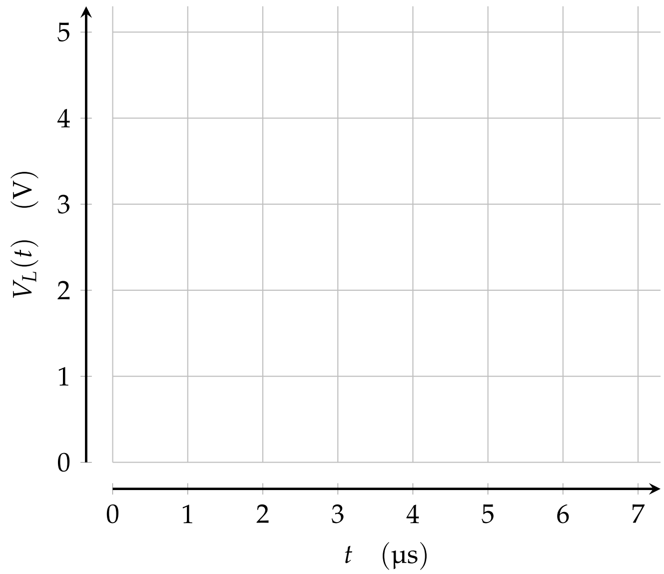
Figure 2: Inductor in series with a voltage source.

Solve for the current $I_L(t)$ and voltage $V_L(t)$ in the circuit over time, in terms of R, L, V_S, t. Note that $I_L(0) = 0\,\text{A}$. Try to solve this equation by inspection. Otherwise, you can use the following integral for the particular solution (with the proper values and functions):

$$e^{-st} \int e^{st} b(t) dt$$

(c) **Suppose $R = 500\,\Omega, L = 1\,\text{mH}, V_S = 5\,\text{V}$. Plot the current through and voltage across the inductor ($I_L(t), V_L(t)$), as these quantities evolve over time.**





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