Discussion 2A

1. Current, Power, and Energy for a Capacitance (Hambley Example 3.3)

Suppose that the voltage waveform shown in Figure 1 is applied to a 10-μF capacitance.

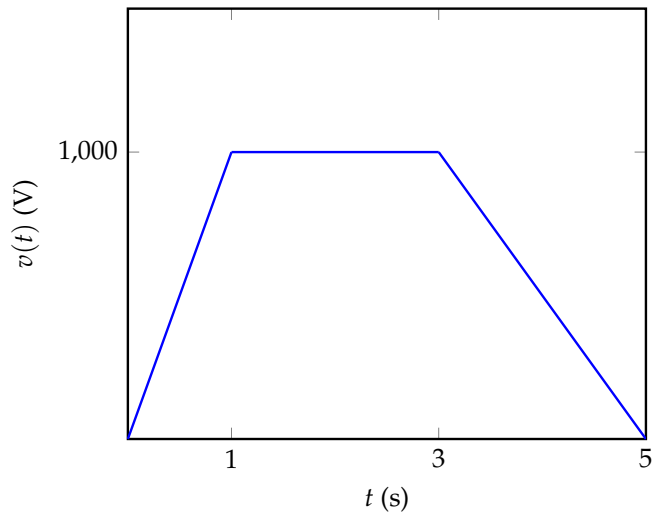

Figure 1: Plot of $v(t)$

Find and plot the current, the power delivered, and the energy stored for time between 0 and 5 s.

2. Analyzing an RC Circuit with a Constant Source (Adapted from Hambley Example 4.4)

Assume you are given the following circuit, where the capacitor is initially charged so that $v_C(t) = 1V$.

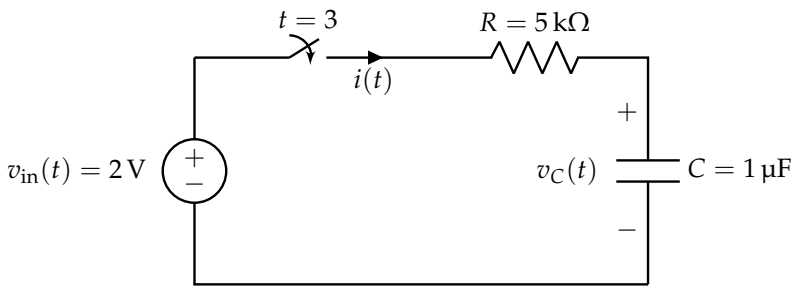

Figure 2

(a) **Set up a differential equation for the voltage $v_C(t)$ across the capacitor in the form:**

$$\frac{\mathrm{d}v_C(t)}{\mathrm{d}t} = \lambda v_C(t) + u(t) \tag{1}$$

(b) **Solve for the voltage $v_C(t)$ using the homogeneous and particular solution method, with initial time $t_0 = 3$.** *(HINT: Recall, that $v_C(t)$ is composed of a homogeneous and a particular solution. First, use your work from the previous part to find the homogeneous solution form. Then, to solve for both the unkown coefficient and the particular solution, find the initial condition of $v_C(t)$ using steady-state properties. As an extra check, you can always plug your final solution back into the differential equation to confirm.)*

3. Analyzing an RC Circuit with a Sinusoidal Source (Adapted from Hambley Example 4.4)

Assume you are given the following circuit, where the capacitor is initially charged so that $v_C(t) = 1V$.

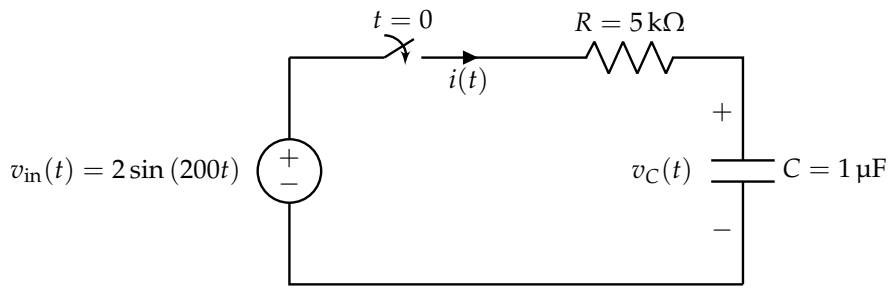

Figure 3

(a) **Set up a differential equation for the voltage $v_C(t)$ across the capacitor in the form:**

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} + av(t) = b(t) \tag{2}$$

(b) **What is the initial condition of $v(t)$? In other words, what is $v(0)$?**

(c) **Solve for the voltage $v(t)$ through the circuit. Also, identify the transient response (homogeneous solution) and the forced response (particular solution) of $v(t)$.** You may directly use the fact that the solution to a differential equation in the same form as Equation 2 is:

$$v(t) = A\mathrm{e}^{-a(t-t_0)} + \mathrm{e}^{-at}\int_{t_0}^{t-t_0} \mathrm{e}^{at'} b(t')\,\mathrm{d}t' \tag{3}$$

(d) **(OPTIONAL) Solve for the current $i(t)$ through the circuit.**

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