EECS 16B Designing Information Devices and Systems II
Spring 2021 Discussion Worksheet **Discussion 1B**

1 KVL/KCL Review

Kirchhoff's Circuit Laws are two important laws used for analyzing circuits. Kirchhoff's Current Law (KCL) says that the sum of all currents entering a node must equal 0. For example, in Figure 1, the sum of all currents entering node 1 is $I_1 - I_2 - I_3 = 0$. Assuming that I_1 and I_3 are known, we can easily obtain a solvable equation for V_x by applying Ohm's law: $I_1 - \frac{V_x}{R_1} - I_3 = 0$.

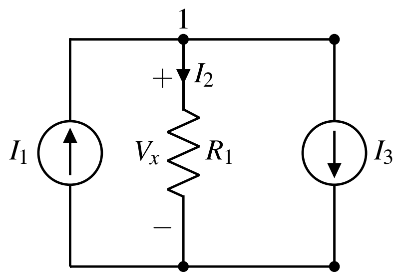


Figure 1: KCL Circuit

Kirchhoff's Voltage Law (KVL) states that the sum of all voltages in a circuit loop must equal 0. To apply KVL to the circuit shown in Figure 2, we can add up voltages in the loop in the counterclockwise direction, which yields $-V_1 + V_x + V_y = 0$. Using the relationships $V_x = i \cdot R_1$ and $i = I_1$, we can solve for all unknowns in this circuit. You can use these two laws to solve any circuit that is planar and linear.

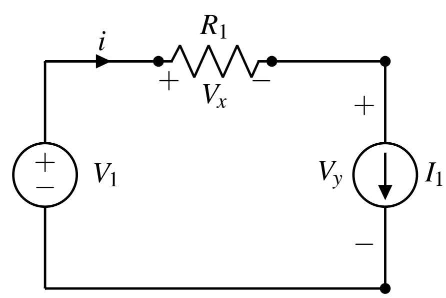


Figure 2: KVL Circuit

If you would like to review these concepts more in-depth, you can check out the EECS16A Fall 2020 course notes.

2 Op-amp Review

Figure 3 shows the equivalent model of an op-amp. It is important to note that this is the general model of an op-amp, so our op-amp golden rules cannot be applied to this unless certain conditions are met.

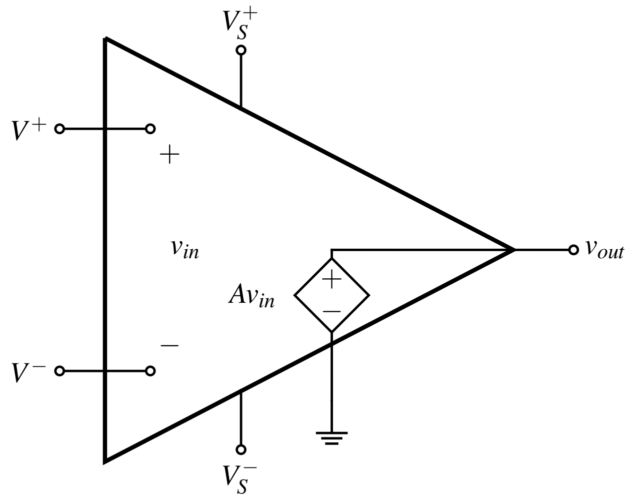


Figure 3: General Op-Amp Model

Conditions Required for the Golden Rules:

(a) $R_{in} \to \infty$

(b) $R_{out} \to 0$

(c) $A \to \infty$

(d) The op-amp must be operated in negative feedback.

When conditions (a)-(c) are met, the op-amp is considered ideal. Figure 4 shows an ideal op-amp in negative feedback, which can be analyzed using the Golden Rules.

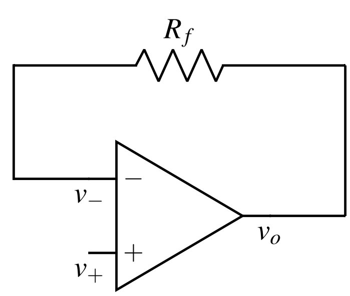


Figure 4: Ideal Op-Amp in Negative Feedback

Golden Rules of ideal op-amps in negative feedback:

(a) No current can flow into the input terminals ($I_- = 0$ and $I_+ = 0$). (This property follows from condition (a) and does not require negative feedback.)

(b) The $(+)$ and $(-)$ terminals are at the same voltage ($V_+ = V_-$).

If you would like to review these concepts more in-depth, you can check out op-amp introduction and op-amp negative feedback from the EECS16A course notes.

1. KVL/KCL Review

Use Kirchhoff's Laws on the circuit below to find V_x in terms of V_{in}, R_1, R_2, R_3.

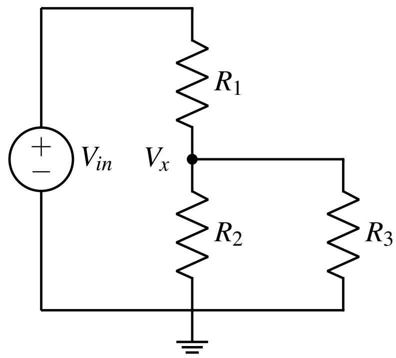


Figure 5: Example Circuit

(a) What is V_x?

(b) As $R_3 \to \infty$, what is V_x? What is the name we used for this type of circuit?

2. Op-Amp Summer

Consider the following circuit (assume the op-amp is ideal):

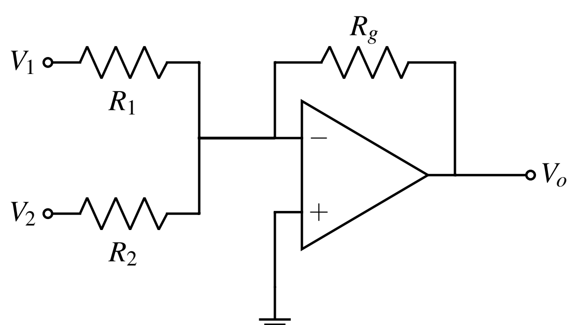


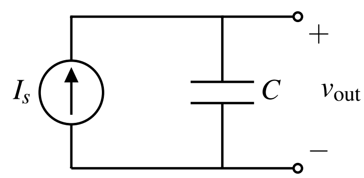
Figure 6: Op-amp Summer

What is the output V_o in terms of V_1 and V_2? You may assume that R_1, R_2, and R_g are known.

3. Current Sources And Capacitors (The following problem has been adapted from EECS16A Fall 20 Disc 9A.)

Recall charge has units of Coulombs (C), and capacitance is measured in Farads (F) $= \frac{\text{Coulomb}}{\text{Volt}}$.
It may also help to note metric prefix examples: $3\,\mu\text{F} = 3 \times 10^{-6}\text{F}$.

Given the circuit below, find an expression for $v_{\text{out}}(t)$ in terms of I_s, C, V_0, and t, where V_0 is the initial voltage across the capacitor at $t = 0$.



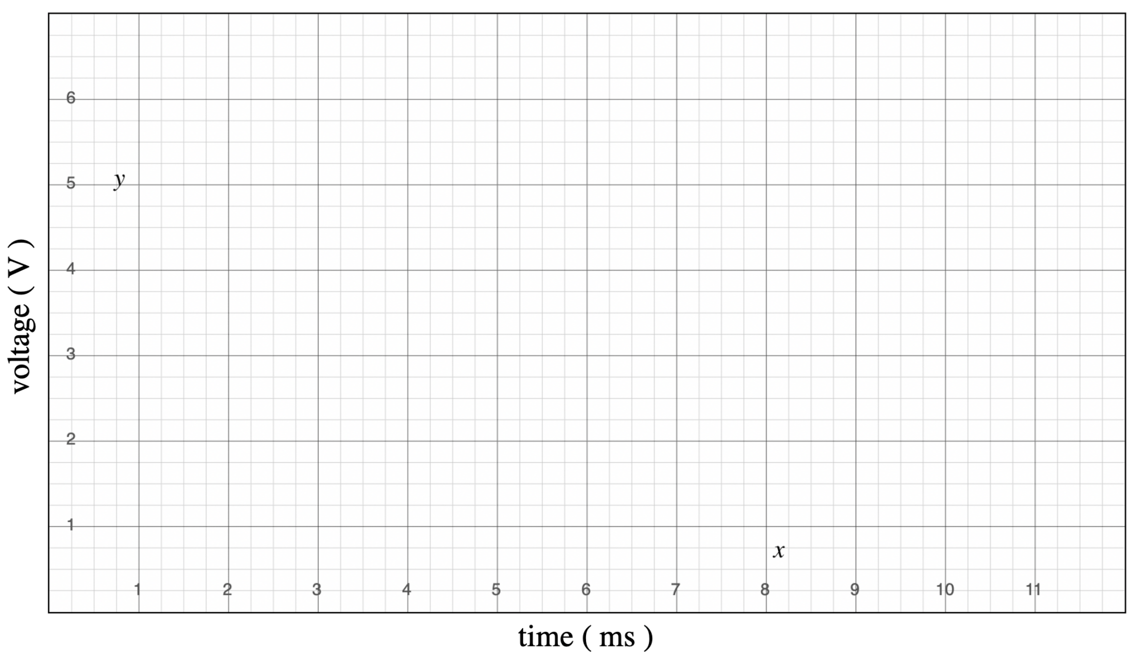
Then plot the function $v_{\text{out}}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\,\mu\text{F}$.

(a) Capacitor is initially uncharged $V_0 = 0$ at $t = 0$.

(b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.

(c) **Practice:** Swap this capacitor for one with half the capacitance $C = 1\,\mu\text{F}$, which is initially uncharged $V_0 = 0$ at $t = 0$.

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.



4. Linear Algebra Review

For the following matrices, find the following properties:

i. What is the column space of the matrix?

ii. What is the null space of the matrix?

iii. What are the eigenvalues and corresponding eigenspaces for the matrix?

(a) $\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$

For this matrix you are told that the eigenvalues are: 2, 1, and 0.