EECS 16B Designing Information Systems and Devices II UC Berkeley Spring 2024

Midterm 2

Exam Location: In Person

PRINT your student ID: ________________________________

PRINT AND SIGN your name: ____________, ____________ ____________
(last) (first) (sign)

PRINT your discussion sections and (u)GSIs (the ones you attend): ________________

Row Number: ____________ Seat Number: ____________

Name and SID of the person to your left: ________________________________

Name and SID of the person to your right: ________________________________

Name and SID of the person in front of you: ________________________________

Name and SID of the person behind you: ________________________________

1. **Honor Code (0 pts.)**

We treat all our students with utmost trust and respect, and expect students to return the same trust and respect. In EECS16B we will have ***zero-tolerance*** for academic dishonesty. There will be ***dire consequences*** for students that violate that trust and the Berkeley code of conduct. Both professors are committed to enforcing academic honesty, and ***dishonesty cases will be punished in their fullest -- no excuses or special circumstances will be considered***. Always seek help, never cheat.

I acknowledge: ________________________ (signature)

2. **What is your favorite topic of the course so far? (0 pts.)**

Do not turn this page until the proctor tells you to do so.
You can work on the above problems before time starts.

3. MCQ (24 pts.)

For all questions, fill in boxes for all answers that apply. There can be more than one correct answer.

(a) (4 pts.) Suppose we have discrete-time LTI model:

$$\vec{x}[i+1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \vec{x}[i] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[i] \tag{1}$$

Which of the following states are *reachable* if our initial state is $\vec{x}[0] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$?

☐ $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$ ☐ $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ ☐ $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ ☐ $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(b) (4 pts.) Suppose we have a continuous-time LTI model $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + \vec{b}u(t)$ where the eigenvalues of A are $-\frac{1}{2} + \frac{j}{2}, -\frac{1}{2} - \frac{j}{2}, -1$. Given this, **which of the following are true?**

- ☐ The system is BIBO stable
- ☐ The system is marginally stable
- ☐ The system is unstable
- ☐ The system *may* exhibit oscillatory state response

(c) (4 pts.) Suppose we have a discrete-time LTI model $\vec{x}[i+1] = A\vec{x}[i] + \vec{b}u[i]$ where the eigenvalues of A are $-\frac{1}{2} + \frac{j}{2}, -\frac{1}{2} - \frac{j}{2}, -1$. Given this, **which of the following are true?**

- ☐ The system is BIBO stable
- ☐ The system is marginally stable
- ☐ The system is unstable
- ☐ The system *may* exhibit oscillatory state response

(d) (4 pts.) The controllability matrix $C \in \mathbf{R}^{n\times n}$ for a discrete-time LTI system $\vec{x}[i+1] = A\vec{x}[i] + \vec{b}u[i]$ (where $A \in \mathbf{R}^{n\times n}$ and $\vec{b} \in \mathbf{R}^{n\times 1}$) is full rank ($\text{rank}(C) = n$). **Which of the following options are always true?**

- ☐ The system is unstable
- ☐ The system is uncontrollable
- ☐ The arbitrary state $\vec{x}^* \in \mathbf{R}^{n\times 1}$ is reachable in $k < n$ steps.
- ☐ The arbitrary state $\vec{x}^* \in \mathbf{R}^{n\times 1}$ is reachable in n or more steps.

(e) (4 pts.) Consider a symmetric matrix $A \in \mathbf{R}^{n\times n}$ with all eigenvalues such that $|\lambda_i| < 1 \; \forall i \in [1, 2, \cdots, n]$. **Which of the following are true?**

- ☐ A discrete-time LTI system with state transition matrix A *may* exhibit oscillatory state response.
- ☐ A discrete-time LTI system with state transition matrix A is *always* BIBO stable.
- ☐ A continuous-time LTI system with state matrix A *may* exhibit oscillatory state response.
- ☐ A continuous-time LTI system with state matrix A is *always* BIBO stable.

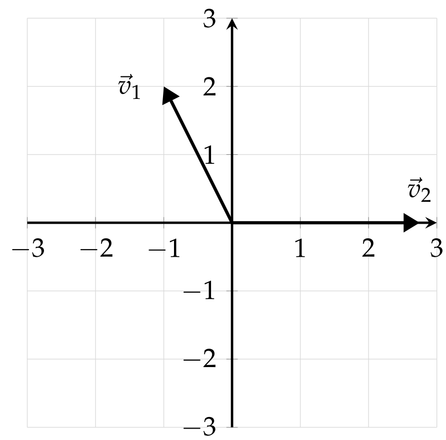
(f) (4 pts.) Suppose you are given a matrix $D \in \mathbb{R}^{5\times 2}$. $D^\top D$ has eigenvalues $\lambda_1 = 5, \lambda_2 = 0$. **Mark all that are true:**

- ☐ $DD^\top$ is not invertible.
- ☐ $D^\top D$ is not invertible.
- ☐ $D^\top D = DD^\top$.
- ☐ The trace of $D^\top D$ (which is the sum of its diagonal elements) is 5.

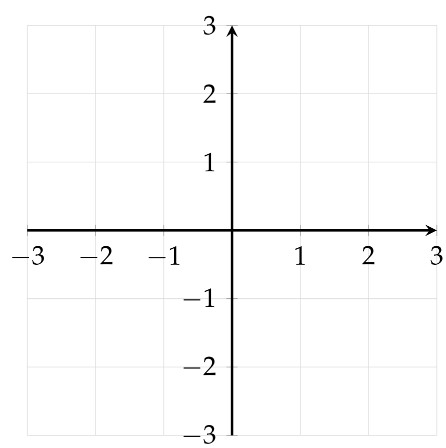
4. Visual Gram-Schmidt (28 pts.)

(a) (12 pts.) **Given two vectors $\vec{q}, \vec{v} \in \mathbf{R}^n$, where $\|\vec{q}\| = 1$, show that $\vec{q}$ and $\vec{v} - \vec{q}^T\vec{v}\vec{q}$ are orthogonal.**

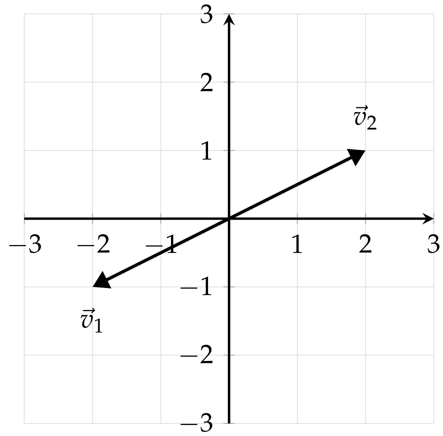
(b) (8 pts.) You perform the Gram-Schmidt procedure on the following vectors $(\vec{v}_1, \vec{v}_2)$ as shown in the plot below:



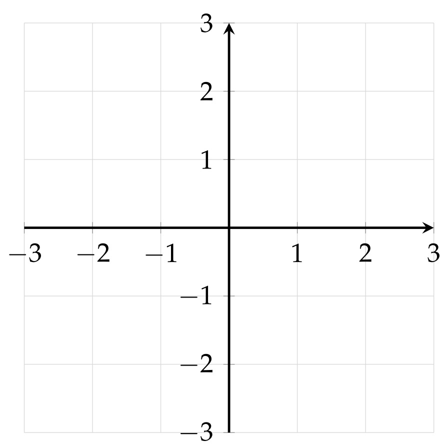
On the provided plot below, **draw and label the orthogonal vectors** $(\vec{q}_1, \vec{q}_2)$ **outputted by the Gram-Schmidt algorithm performed in order of their numerical subscripts (i.e. starting with** v_1**).** If any vectors are $\vec{0}$, draw a dot at $(0, 0)$ and still label it.



(c) (8 pts.) Now, suppose we start with the vectors $(\vec{v}_1, \vec{v}_2)$ as shown in the plot below:

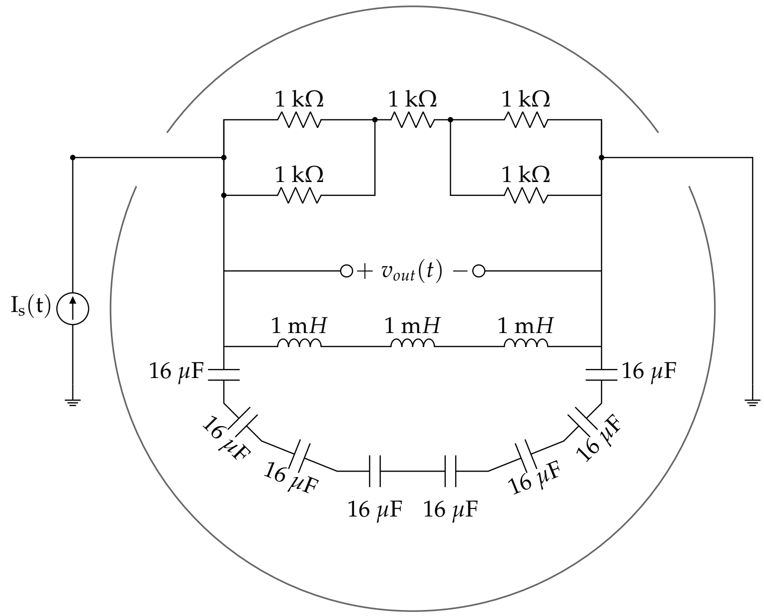


On the provided plot below, **draw and label the vectors $(\vec{q}_1, \vec{q}_2)$ outputted by the Gram-Schmidt algorithm done on the set $(\vec{v}_1, \vec{v}_2)$ in order of their numerical subscripts (i.e. starting Gram-Schmidt with v_1)**. If any vectors are $\vec{0}$, draw a dot at $(0, 0)$ and still label it.

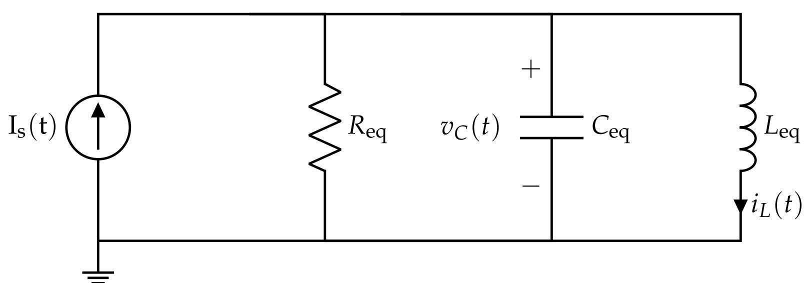


5. The exam is again smiling at you. Be happy! (36 pts.)

Our friend is getting older; we have added a **moustache**. Keep calm and smile on, and recall *Divide et impera* ("divide and conquer").



The above circuit can be simplified to the circuit below:



(a) (6 pts.) **Find the numerical values of R_{eq}, and C_{eq}, L_{eq}, for the equivalent model above.**

$R_{eq} =$	$C_{eq} =$	$L_{eq} =$

(b) (12 pts.) The above circuit solution can be expressed as the following vector differential equation.

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = A\vec{x}(t) + \vec{b}I_S(t) \tag{2}$$

Let $\vec{x}(t) = \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix}$, express A and $\vec{b}$ symbolically in terms of R_{eq}, C_{eq}, and L_{eq}. Show your work, and write your final answer in the box.

$A =$	$b =$

(c) (18 pts.)

Now, consider a new system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = A\vec{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} V_{\text{in}}(t) \tag{3}$$

A has the form $\begin{bmatrix} 2 & a \\ 0 & 0.5 \end{bmatrix}$ where $a \neq 0$, $a \in \mathbf{R}$ with eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

What is a? Is the system BIBO stable? Is the system controllable? Show your work, and write your final answer in the box.

$a =$

BIBO Stable? (Circle One): Yes or No

Controllable? (Circle One): Yes or No

6. Stability and Control (32 pts.)

For all of the following parts, suppose we have a discrete system $\vec{x}[i+1] = A\vec{x}[i] + \vec{b}u[i]$, where $\vec{x}[i] \in \mathbb{R}^2, u[i] \in \mathbb{R}, A \in \mathbb{R}^{2\times 2}$, and $\vec{b} \in \mathbb{R}^2$.

(a) (8 pts.) Suppose that $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. **Is the system BIBO stable? Is the system controllable? Show your work, and write your final answer in the box.**

BIBO Stable? (Circle One): Yes or No Controllable? (Circle One): Yes or No

(b) (8 pts.) Suppose $A = \begin{bmatrix} 0.6 & 0.5 \\ 0 & a \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 0.2 \\ 0.9 \end{bmatrix}$. **Assuming $a \in \mathbf{R}$, for what values of a is the system BIBO stable? Show your work, and write your final answer in the box.**

a is in the range:

(c) (16 pts.) Suppose $A = \begin{bmatrix} 2 & 0 \\ 0 & 0.8 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. We want to stabilize the system with the following feedback controller:

$$u[i] = \begin{bmatrix} f & 0 \end{bmatrix} \vec{x}[i]$$

Assuming $f \in \mathbf{R}$, for what values of f is the system stable? Show your work, and write your final answer in the box.

f is in the range:

7. Symmetric Matrices (24 pts.)

(a) (4 pts.)
Let $A \in \mathbf{R}^{m \times n}$. Show that AA^T is symmetric.

(b) (10 pts.)
Let $A \in \mathbf{R}^{3\times 3}$. A is diagonalizable with eigenvalues j, −j, and 1. **What are the eigenvalues of $A^2 = AA$? Show your work, and write your final answer in the box. If eigenvalues are repeated, list them multiple times.**

$\lambda_1 =$	$\lambda_2 =$	$\lambda_3 =$

(c) (10 pts.)
$A \in \mathbf{R}^{2\times 2}$ is a **symmetric** matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$ and eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} a \\ 1 \end{bmatrix}$.
What is a? Show your work, and write your final answer in the box.

$a =$

[Doodle page! Draw us something if you want or give us suggestions or complaints. You can also use this page to report anything suspicious that you might have noticed.

If needed, you can also use this space to work on problems. But if you want the work on this page to be graded, make sure you tell us on the problem's main page.]

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

