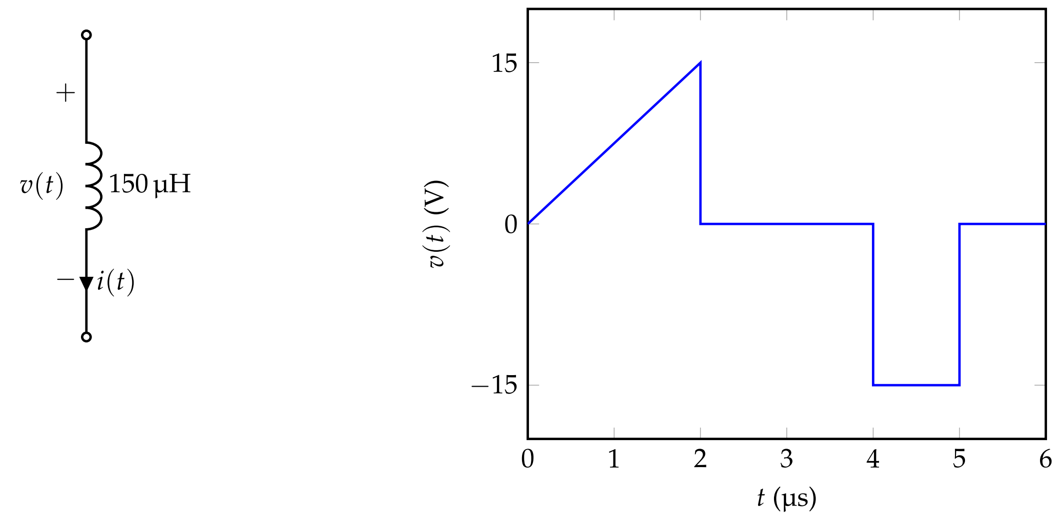
1. Determining Current for an Inductor (Hambley Exercise 3.7)

The voltage across a 150-μH inductance is shown in Figure 1. The initial current is $i(0) = 0$.



(a) Example Circuit

(b) Plot of $v(t)$

Figure 1

Find and plot the current $i(t)$ to scale versus time. Assume that the references for $v(t)$ and $i(t)$ have the passive configuration (current enters through the (+) terminal of the passive component).

Solution: Here we want to use the equation

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t')\,\mathrm{d}t' + i(t_0) \tag{1}$$

For the interval where t is between 0 and 2 μs,

$$\begin{aligned} i(t) &= \frac{1}{150}\int_{0}^{t} 7.5t'\,\mathrm{d}t' + i(0) && (2)\\ &= \frac{1}{150}\left[3.75t^2\right]\Big|_0^t && (3)\\ &= \frac{3.75}{150}t^2 && (4)\\ &= 0.025t^2\,\mathrm{A} && (5) \end{aligned}$$

Note: Let's verify what our units are for the equation.

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t')\,\mathrm{d}t' + i(t_0) \tag{6}$$

Since we are integrating voltage over time (which is given to us in µs), the units for $\int_{t_0}^{t} v(t')\,\mathrm{d}t'$ will be V µs. The coefficient of $\frac{1}{L}$ will have the units $\frac{1}{\mu\mathrm{H}}$. Combining these units, we have $\frac{\mathrm{V}\,\mu\mathrm{s}}{\mu\mathrm{H}} = \frac{\mathrm{V\,s}}{\mathrm{H}} = \mathrm{A}$.

Therefore, our derived equation for current is in amperes. The same will apply for the following intervals.

For the interval between 2 and 4 μs, the voltage is 0 meaning that the current remains constant during this time. Since the current of an inductor cannot change instantaneously, the current will remain constant at the value at $t = 2\mu$s, which is 0.1A.

For the last interval between 4 and 5, we will apply the same equation. This time we want to integrate starting from $t_0 = 4\mu$s.

$$i(t) = \frac{1}{150}\int_4^t -15\,\mathrm{d}t' + i(4) \tag{7}$$

$$= \frac{1}{150}\left[-15t\right]\Big|_4^t + 0.1 \tag{8}$$

$$= -\frac{15}{150}(t-4) + 0.1 \tag{9}$$

$$= -0.1\,(t-5)\ \mathrm{A} \tag{10}$$

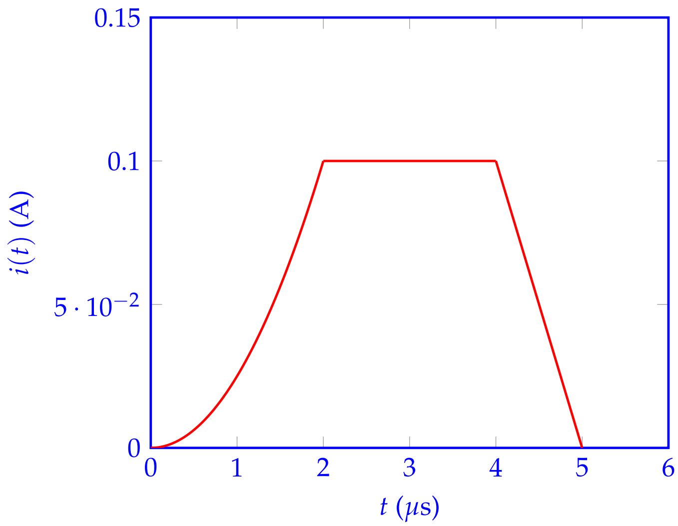


Figure 2: Plot of $i(t)$

2. Calculating Equivalent Inductance (Hambley Exercise 3.10)

Find the equivalent inductance for the circuit shown in Figure 3.

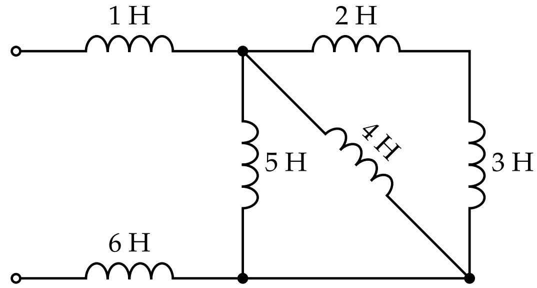


Figure 3: Inductor Circuit

Solution: Recall, that the inductor equivalence equations for series and parallel are analagous to that of resistors.

First, start by noticing that that the 2H and 3H inductors are in series and we can treat those inductors as a single inductor with $2 + 3 = 5$H inductance:

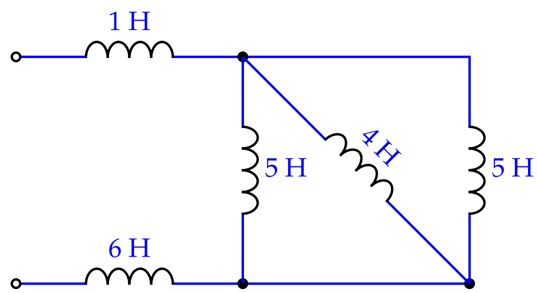


Figure 4

Then, notice that the new 5H inductor is in parallel with the 4H and other 5H inductors. Applying the equation for inductors in parallel,

$$\frac{1}{L_{\text{eq}}} = \frac{1}{5} + \frac{1}{4} + \frac{1}{5} \tag{11}$$

$$= \frac{13}{20} \tag{12}$$

$$L_{\text{eq}} = \frac{20}{13} \tag{13}$$

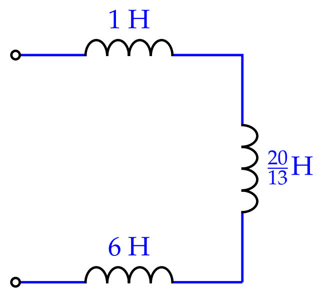


Figure 5

Finally, these three inductors are in series, so we can add their inductances together:

$$L_{\text{eq}} = 1 + \frac{20}{13} + 6 \tag{14}$$
$$= \frac{111}{13}\text{H} \tag{15}$$
$$\approx 8.54\text{H} \tag{16}$$

3. Voltage, Power, and Energy for an Inductance (Hambley Example 3.6)

The current through a 5 H inductance is shown in Figure 6.

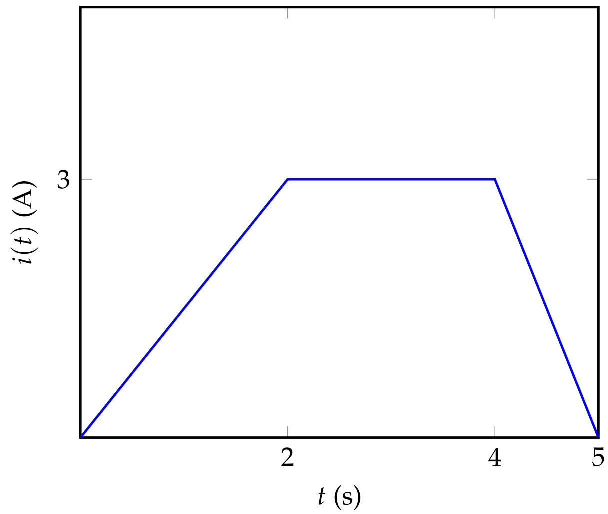


Figure 6: Plot of $i(t)$

Plot the voltage, power and stored energy to scale versus time for t between 0 and 5 s

Solution: We will first compute the voltage using the equation

$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} \tag{17}$$

The time derivative of the current is the slope of the current versus time, which we can read from the provided graph. Between 0 and 2 s, we have $\frac{\mathrm{d}i(t)}{\mathrm{d}t} = 1.5\,\frac{\mathrm{A}}{\mathrm{s}}$ and thus $v = 7.5\,\mathrm{V}$. Between 2 and 4 s, $\frac{\mathrm{d}i(t)}{\mathrm{d}t} = 0\,\frac{\mathrm{A}}{\mathrm{s}}$ and therefore $v = 0\,\mathrm{V}$. In the last interval between 4 and 5 s, $\frac{\mathrm{d}i(t)}{\mathrm{d}t} = -3\,\frac{\mathrm{A}}{\mathrm{s}}$ and $v = -15\,\mathrm{V}$.

This results in the following graph

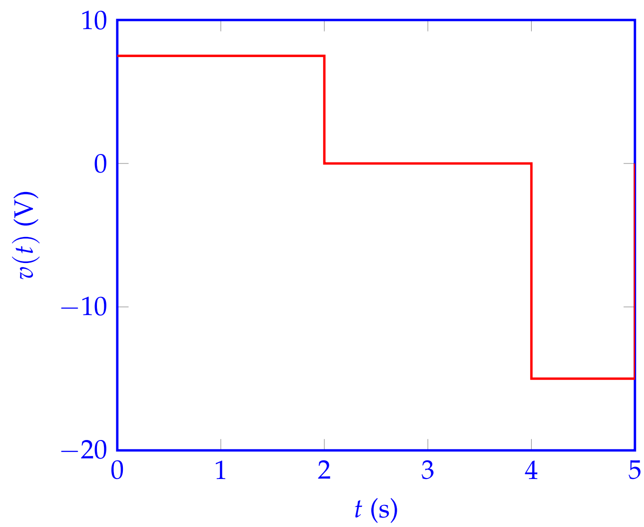


Figure 7: Plot of $v(t)$

We can then solve for the power using the equation:

$$p(t) = v(t)i(t) \tag{18}$$

which would give us the following graph

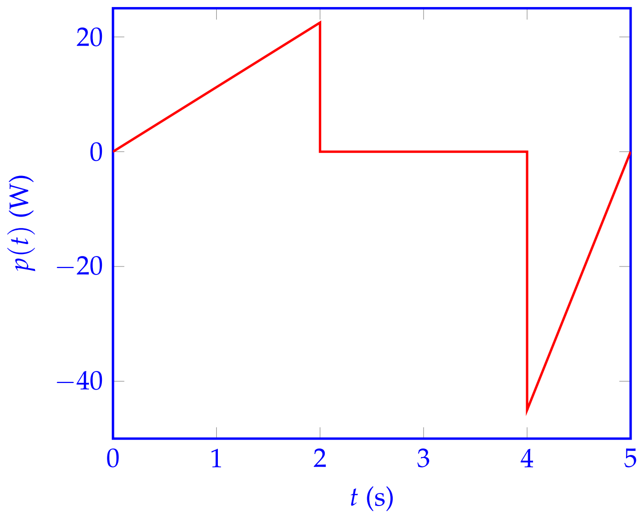


Figure 8: Plot of $p(t)$

Lastly, we need to solve for the stored energy as a function of time which uses the equation

$$w(t) = \frac{1}{2}Li^2(t) \tag{19}$$

This creates the following plot

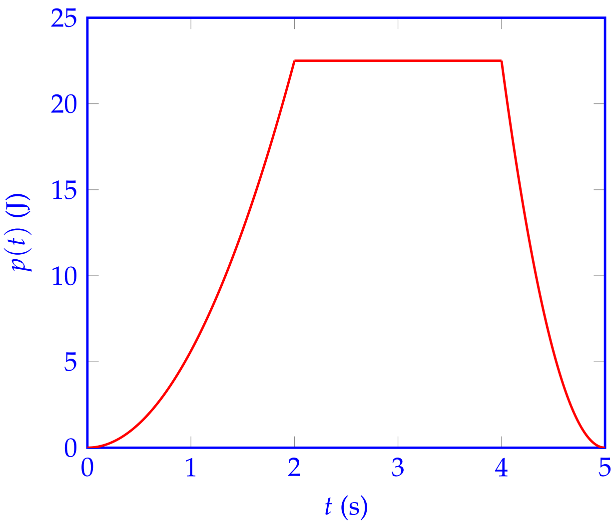


Figure 9

4. Steady-State Analysis (Hambley Example 4.1)

Find v_x and i_x for the circuit shown in Figure 10 for $t >> 0$.

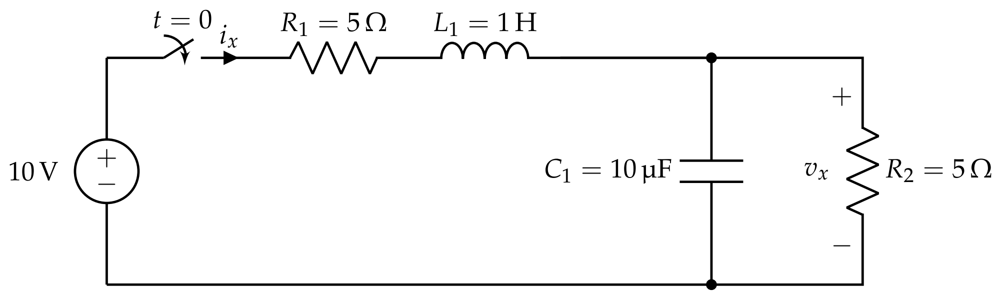


Figure 10

Solution: After the switch has been closed for a long time, we expect the transient response to have decayed to 0. This means that the circuit is operating in dc steady-state conditions.

An inductor in steady-state is equivalent to a wire or a short circuit. This is because current is now constant, meaning its time derivative is 0. Therefore, by

$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} = L \times 0 = 0\,\text{V} \tag{20}$$

The voltage across the inductor is now 0, meaning that we can treat it as a short circuit.

A capacitor in steady-state is equivalent to an open circuit. This is because the voltage is constant and its time derivative is 0. Therefore, by the capacitor element equation:

$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t} = C \times 0 = 0\,\text{A} \tag{21}$$

The current through the capacitor is 0, meaning that is essentially an open circuit.

We can now simplify our circuit to look like the following:

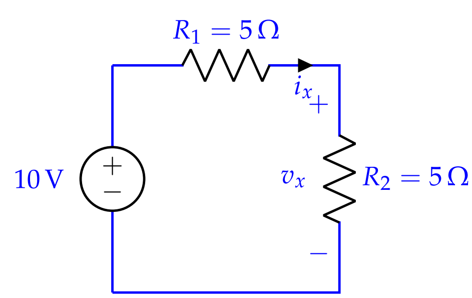


Figure 11: Equivalent Circuit

Recognizing that the resistors are in series, the overall current will be:

$$i_x = \frac{10}{R_1 + R_2} = 1\,\text{A} \tag{22}$$

The voltage v_x would then be:

$$v_x = R_2 i_x = 5\,\text{V} \tag{23}$$

You can also solve for v_x by recognizing that the equivalent circuit is a voltage divider, which would yield:

$$v_x = \frac{R_2}{R_1 + R_2} 10 = 5\,\text{V} \tag{24}$$