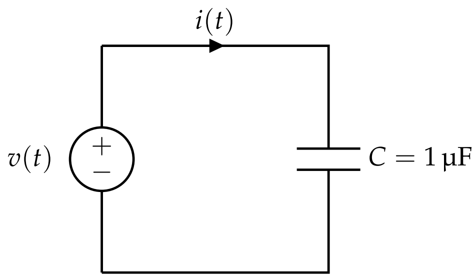
EECS 16B Designing Information Systems and Devices II UC Berkeley Fall 2022

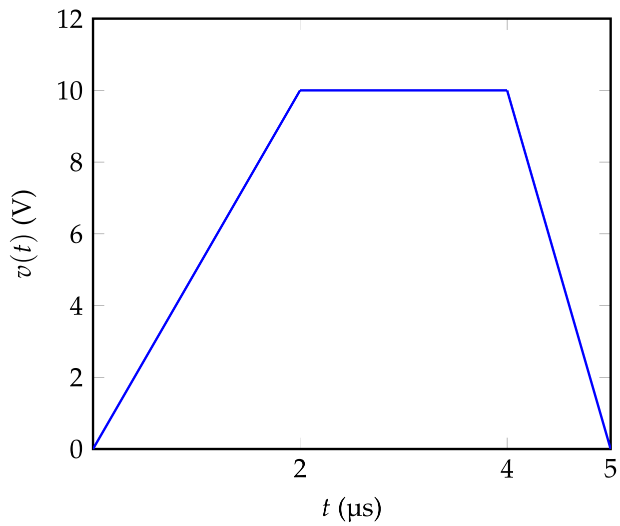
Discussion 1A

1. **Determining Current for a Capacitance Given Voltage (Hambley Example 3.1)**

   Supposed that the voltage $v(t)$ shown in Figure 1b is applied to a $1\,\mu\text{F}$ capacitance.



(a) Example Circuit



(b) Plot of $v(t)$

Figure 1

Plot the stored charge ($q(t)$) and the current ($i(t)$) through the capacitor versus time.

Solution:

Recall that the charge stored on a capacitor is given by the equation

$$q(t) = Cv(t) \tag{1}$$

Since $C = 1\,\mu\text{F} = 1 \times 10^{-6}\,\text{F}$, we have that

$$q(t) = 10^{-6} v(t) \tag{2}$$

Thus the graph for the stored charge would be the following

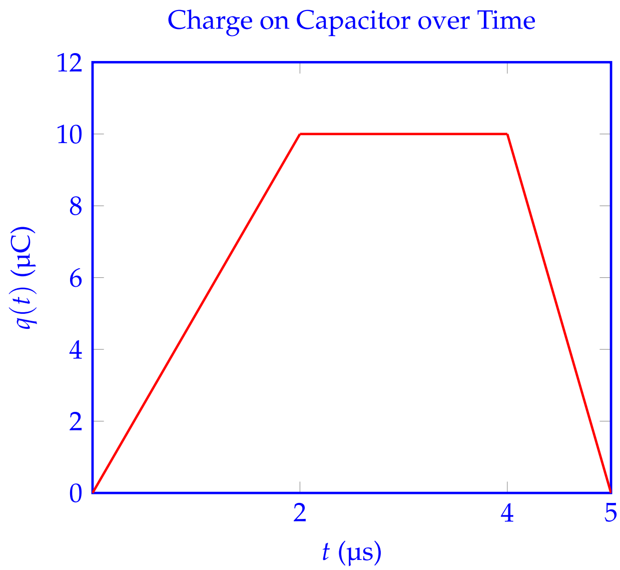


Figure 2: Plot of $q(t)$

Next, we are asked to solve for the current through the capacitor versus time. The relationship between the current through a capacitor and the voltage across the given capacitor is given by the equation:

$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t} \tag{3}$$

Which states that current is proportional to the the derivative of voltage with respect to time. The derivative of the voltage is just the slope of the voltage vs. time graph.

For time t between 0 and 2 μs, we have that the derivative is

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = \frac{10\,\mathrm{V}}{2 \times 10^{-6}\,\mathrm{s}} = 5 \times 10^{6}\,\frac{\mathrm{V}}{\mathrm{s}} \tag{4}$$

Then,

$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t} = 10^{-6} \times 5 \times 10^{6} = 5\,\mathrm{A} \tag{5}$$

For the interval between 2 and 4 μs, the voltage is constant meaning that $\frac{\mathrm{d}v(t)}{\mathrm{d}t}$ and therefore, $i(t) = 0\,\mathrm{A}$.

Repeating the same calculation for the interval between 4 and 5 μs, we have

$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t} = 10^{-6} \times \frac{-10\,\mathrm{V}}{1 \times 10^{-6}\,\mathrm{s}} = 10^{-6} \times -10^{7} = -10\,\mathrm{A} \tag{6}$$

Plotting this, we have,

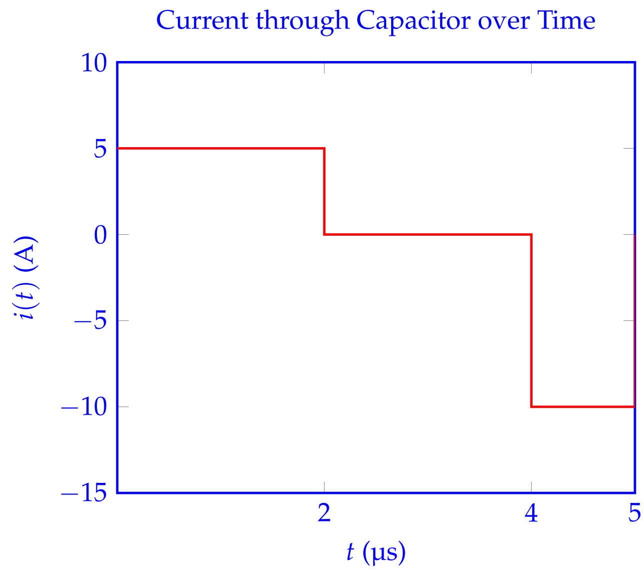


Figure 3: Plot of $i(t)$

2. Determining Voltage for a Capacitance Given Current (Hambley Example 3.2)

After t_0 the current in a 0.1 μF capacitor is given by

$$i(t) = 0.5 \sin 10^4 t \tag{7}$$

(The argument of the sin function is in radians.) The initial charge on the capacitor is $q(0) = 0$.

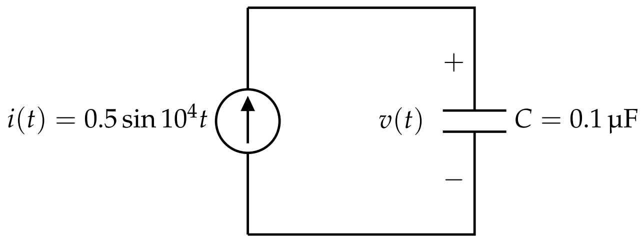


Figure 4: Example Circuit

Plot $i(t)$, $q(t)$, and $v(t)$ to scale versus time.

Solution:

Recall the following equation

$$q(t) = \int_0^t i(t)dt + q(0) \tag{8}$$

We can directly use this equation to solve for the charge on the capacitor.

$$\begin{aligned} q(t) &= \int_0^t i(t)dt + q(0) && (9) \\ &= \int_0^t 0.5 \sin 10^4 t dt && (10) \\ &= -0.5 \times \left[10^{-4} \cos 10^4 t\right]\Big|_0^t && (11) \\ &= 0.5 \times 10^{-4} \left[1 - \cos 10^4 t\right] && (12) \end{aligned}$$

Using the relationship between voltage and charge of a capacitor, we can solve for $v(t)$

$$v(t) = \frac{q(t)}{C} = \frac{q(t)}{10^{-7}} = 500 \left[1 - \cos 10^4 t\right] \tag{13}$$

Here are the plots of $i(t)$, $q(t)$, and $v(t)$

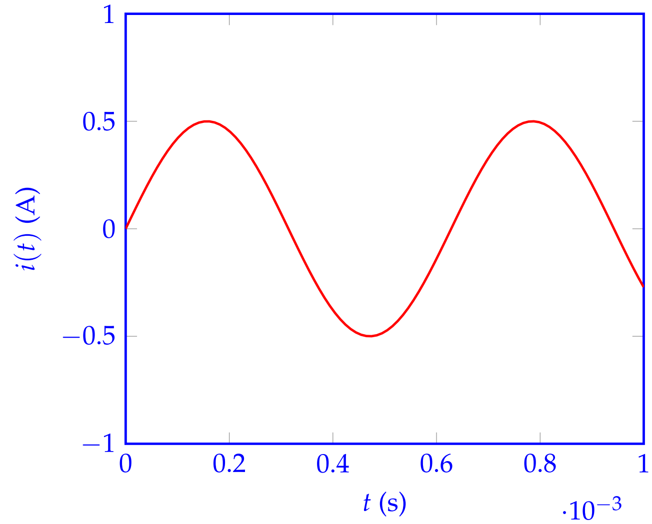


Figure 5: Plot of $i(t)$

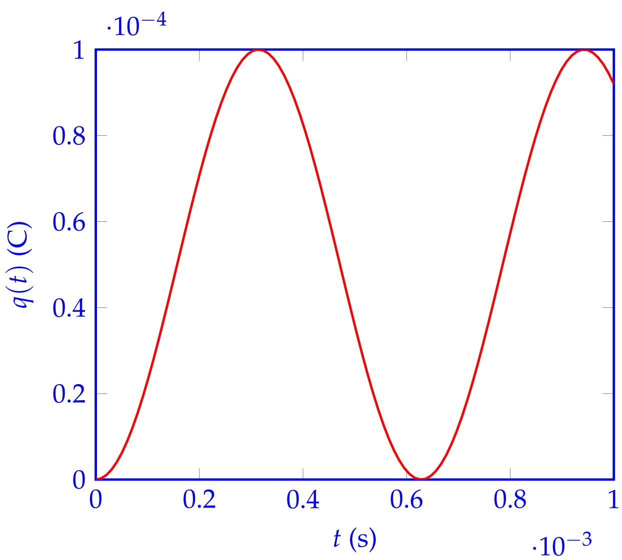


Figure 6: Plot of $q(t)$

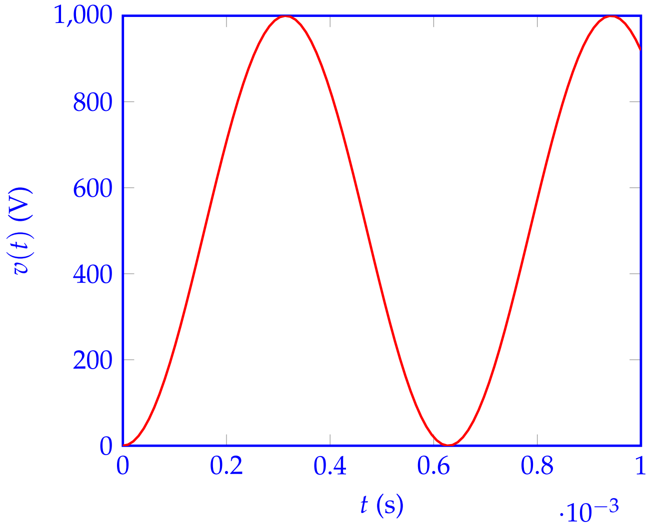


Figure 7: Plot of $v(t)$

3. Current, Power, and Energy for a Capacitance (Hambley Example 3.3)

Suppose that the voltage waveform shown in Figure 8 is applied to a 10-μF capacitance.

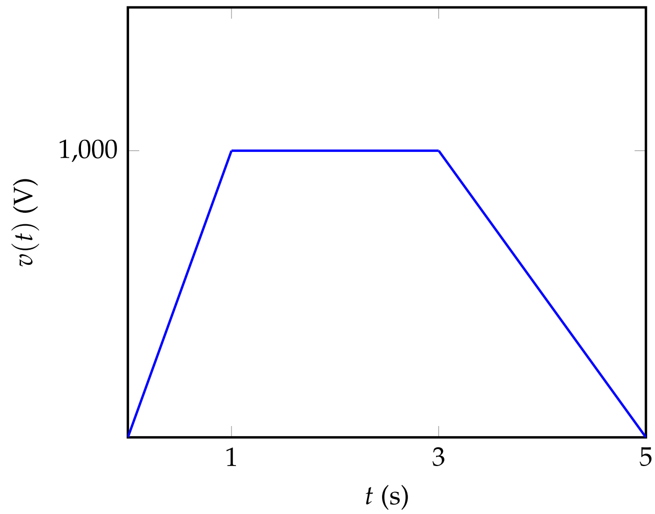


Figure 8: Plot of $v(t)$

Find and plot the current, the power delivered, and the energy stored for time between 0 and 5 s.

Solution:

First, we write expressions for the voltage as a function of time:

$$v(t) = \begin{cases} 1000t\,\mathrm{V} & 0 < t < 1 \\ 1000\,\mathrm{V} & 1 < t < 3 \\ 500(5-t)\,\mathrm{V} & 3 < t < 5 \end{cases}$$

Then, using the equation

$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t} \tag{14}$$

We can obtain expressions for the current

$$i(t) = \begin{cases} 10 \times 10^{-3}\,\mathrm{A} & 0 < t < 1 \\ 0\,\mathrm{A} & 1 < t < 3 \\ -5 \times 10^{-3}\,\mathrm{A} & 3 < t < 5 \end{cases}$$

Using the equation

$$p(t) = v(t)i(t) \tag{15}$$

We can obtain expressions for power

$$p(t) = \begin{cases} 10t\,\mathrm{W} & 0 < t < 1 \\ 0\,\mathrm{W} & 1 < t < 3 \\ 2.5(t-5)\,\mathrm{W} & 3 < t < 5 \end{cases}$$

Lastly, using the equation

$$w(t) = \frac{1}{2}Cv^2(t) \tag{16}$$

$$w(t) = \begin{cases} 5t^2\,\mathrm{J} & 0 < t < 1 \\ 5\,\mathrm{J} & 1 < t < 3 \\ 1.25(5-t)^2\,\mathrm{J} & 3 < t < 5 \end{cases}$$

The plots for $i(t)$, $p(t)$, and $w(t)$ are as follows

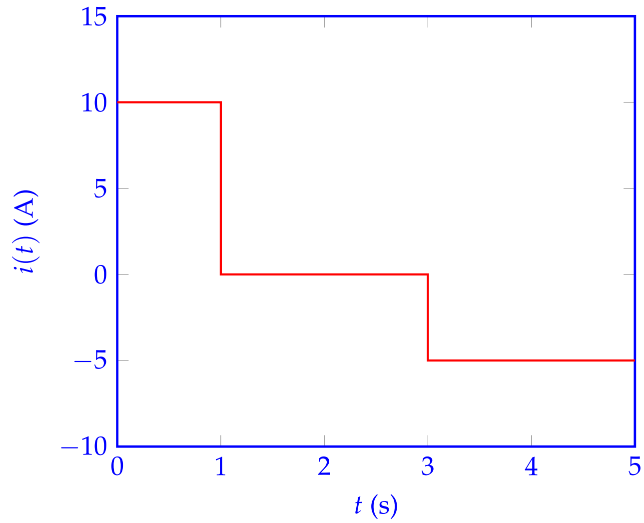


Figure 9: Plot of $i(t)$

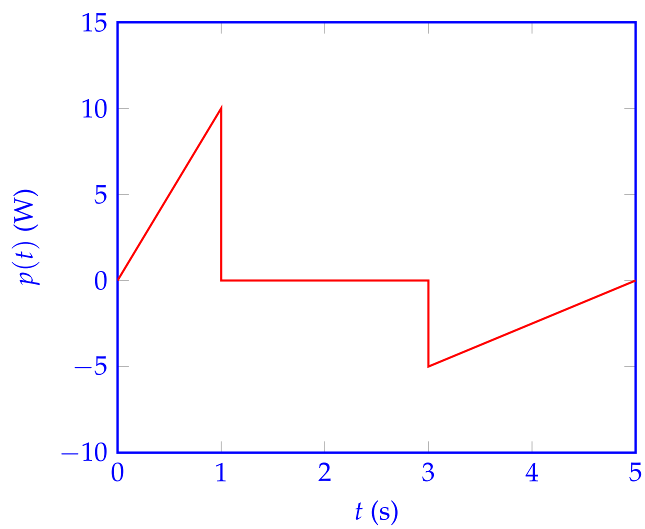


Figure 10: Plot of $p(t)$

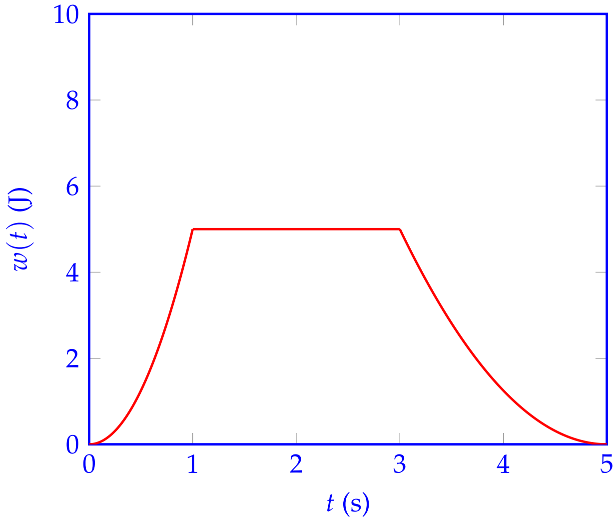


Figure 11: Plot of $w(t)$

